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THEORY AND IMPLEMENTATION OF A FUZZY CONTROL SCHEME FOR PENDUBOT

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Abstract: To deal with the swinging up and balancing control for an under-actuated robot, Pendubot, a new fuzzy logical control method is proposed in this paper. Two separate fuzzy reasoning control laws are employed for the tasks of swinging up and balance control. The proposed control scheme is tested by both simulations and hardware experiments. Hardware experimental results show the applicability of the proposed scheme. *Copyright © 2002 IFAC*

Keywords: Fuzzy logical control, reasoning, under-actuated robot, implementation.

1. INTRODUCTION

Interest in studying the under-actuated mechanical systems is motivated by their role as a class of strongly nonlinear systems where complex internal dynamics, nonholonomic behavior, and lack of feedback linearizability are often exhibited, for which traditional nonlinear control methods are insufficient and new approaches must be developed. Frequently cited applications include saving weight and energy by using fewer actuators and gaining fault tolerance to actuator failure. The challenge of solving control problems associated with this class of systems will also stimulate new results in robot control theory in the years to come. The Pendubot is a benchmark system for under-actuated robot manipulators, consisting of a double pendulum with an actuator at only the first joint. Using the Pendubot, one can mainly investigate the set-point regulation, including swinging up and balancing.

The problem of swinging the Pendubot from the open loop stable configuration $q_1 = -\pi/2, \quad q_2 = 0$ to any of the inverted equilibria and then balancing it is an interesting topic because of the strong nonlinearity and dynamic coupling between the links. Earlier work that deals with control of swinging up and balancing the Pendubot is described by Spong and Block (1995), who applied partial feedback linearization approach to swing up the two links and applied state feedback controller to balance the Pendubot at the unstable upright position. Since then, various control schemes have been developed, for example, an energy-based method (Yoshida, 1999), the robust adaptive control algorithm (Lee, 1996) or hybrid robust control algorithm (Fierro, 1999).

Although some interesting techniques and results have been presented in the above mentioned publications, the control of under-actuated systems still remains an open problem. For instance, most of the control schemes mentioned were mainly developed for two-link arms. For the general case, they either failed to provide a thorough analysis of the overall system stability or assumed that gravitation forces do not act on the passive joints. Furthermore, the precise knowledge of dynamic model is generally required. In addition, some schemes may not be implementable in real systems. Therefore, comprehensive theory for control of under-actuated systems remains unavailable.

Fuzzy logic control techniques, as an alternative approach, were originally advocated by Zadeh (1979), as a means of both collecting human knowledge and experience and dealing with uncertainties in the control process. It has become a very popular tool in control engineering. Therefore, aiming at the above mentioned control problems for the under-actuated mechanical nonlinear systems, Sanchez and Vidolov

(1998) designed a MIMO fuzzy PD controller to swing up and balance an under-actuated robot. Yi (2000) and Marcio (2000) proposed control strategies based on genetic algorithm and Takagi-Sugeno fuzzy methodology to control single or double pendulum robot. We should mention that only simulation results are used to validate these control schemes and there are no experimental results, which should be delivered if the fuzzy control schemes are employed.

Focusing on the Pendubot, a new fuzzy logic control method is proposed in this paper. Two separate fuzzy reasoning control laws are employed for the tasks of swinging up and balance control. The main contributions are two folds: 1) a fuzzy control scheme is derived to swing up and balance the Pendubot at the top position; 2) this scheme is implemented in the real system.

This paper is organized as the following: Section 2 gives the description of the Pendubot; Section 3 derives the control scheme for swinging up and balancing; Section 4 describes the simulations and experiments; Section 5 concludes the results.

2. DESCRIPTION OF PENDUBOT

The Pendubot, short for **Pendu**lum Ro**bot**, is an electro-mechanical (or Mechatronic) system, consisting of two rigid links interconnected by revolute joints as illustrated in Fig. 2.1. The first joint is actuated by a DC-motor and the second joint is unactuated. Thus, this is a strong nonlinear system.

