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A HYBRID FINITE TIME VARIABLE STRUCTURE CONTROLLER FOR RIGID ROBOTIC MANIPULATORS

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Abstract: In this paper, a global variable structure relay control scheme with finite time convergence is proposed for multi-link rigid robotic manipulator systems with uncertain dynamics. For general finite time variable structure controllers, the control signal may tend to infinity when the initial states of the system are in some specified areas, causing the singularity problem. This paper gives a design approach for finite time tracking control by using a relay control method so that the boundedness of the control signal is guaranteed and the singularity phenomenon is avoided.

Keywords: Multi-link rigid robotic manipulator, Finite time convergence, Variable structure control.

1.INTRODUCTION

Variable structure control approach has been widely used to deal with uncertain dynamical systems and successfully applied to the rigid robotic manipulator systems (Utkin, 1992; Zinober, 1990;Yeung and Chen, 1988; Slotine and Sastry,1983; Fu and Liao, 1990; Young, 1988). In general, the design procedure of variable structure control is first to design a switching plane in which the error dynamics asymptotically converges to zero under the action of the given control. Since the switching plane usually is a linear one, the tracking error at most converges to zero exponentially. In practice, the finite time tracking of the target is required for the rigid robotic manipulators, i.e. the tracking error reaches zero in given finite time. To get fast tracking error convergence on the sliding mode, a terminal sliding mode control scheme has been proposed in (Wu et al.,1998; Wu et al. 2001; Yu et al.1999; Man and Yu, 1997) by employing a nonlinear switching surface, and a solution of the finite time tracking problem is derived. For multi-link rigid robotic manipulator systems with uncertainties, a robust MIMO terminal sliding mode control scheme has been developed in (Man et al., 1994) based on a nonlinear switching surface and the finite time convergence is reached by suitably designing the controller and the switching plane variables, and the output tracking error can then converge to zero in finite time on the terminal sliding mode. However, the control signal given in (Man et al., 1994) can only be guaranteed bounded on the terminal sliding mode surfaces. In transient process to the nonlinear switching surfaces, the singularity of the closed-loop systems may occur if the initial states of the error systems are in some specified areas, resulting in the unboundedness of the control law.

In this paper, to avoid the singularity existed in the finite time control problem, a relay control approach is proposed for multi-link rigid robotic manipulator systems with uncertainties by using a two-phase

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control scheme in which a slow linear switching plane and a fast nonlinear switching surface are introduced. The use of the slow linear switching surface is to transfer the trajectories of the error dynamics to a specified open region constructed by a series of manipulations as given in (Wu et al., 1998) so that the trajectories in the region continuously move towards the fast nonlinear switching surface without incurring the singularity. Once the trajectory enters the region, the terminal sliding mode control is activated. We will show that the globally asymptotic stability of the closed-loop system is guaranteed, and the output tracking error can reach zero in finite time. The proposed finite time controller is robust in the system uncertainties.

2.PROBLEM FORMULATION

The dynamic equation of an n-joint robotic manipulator system can be described by the following form

$$M(q)\ddot{q}+F(q,\dot{q})\dot{q}+G(q)=u(t)\,, \qquad (2.1)$$

where $q(t)$ is an n-order vector of joint angular positions, $u(t)$ is n-order vector of applied joint torques, $M(q)$ is $n\times n$ symmetric positive definite inertia matrix, $F(q,\dot{q})\dot{q}$ represents the $n\times1$ vector of coriolis and centrifugal torques, and $G(q)$ is the $n\times1$ vector of gravitational torques. Here $u(t),q,\dot{q}$ is measurable respectively. For system (2.1), the following assumptions are made:

A1). There exists a known positive constant $a_1>0$ such that the minimum eigenvalue of $M(q)^{-1}$ satisfies $\lambda_{\min}[M(q)^{-1}]\ge a_1>0$.

A2). An upper bound $a_2>0$ of $\| M(q)^{-1} \|$ is known.

A3). $\|F(q,\dot{q})\dot{q}+G(q)\|<H(q,\dot{q}),\quad H(q,\dot{q})$ is a known continuous positive function.

According to the practical robotic manipulators, the assumptions *A1)-A3)* are reasonable. In fact, much stronger conditions are required in literature Singh (1985), Grimm (1990), Leung et al. (1991). In order to design the variable structure relay controller to make the state $q,\dot{q}$ converge to the ideal reference state, for system (2.1), let $x=[q^T,\dot{q}^T]^T$. Then system (2.1) can be expressed by the following dynamics

$$\dot{x}=\begin{bmatrix}\dot{q}\\ M(q)^{-1}[-F(q,\dot{q})\dot{q}-G(q)]\end{bmatrix}+\begin{bmatrix}0\\ I\end{bmatrix}M(q)^{-1}u(t) \quad (2.2)$$

The reference model is chosen as

$$\begin{bmatrix}\dot{q}_m\\ \ddot{q}_m\end{bmatrix}=\begin{bmatrix}0 & I\\ R & Q\end{bmatrix}\begin{bmatrix}q_m\\ \dot{q}_m\end{bmatrix}+\begin{bmatrix}0\\ B_1\end{bmatrix}r(t)\underline{\underline{\Delta}}Ax_m+B_m r(t) \quad (2.3)$$

where matrices R,Q,B_1 are constant such that system (2.3) is stable. It is equivalent to the following

$$\ddot{q}_m=Rq_m+Q\dot{q}_m+B_1 r(t).$$

Here $r(t),q_m(t),\dot{q}_m(t)$ are measurable vector signals. Define a tracking error as

$$\varepsilon(t)=q(t)-q_m(t)\underline{\underline{\Delta}}e_1(t),\ \dot{\varepsilon}(t)=\dot{q}(t)-\dot{q}_m(t)\underline{\underline{\Delta}}e_2(t),$$
$$e(t)=[\varepsilon(t)^T,\dot{\varepsilon}(t)^T]^T \qquad (2.4)$$

From (2.2) and (2.3), we can get

$$\dot{e}(t)=Ae(t)+B[M(q)^{-1}u(t)+g(q,\dot{q},r)] \qquad (2.5)$$

where $B=[0,I]^T$,

$$g(q,\dot{q},r)=M(q)^{-1}[-F(q,\dot{q})\dot{q}-G(q)]-Rq-Q\dot{q}-B_1 r(t) \qquad (2.6)$$

In (Man et al., 1994), a finite time terminal sliding mode controller is proposed for a nonlinear uncertain system with a structure similar to (2.5). However, in the transient process to the nonlinear switching surface, if accidentally the initial values of the system state are in some specified region, the control signal may be unbounded, causing the system to be singular. In the following, we will give a variable structure relay control scheme to avoid the singular phenomenon. The scheme uses a two phase control: one phase is a pre-terminal sliding mode control that transfers the trajectory to a given open region in which the terminal sliding mode control is not singular. Inside the region, the other phase — the terminal sliding mode control takes place bringing the current state of the trajectory to the origin in finite time.

For the above goal, let us construct an augmented linear system as

$$\dot{z}=Az+Bv \qquad (2.7)$$

where $v(t)$ is to be determined in the sequel, and

$z=[\varsigma(t)^T,\dot{\varsigma}(t)^T]^T$. From the definition of A, B it is known that (A, B) is completely controllable. Define

$$\eta(t)=e(t)-z(t)=[(\varepsilon(t)-\varsigma(t))^T,(\dot{\varepsilon}(t)-\dot{\varsigma}(t))^T]^T \quad (2.8)$$