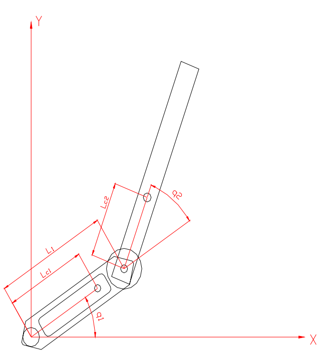


Fig. 2.1 Coordinate description of the Pendubot.

The equations of motion for the Pendubot can be written in matrix:

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \begin{bmatrix} \tau \\ 0 \end{bmatrix} \qquad (2.1)$$

where, $D(q)$ is symmetric and positive define, and

$$D(q) = \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix}$$

$$d_{11} = m_1 l_{c1}^2 + m_2(l_1^2 + l_{c2}^2 + 2l_1 l_{c2}\cos q_2) + I_1 + I_2$$

$$d_{12} = d_{21} = m_2(l_{c2}^2 + l_1 l_{c2}\cos q_2) + I_2$$

$$d_{22} = m_2 l_{c2}^2 + I_2$$

$C(q,\dot{q})$ has been chosen such that $\dot{D} - 2C$ is skew symmetric.

$$C(q,\dot{q}) = \begin{bmatrix} h\dot{q}_2 & h\dot{q}_2 + h\dot{q}_1 \\ -h\dot{q}_1 & 0 \end{bmatrix}$$

$$h = -m_2 l_1 l_{c2} \sin q_2$$

$$g(q) = \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix}$$

$$\phi_1 = (m_1 l_{c1} + m_2 l_1) g \cos q_1 + m_2 l_{c2} g \cos(q_1 + q_2)$$

$$\phi_2 = m_2 g l_{c2} \cos(q_1 + q_2)$$

where

m_1 and m_2: the total mass of both links;

l_1 and l_2 : the length of both links;

l_{c1} and l_{c2}: the distance to the center of mass of both links;

I_1 and I_2: the moment of inertia of both links about their centroid;

g : the acceleration of gravity.

Note *0* in the vector on the right side of equation (2.1) indicates the absence of an actuator at the second joint, τ is the vector of torque applied to the links and q is the vector of joint angle positions.

3. FUZZY CONTROL SCHEME

3.1 Introduction of Fuzzy Control

As introduced by L. Zadeh in its seminal papers, fuzzy logic and approximate reasoning provide a framework for formalizing intuition and common sense knowledge. In the field of process control, these basic tools have been used to formalize experienced operator knowledge into fuzzy rule bases (FRB) giving rise to fuzzy logic controllers via the fuzzy machinery or the fuzzy associative memory model. The FRB proposed in the literature to describe a multiple-input-multiple-output (MIMO) plant controller are usually stated as a collection of If-Then rules such as:

IF x_1 is F_1^j, …, and x_n is F_n^j,
Then ν_1 is G_1^j, …, and ν_q is G_q^j

with j=1,2, …, M, the x_i and ν_k denote the linguistic input and output variables respectively, while F_i^j and G_k^j stand for their linguistic qualifications which are associated with fuzzy subsets. In such a knowledge representation, each if-part of a rule can be viewed as a description of a particular process state to which a collection of linguistically specified control actions is associated. Therefore, the number of rules depends directly upon the number of couples {process state, control actions} an experienced operator can enumerate.

3.2 Swinging Up Control Scheme

The control problem we consider is described in the introduction section and illustrated in Fig. 3.1.

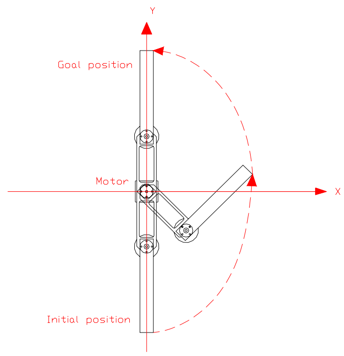


Fig. 3.1 Swing-up and balancing control problem

We define the joint states as the crisp inputs and the torque as the crisp output of fuzzy control system. From the point view of fuzzy logic control, the first step is fuzzification, which is a mapping from the observed input to the fuzzy sets defined in corresponding universes of discourse.

SS represents the shoulder state that is made up with SE and RSE, that is

SS={SE, RSE}.

The angular shoulder error (SE) is defined as: $se = \pi/2 - q_1$. The rate of change of SE (RSE), at time t, is defined as

$$
\begin{aligned}
rse &= \frac{se(t) - se(t-h)}{h} \\
&= \frac{[\pi/2 - q_1(t)] - [\pi/2 - q_1(t-h)]}{h} \\
&= -\frac{q_1(t) - q_1(t-h)}{h} = -\dot{q}_1
\end{aligned}
$$

where h is the sampling period (in our experiment, the sampling period used to approximate the rate of changes of the output variables is set to 0.005s).

ES represents the elbow state that is made up with EE and REE, that is

ES={EE, REE}.

The angular elbow error (EE) is defined as: $ee = \pi/2 - (q_1 + q_2)$. The rate of change of EE (REE), at time t, is defined as

$$ree = \frac{ee(t) - ee(t-h)}{h} = -\dot{q}_1 - \dot{q}_2$$

Not only for the shoulder state (SS) but also for the elbow state (ES), they are separately made up with two atomic variables the angular error (se or ee) and its rate of change (rse or ree). Each of these atomic variables is associated with two fuzzy sets that are labeled ***positive*** and ***negative***.

Let Σ denotes one of these atomic variables, then, for any $x \in R$, its related fuzzy sets are given by:

$$\Sigma P(x) + \Sigma N(x) = 1 \qquad (3.1)$$

$$\Sigma P(x) = \frac{1 + \tanh(kx)}{2} \qquad (3.2)$$

This is our fuzzification. Now we consider the joint shoulder as an example to derive the fuzzy control law. Any real value of se (resp. rse) is given a non zero grade of membership to both input fuzzy sets. A control strategy for the highest priority goal is stated as:

Rule1: **IF** "SS is SSP" **THEN** force τ is Positive
Rule 2: **IF** "SS is SSN" **THEN** force τ is Negative

The fuzzy sets SSP and SSN of SS are defined using a quasi-linear-mean aggregating operator proposed by Fodor, that is the membership grade of any pair {se, rse} to SSP and SSN is given by:

$$SSP(se, rse) = \mu_s SEP(se) + (1 - \mu_s) RSEP(rse) \qquad (3.3)$$

$$SSN(se, rse) = \mu_s SEN(se) + (1 - \mu_s) RSEN(rse) \qquad (3.4)$$

where the weighting parameter $\mu_s \in [0,1]$ is drawn from the unit interval. It is used to give more or less importance to one of the two variables. These two rules, having preconditions that cover any range of sensor readings, are given the same meaning: the more true the precondition the more positive (resp. negative) the conclusion. Therefore, we use the following output fuzzy sets as the system output:

$$FP(\zeta) = \begin{cases} \zeta & if \zeta \in [0,1] \\ 0 & if \zeta \in [-1,0[\end{cases} \quad FN(\zeta) = \begin{cases} |\zeta| & if \zeta \in [-1,0] \\ 0 & if \zeta \in]0,1] \end{cases}$$

where ζ is related to the applied force, τ, by a scaling factor G_1 such that $\tau = G_1 \zeta$, and a slight modification of the reasoning method originally proposed by Tsukamoto, the defuzzified output is simply given by:

$$S(se, rse) = G_1 \zeta = G_1 [SSP(se, rse) - SSN(se, rse)] \qquad (3.5)$$

Using equations (3.1) - (3.4), we can get:

$$
\begin{aligned}
& S(se, rse) = S(se_1, se_2) \\
& = SSP(se, rse) - SSN(se, rse) \\
& = \mu_s SEP(se) + (1 - \mu_s) RSEP(rse) \\
& - [\mu_s SEN(se) + (1 - \mu_s) RSEN(rse)] \\
& = \mu_s \tanh(k_1 se) + (1 - \mu_s) \tanh(k_2 rse) \\
& = \mu_s \tanh(se_1) + (1 - \mu_s) \tanh(se_2) \qquad (3.6)
\end{aligned}
$$