(2.5) and (2.8) gives rise to

$$\dot{\eta}(t)=A\eta(t)+B[M(q)^{-1}u(t)+g(q,\dot{q},r)-v(t)] \quad (2.9)$$

Define again $v(t)$ as

$$v(t)=B^T Exp(-A^T t)G_c^{-T}(t_0,t_f)\exp(At_f)z(t_f) - \exp(-At_0)z(t_0), \quad (2.10)$$

where $G_c(t_0,t_f)$ is the controllability grammian matrix of linear system (2.7) with the form

$$G_c(t_0,t_f)=\int_{t_0}^{t_f}\exp(-A\tau)bb^T\exp(-A^T\tau)d\tau$$

and $z(t_0), z(t_f)$ are the initial state and final state respectively. The controllability means that the matrix $G_c(t_0,t_f)$ is nonsingular for all t_f. Based on the knowledge of the linear system theory, under the control law $v(t)$ given by (2.10), *z(t)* starting from arbitrary initial state vector $z(t_0)$ is transferred to any preset final state $z(t_f)$ in finite time t_f. Let us consider (2.8) and (2.9). If the controller $u(t)$ is designed to drive $\eta(t)$ to tend to zero, i.e. there exists t_f such that when $t \ge t_f$,

$$|e(t)-z(t)|<\varepsilon_0, \quad |e(t_f)-z(t_f)|<\varepsilon_0$$

where ε_0 is a sufficiently small, then $e(t)$ enters a open small region of $z(t_f)$. In what follows, the first phase control to make the trajectory of (2.5) from any initial state reach a neighborhood of $z(t_f)$ is firstly proposed. For this goal, define a slower switching plane as

$$s_i=d_i(\dot{\varepsilon}_i-\dot{\varsigma}_i)+c_i(\varepsilon_i-\varsigma_i) \quad (2.11)$$

where $\varsigma_i, \varepsilon_i, c_i\ d_i$ are components of $\varsigma(t)$, $\varepsilon(t)$ and positive constants respectively. Define

$$S_1=[s_1,s_2,...,s_n]^T, \quad C=diag(c_1,c_2,...,c_n),$$
$$D=diag(d_1,d_2,...,d_n)$$

then we have

$$\dot{S}_1=[C,D]\eta(t)\underline{\underline{\Delta}}C\eta_1+D\eta_2$$

In order to make $e(t)$ reach zero in finite time along a specified nonlinear surface so that the completely tracking of the output is realized, let us define a fast nonlinear switching surface as

$$s_i^*=d_i^*\dot{\varepsilon}_i+c_i^*\varepsilon_i^{p/q} \quad (2.12)$$

where $\varepsilon_i, c_i^*\ d_i^*$ are components of $\varepsilon(t)$ and positive constants, and p, q are odd positive integers satisfying $p<q$. Define

$$S_2=[s_1^*,s_2^*,...,s_n^*]^T. \quad D^*=diag[d_1^*,d_2^*,...,d_n^*],$$
$$C^*=diag[c_1^*,c_2^*,...,c_n^*].$$

We now outline the main design steps of the variable structure relay controller.

1). According to (2.9) and (2.11), design a variable structure control such that the trajectory of (2.5) enters a neighborhood of $z(t_f)$ in a given finite time , i.e. $\eta(t)$ tends to a small neighborhood of the origin.

2). Design the finite time variable structure control such that the trajectory of the system (2.5) starting from a small neighborhood of $z(t_f)$ firstly reaches nonlinear switching surface $s_i=0$, then moves to zero in finite time along this surface.

Since any trajectory of (2.5) moves under the first variable structure control then continuously goes to zero in finite time with the other control law, this control scheme is called the variable structure relay control.