In the above equation (3.5), se_1, se_2 stand for $k_1 se$, $k_2 rse$.

For the joint elbow, same as the derivation of (3.6), we can also get:

$$
\begin{aligned}
E(ee, ree) &= E(ee_1, ee_2) \\
&= ESP(ee, ree) - ESN(ee, ree) \\
&= \mu_e \tanh(k_3 ee) + (1-\mu_e)\tanh(k_4 ree) \\
&= \mu_e \tanh(ee_1) + (1-\mu_e)\tanh(ee_2) \qquad (3.7)
\end{aligned}
$$

where ee_1, ee_2 stand for $k_3 ee$, $k_4 ree$.

In order to swing up the Pendubot and let the two links arrive at the goal position (top) at the same time, we should use the following swing-up control phase:

IF "SS is SSP" **ALSO IF** "ES is ESP" **THEN** torque is Positive;
IF "SS is SSN" **ALSO IF** "ES is ESN" **THEN** torque is Negative

where the logical connective "ALSO" is used to explicitly give more importance to the joint variable SS than to the joint variable ES in this swing-up controller. Here "ALSO" is again a quasi-linear-mean operator, with $\lambda > 0.5$ as its parameter. The rule firing strengths are thus given by:

$$
\begin{aligned}
OSP &= \lambda SSP(se, rse) + (1-\lambda) ESP(ee, ree) \\
OSN &= \lambda SSN(se, rse) + (1-\lambda) ESN(ee, ree)
\end{aligned}
$$

where OS is overall joint variable used to describe the system state including the shoulder joint states and the elbow joint states.

Thus as the mentioned before around equation (3.5), using the already given reasoning method to combine the two joint states, and fuzzy sets for the torque variable, the control torque is given by:

$$
\begin{aligned}
\Gamma &= G[OSP - OSN] \\
&= G[\lambda(SSP(se, rse) - SSN(se, rse)) \\
&\quad + (1-\lambda)(ESP(ee, ree) - ESN(ee, ree))] \\
&= G[\lambda S(se_1, se_2) + (1-\lambda) E(ee_1, ee_2)] \qquad (3.8)
\end{aligned}
$$

where $S(se_1, se_2), E(ee_1, ee_2)$ are defined as equations (3.6) and (3.7).

To fine tune the swing up control, the fuzzy control parameters $\mu_e, \mu_s, k_1, k_2, k_3, k_4, \lambda,$ and G were adjusted. Correct control parameters were found that quickly swing the second link and slowly into its equilibrium position so that the balancing controller can catch and balance the link. Table 3.1 lists all the control parameters.

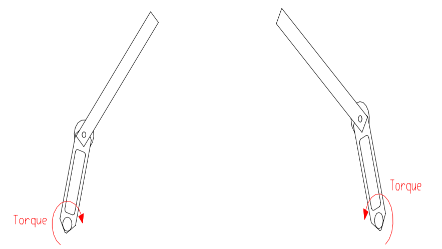
Table 3.1 Parameters for swinging up controller

μ_s	μ_e	k_1	k_2	k_3	k_4	λ	G
0.5	0.5	1.0	0.167	1.0	0.1	0.325	35.0

3.3 Balancing Control Scheme

The control for balancing the Pendubot is very similar to the classical cart-pole inverted pendulum problem. A balancing configuration is defined to be any configuration where $|se| \cong 0$ and $|ee| \cong 0$. The highest priority goal is now to keep the second link balanced at the top position. In order to implement this goal, we design a PD-like fuzzy balancing controller. The rule base is almost contrary to the swing-up controller's rule base because the goals of both controllers are different.