3. CONTROLLER DESIGN

Let us design the first phase controller for system (2.9) by using the linear switching function (2.11), which is given by

$$u_i(t)=-\frac{\beta s_i}{\|S_1\|}\rho(t), \ u(t)=[u_1,u_2,...,u_n]^T \quad (3.1)$$

$$\rho(t)=\delta_1\|\eta(t)\|+\delta_2[H(q,\dot{q})+\|DRq\| + \|DQ\dot{q}\|+\|DB_1 r(t)\|+\|Dv(t)\|+1] \quad (3.2)$$

where $\beta, \delta_1, \delta_2$ are positive numbers to be determined. In what follows let us consider the closed-loop (2.9) and (3.1). Choose a candidate Lyapunov function as

$$\dot{V}_1=\frac{1}{2}S_1^T S_1 \quad (3.3)$$

The time derivative of V_1 along (2.9) and (3.1) is as

$$\dot{V}_1 = S_1^T \dot{S}_1 = S_1^T [C(\dot{\varepsilon} - \dot{\varsigma}) + D(\ddot{\varepsilon} - \ddot{\varsigma})] = S_1^T [C, D]\dot{\eta}$$
$$= S_1^T [C, D][A\eta(t) + B(M(q)^{-1} u(t)$$
$$+ g(q, \dot{q}, u, r) - v(t))]$$
$$= S_1^T [(DR, C + DQ)\eta(t) + DM(q)^{-1} u(t)$$
$$+ Dg(q, \dot{q}, t) - Dv(t)]$$
$$\leq \delta_1 \|S_1^T\| \|\eta(t)\| + S_1^T DM(q)^{-1} u(t)$$
$$+ \|S_1^T\| \|DM(q)^{-1}\| H(q, \dot{q}) + \|S_1\| [\|DRq\|$$
$$+ \|DQ\dot{q}\| + \|DB_1 r(t)\| + \|Dv(t)\|]. \quad (3.4)$$

where $\delta_1 \geq \|(DR, C + DQ)\|$ are chosen positive constants. For simplicity, let us select $D = dI_{n\times n}, d > 0.$ According to the control law (3.1) it can be obtained that

$$S_1^T DM(q)^{-1} u(t) = -S_1^T dM(q)^{-1} \beta S_1 \frac{1}{\|S_1\|} \rho(t)$$
$$< -d\beta a_1 \|S_1\| \rho(t)$$

$$= -d\beta a_1 \|S_1\| [\delta_1 \|\eta(t)\|$$
$$+ \delta_2 (H(q, \dot{q}) + \|Rq\|$$
$$+ \|Q\dot{q}\| + \|B_1 r(t)\| + \|v(t)\| + 1)],$$

$$\|S_1^T\| \|DM(q)^{-1}\| H(q, \dot{q}) + \|S_1\| [\|DRq\|$$
$$+ \|DQ\dot{q}\| + \|DB_1 r(t)\| + \|Dv(t)\|] \quad (3.5)$$
$$\leq \delta_2 \|S_1\| [H(q, \dot{q}) + \|Rq\| + \|Q\dot{q}\| + \|B_1 r(t)\| + \|v(t)\|$$

where $\delta_2 > d \max[a_2, 1]$. If β is chosen to satisfy $d\beta\alpha_1 > 1$, substituting (3.5) into (3.4), we have

$$\dot{V}_1 < -d\beta a_1 \|S_1\| \quad (3.6)$$

The Lyapunov function (3.3) together with inequality (3.6) guarantees that $S_1(t)$ tends to zero in finite time and the trajectories of the systems (2.5), (2.7) and (2.9) are restricted to move along $S_1 = 0$. Therefore, on $S_1 = 0$, from (2.11) we get

$$\eta_2 = \dot{\varepsilon}(t) - \dot{\varsigma}(t) = -D^{-1} C(\varepsilon(t) - \varsigma(t)) = -D^{-1} C\eta_1 \quad (3.7)$$