In order to keep both links balanced at the unstable configuration, the motor should provided the appropriate torque with the correct direction and magnitude although this torque is too small to close to zero. At the top unstable position, there are four possible situations for the two links, and two typical configurations of them are illustrated in Fig. 3.2.



$q_1 < \pi/2,\ q_2 < 0$ $\qquad$ $q_1 > \pi/2,\ q_2 > 0$
$se > 0,\ ee > 0$ $\qquad$ $se < 0,\ ee < 0$
$\tau < 0$ $\qquad$ $\tau > 0$

Fig. 3.2 Two typical configurations

In the control system, there are two kinds of inputs: the error signals se, ee and their rate of change rse, ree; there is only one output, the torque τ actuated link one. We just put two scaling factors to them, that is, $k_{p1} se, k_{d1} rse$ and $k_{p2} ee, k_{d2} ree$, which are like to the PD controller. After the error and the rate of change, such as se and rse, are correspond with each other by the scaling factors k_{p1}, k_{d1}, we can simply depict the "SSP" and "SSN" as:

IF "se is Positive" **AND IF** "rse is Positive" **THEN** "SS is SSP"
IF "se is Negative" **AND IF** "rse is Negative" **THEN** "SS is SSN"

Here, "AND" is the Zadeh's logical "AND". As the values of se and rse are real numbers, we can use mathematic form to express these rules, such as:

$$
SS = k_{p1} se + k_{d1} rse \qquad (3.9)
$$

which has included both rules “SSP” and “SSN”. Similarly, the elbow states can also expressed as the following:

$$ES = k_{p2}ee + k_{d2}ree \qquad (3.10)$$

Because in the balancing control the highest priority goal is to keep the second link balanced, the elbow states are more important than the shoulder states. Thus, the following two rules are used to design a fuzzy PD-like balancing controller:

IF “ES is ESN” **ALSO IF** “SS is SSN” **THEN** torque is Positive
IF “ES is ESP” **ALSO IF** “SS is SSP” **THEN** torque is negative

where the logical connective “ALSO” is used to give more importance to the joint elbow variable “ES” than to the joint shoulder variable “SS”. Here, “ALSO” is again a quasi-linear-mean operator, with $\lambda > 0.5$ as its parameter. We chose 0.75 for the λ. Therefore, the balancing control law is:

$$\begin{aligned}\Gamma &= -[\lambda ES + (1-\lambda)SS] \\ &= -\lambda(k_{p2}ee + k_{d2}ree) - (1-\lambda)(k_{p1}se + k_{d1}rse) \qquad (3.11)\end{aligned}$$

Table 3.2 gives all parameters of balancing controller, i.e. $k_{d1}, k_{p1}, k_{d2}, k_{p2,}$ and λ.

Table 3.2 Parameters for balancing controller

k_{d1}	k_{p1}	k_{d2}	k_{p2}	λ
4.25	0.88	2.75	21.66	0.75

4. SIMULATIONS AND EXPERIMENTS

This section presents the simulation and the real-time experiment results for the proposed fuzzy controllers, using the Pendubot Model P-2 as the test machine. The other important component of the Pendubot’s hardware is its controller. The controller is a digital implementation of the algorithms proposed in section 3. We use Microsoft C++ 7.0 to write the controlling program. For the details regarding the experimental implementation, please refer to the reference (Ma, 2001).

Fig. 4.1 illustrates the simulation diagram. The simulation results is omitted to save the space. Experimental results are shown in Figures 4.2 and 4.3 From Figures 4.2 - 4.4, we can conclude: link two is always following link two before around 0.2sec; the torque of the system is within the permitted range which is from -10N.m to 10N.m; during balancing the Pendubot at the upright unstable position, the torque is close but not equal to zero, which indicates both links actually have a little dither and the balancing controller keeps them balancing at that unstable position; it is confirmed the validity of the proposed new approach.