By the definition of the matrix D, C, (3.7) implies that $\varepsilon(t) - \varsigma(t)$ tends to zero exponentially. Meanwhile (3.7) shows that $\eta_1(t)$ as well as $\eta_2(t)$ tends to zero exponentially under the condition $S_1 = 0$. This guarantees by (2.8) that $e(t) - z(t)$ enters a sufficiently small neighborhood of zero, i.e. for any $\varepsilon_0 > 0$, there exist $t_f > 0$ such that $t > t_f$, $\|e(t) - z(t)\| < \varepsilon_0$. For any other solution $\bar{e}(t)$ of (2.5) with different initial state from $e(t)$, it can be obtained that $\|\bar{e}(t) - z(t)\|$ tends to zero exponentially. Then for $\varepsilon_0 > 0$, there exists $\bar{t} > 0$, when $t > \bar{t}$, $\|\bar{e}(t) - z(t)\| < \varepsilon_0$. Without lose of generality, let $t_f > \bar{t}$, then $\|\bar{e}(t_f) - z(t_f)\| < \varepsilon_0$. From (Wu et al., 1998, Lemmas 3,4,5), it has been proved that one can always construct an open region Ω such that the control law $u(t)$ and the trajectory maintains bounded when the trajectory continuously moves outside this region and reaches the fast nonlinear switching surface $S_2 = 0$. Because for the system (2.7), the first terminal $z(t_f)$ can be selected arbitrarily, without loss of generality, let $z(t_f)$ belongs to Ω. As a special case, one can let $z(t_f) \in \{S_2 = 0\} \cap \Omega$.. Since Ω is an open set and $\|e(t_f) - z(t_f)\|$ can be make sufficiently small, $e(t_f)$ also belongs to Ω.

For any solution $e(t)$ of (2.5), in the first phase control with the controller (3.1), after finite time T_1, $S_1 = 0$ is reached. By finite time T_2, $e(t)\,(t \geq T_1 + T_2)$ enters Ω. Once $e(t) \in \Omega$, the second phase controller is taken, i.e. the control law is switched to the terminal sliding mode controller with the form

$$u_i(t) = -\frac{\beta^* s_i^*}{\|S_2\|} \rho^*(t) \quad (3.8)$$

where β^* is a positive number and $\rho^*(t)$ is as

$$\rho^*(t) = \left\| C^* \frac{p}{q} diag(\varepsilon_i^{\,p/q-1}) \dot{\varepsilon} \right\|$$
$$+ \|D^* diag(0, I)[Ae(t) - Rq - Q\dot{q} - B_1 r\| \quad (3.9)$$
$$+ \|D^* diag(0, I) M(q)^{-1}\| H(q, \dot{q}) + 1$$

Let us analyze the closed-loop (2.5) and (3.8). The Lyapunov function V_2 is taken as

$$V_2 = \frac{1}{2} S_2^T S_2 \quad (3.10)$$

then the time derivative of (3.10) along (2.5) and (3.8) is as

$$\dot{V}_2 = S_2^T \dot{S}_2 = S_2^T \left[C^* \frac{p}{q} diag(\varepsilon_i^{\,p/q-1})\dot{\varepsilon} + D^* \ddot{\varepsilon} \right]$$
$$= S_2^T \left[C^* \frac{p}{q} diag(\varepsilon_i^{\,p/q-1})\dot{\varepsilon} + D^* diag(0, I)\dot{e} \right]$$
$$= S_2^T \left[C^* \frac{p}{q} diag(\varepsilon_i^{\,p/q-1})\dot{\varepsilon} \right] + S_2^T D^* diag(0, I)[Ae(t)$$
$$+ B(M(q)^{-1} u(t) + g(q, \dot{q}, r)]$$
$$\leq \|S_2\| \left\| C^* \frac{p}{q} diag(\varepsilon_i^{\,p/q-1})\dot{\varepsilon} \right\|$$
$$+ \|S_2\| \|D^* diag(0, I)[Ae(t) - Rq - Q\dot{q} - B_1 r\|$$
$$+ \|S_2\| \|D^* diag(0, I) M(q)^{-1}\| H(q, \dot{q}) - \beta^* d^* \|S_2^T\| \rho^*(t)$$
$$\leq -\beta^* d^* \|S_2^T\| \qquad (3.11)$$