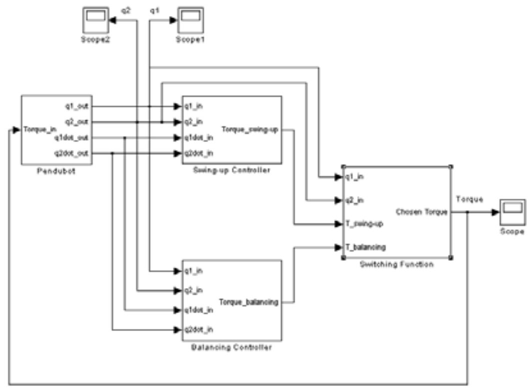


Fig. 4.1 Simulation system

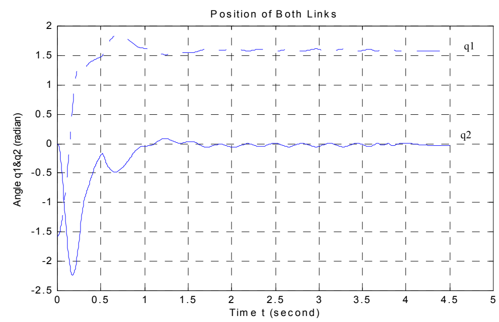


Fig. 4.2 The effect of both angles

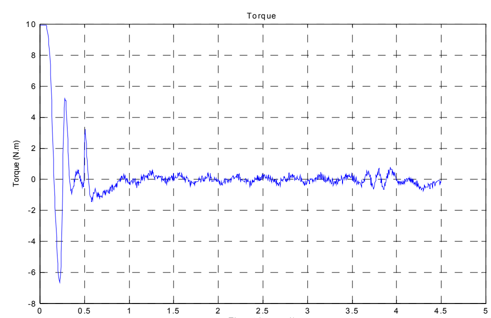


Fig. 4.3 The effect of torque

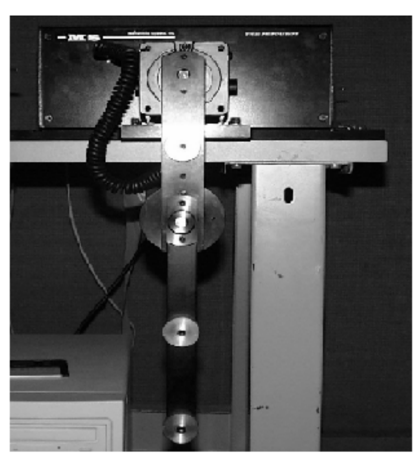


Fig. 4.4 The Pendubot at the rest

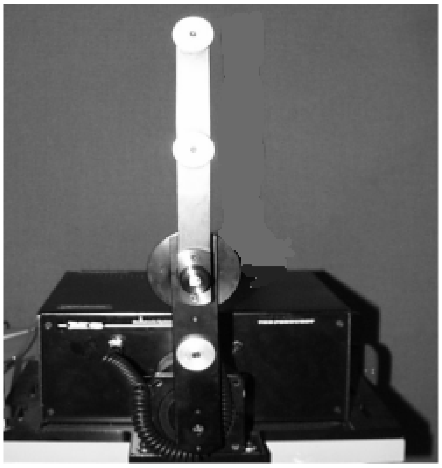


Fig. 4.5 Balancing at the top position

5. CONCLUSIONS

In our study, a fuzzy scheme for swinging up and balancing control is proposed and implemented in the actual under-actuated mechanism: the Pendubot. As an alternative approach to the previous classical methods, the real-time implementation shows the success of the application and feasibility of the fuzzy control method to the nonlinear systems.

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