here without loss of generality choosing, $d_i^* = d^*$, $i = 1,2,..., n$ (3.10) and (3.11) implies that $S_2(t)$ tends to zero in finite time, i.e., $e(t)$ reaches the fast nonlinear switching surface $S_2 = 0$ in finite time. According to (2.12), on the surface $S_2 = 0$, it is obtained that

$$\dot{\varepsilon}(t) = -(D^*)^{-1} C^* \varepsilon^{p/q}(t) \qquad (3.12)$$

where $\varepsilon^{p/q}(t) = [\varepsilon_1^{p/q}, \varepsilon_2^{p/q}, ..., \varepsilon_n^{p/q}]^T$. By means of the selection of the matrices D^*, C^*, since $p < q$, the solution $\varepsilon(t)$ of (3.12) attained zero in finite time and continues to maintain zero forever.

Summarizing the above analysis, we know that the trajectory of the system (2.5) experiences four moving paths from the starting point to the origin:

1). Under the first phase variable structure control law (3.1), the dynamics of (2.9) is driven to the slower switching surface $S_1 = 0$ in finite time T_1.

2). On $S_1 = 0$, $e(t) - z(t)$ tends to zero, i.e., $e(t)$ enters a sufficiently small neighborhood Ω_0 of $z(t_f)$ being included inside Ω in finite time T_2.

3). Once $e(t)$ enters Ω_0, the control law is switched to variable structure controller (3.8) so that $e(t)$ tends to $S_2 = 0$ in finite time T_3.

4). Under the action of the controller (3.8), $e(t)$ continuously moves along $S_2 = 0$ till converging to $e = 0$ in finite time T_4. Therefore, the finite time output tracking is realized.

Since $\varepsilon(t)$ as well as $\dot{\varepsilon}(t)$ goes to zero along $S_2 = 0$ in finite time, $e(t)$ by the definition of (2.4) also tends to zero simultaneously in finite time. These guarantee that $q(t), \dot{q}(t)$ are bounded so that all signals in closed loop of the systems are bounded. We summarize the above analysis in the following theorem.

Theorem: For system (2.1) and the reference model (2.3), if the augmented linear system (2.7) is introduced, the first phase control law (3.1) and the variable structure controller (3.8) are applied, then the closed loop system is asymptotically stable, and the tracking error $e(t)$ reaches zero in finite time.

Remarks: For the definition of the specified region Ω, one can refer to the method given in Wu et al. (1998). For simplicity, we may take $z(t_f)$ on the surface of $S_2 = 0$ such that $e(t_f)$ close to $S_2 = 0$. Therefore, when $e(t)$ continues to move towards $S_2 = 0$ under the second control law (3.8), the $u(t)$ cannot goes to infinite so that the singularity does not occur.

The total time from the initial instant to the final time when $e(t) = 0$ is

$$T = T_1 + T_2 + T_3 + T_4 \qquad (3.13)$$

Now let us consider how to compute the finite time T. Firstly, according to (3.3) and (3.6), one can calculate T_1 which guarantees $S_1 = 0$. Secondly, since dynamics (3.7) is stable exponentially, T_2 can be estimated to satisfy step (2). As to T_3, it is determined by (3.10) and (3.11). Finally, T_4 can be obtained by solving the equation (3.12). Therefore, an upper bound of the finite time guaranteeing the completely tracking has been estimated.

4.CONCLUSION

In this paper, a design scheme of a variable structure relay controller guaranteeing the system global stability and finite time convergence has been proposed for the n-link rigid robotic manipulator systems with unknown parameters and uncertain dynamics. By introducing a two-phase control including a slow switching plane and a fast nonlinear switching surface and the relay method, singularity phenomenon usually associated with the finite time convergence is avoided in the sense that the control signal maintains bounded. The controller with finite time convergence is a kind of fast control with short transient time and can guarantee complete tracking. This is conformable to practical control objective of output tracking of rigid robotic manipulators, for instance, if it is asked that the robotic manipulator tracks a motion object in finite time, the performance with finite time convergence is obviously superior to the one with asymptotically stable.

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