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ROBUST FAULT DIAGNOSIS IN THE PRESENCE OF PROCESS UNCERTAINTIES

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Abstract:
This paper proposes a novel scheme for the generation of primary residual vector (PRV) for sensor or actuator fault detection and isolation (FDI) in multivariate dynamic systems. The PRV, which is used for fault detection purpose, is designed to be insensitive to process uncertainties, including model–plant mismatch (MPM) and process disturbances. To generate the PRV, we do not need a precise system model. Instead, all we need is an estimate of the system model, which may be biased from the true model. Under the condition that the number of process uncertainties is less than the number of outputs, the generated PRV can be made perfectly insensitive to process uncertainties. Even when this condition does not hold, the most important elements in the process uncertainties can still be decorrelated from the PRV. A numerical example to demonstrate the theory is given. The newly proposed approach is compared with existing robust FDI schemes, e.g., the Chow–Willsky scheme. *Copyright ©2002 IFAC*

Keywords: robust sensor or actuator fault detection and isolation, process uncertainties, primary residual vector, structured residual vector, multivariate systems

1. INTRODUCTION

Since the 1970s, tremendous research efforts have been invested into model–based sensor or actuator fault detection and isolation (FDI). Survey papers in this area have been published by Willsky (1976), Gertler (1988), Frank (1990), and Patton et al. (2000). More recent advances have been reviewed by Qin and Li (2001), and Li and Shah (2002).

Most model–based FDI approaches assume that an accurate plant model of the system under consideration is available, and at the same time, they also assume the process disturbances to be zero-mean Gaussian noise. Since model–plant mismatch (MPM) is inevitable and process disturbances can be any functions of time. Their presnece will affect the residuals generated for fault detection. The need for robust FDI schemes which enable the decoupling of MPM and process disturbances from the residuals has been identified. So far there have been very few published studies with respect to robust FDI. Patton and Chen (1992) have developed an observer–based approach toward the removal of process disturbances from the residuals, but the approach does not consider the issue of MPM. Gertler and Kunwer (1995) have proposed a modelling error decoupling method, which decouples the modelling error at each time instant. This method is computationally intensive and not practical. Qin and Li (2001) and Li and Shah (2002) have proposed the subspace identification–based approaches for

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the residual generation in terms of the Chow–Willsky Scheme (1984). They do not identify the individual system matrices. Instead, the residual models are identified directly from training data, thus avoiding the errors in identifying the system matrices. However, the identified residual model is only *asymptotically* consistent with the true residual model. In practice, the model is always trained from finite data samples, therefore modelling errors will inevitably be introduced into the residuals. Assuming that a finite set of different values of the system matrices are available, Lou et al. (1986) have proposed a robust scheme of generating residuals based on the average value of the system matrices. Nevertheless, the effect of MPM can not be completely removed from the residuals using this method.

This paper proposes a robust FDI scheme by extending the original Chow–Willsky scheme (Chow and Willsky, 1984) for sensor or actuator fault detection and isolation, the PRV is made insensitive to process uncertainties, including the MPM and the process disturbances of the system. Using this method, we do not need a precise system model before hand. Instead, only a rough estimation of the system model, which could be biased from the true model, is required. Under the condition that the number of process uncertainties is less than the number of outputs, the robust PRV can be made completely insensitive to the process uncertainties. Even when this condition does not hold, partial decoupling can be carried out, which means that the dominant factors in the process uncertainties can be decorrelated from the PRV.

2. PROBLEM FORMULATION

2.1 *System description*

Assume that the normal behavior of a multivariate dynamic process is represented by the following discrete-time state space model:

$$\begin{cases} \mathbf{x}_{k+1} = \mathbf{A}\mathbf{x}_k + \mathbf{B}\mathbf{u}_k^* + \mathbf{E}\mathbf{d}_k \\ \mathbf{y}_k^* = \mathbf{C}\mathbf{x}_k + \mathbf{o}_k \end{cases} \tag{1}$$

where $\mathbf{u}_k^* \in \Re^l$ and $\mathbf{y}_k^* \in \Re^m$ are *fault–free* process inputs and outputs with dimension l and m respectively; $\mathbf{x}_k \in \Re^n$ is the process state vector; $\mathbf{o}_k \in \Re^m$ is the measurement noise; $\mathbf{d}_k \in \Re^q$ with $1 \leq q \leq n$ is the process disturbances, which can be any unknown function of time, not necessary to be zero–mean; $\mathbf{E} \in \Re^{n\times q}$ consists of q columns of the $n \times n$ identity matrix $\mathbf{I}_n$; $\{\mathbf{A},\ \mathbf{B},\ \mathbf{C}\}$ are unknown time–invariant process matrices with appropriate dimensions. $\mathbf{o}_k$ is assumed to be zero–mean Gaussian–distributed white noise vector with covariance matrix $\mathbf{R}_o$ and to be independent of the initial states $\mathbf{x}_0$ and $\mathbf{d}_k$. Further, the process is assumed to be controllable and observable, which holds in most of the cases.

With the presence of sensor or actuator faults, the observed inputs and outputs can be represented by:

$$\begin{cases} \mathbf{u}_k = \mathbf{u}_k^* + \mathbf{f}_k^u \\ \mathbf{y}_k = \mathbf{y}_k^* + \mathbf{f}_k^y \end{cases} \tag{2}$$

where $\mathbf{f}_k^u \in \Re^l$ and $\mathbf{f}_k^y \in \Re^m$ are input and output sensor faults respectively. In the fault–free case, $\mathbf{f}_k^u$ and $\mathbf{f}_k^y$ are null vectors. If some sensors in inputs and/or outputs are faulty, the corresponding elements in $\mathbf{f}_k^u$ and $\mathbf{f}_k^y$ will be non–zero, while the other elements remain zero.

2.2 *Process Uncertainties*

Since the true value of the system matrices $\{\mathbf{A},\ \mathbf{B}\}$ are never known, we have:

$$\mathbf{A} = \mathbf{A}_\circ + \delta\mathbf{A}, \quad \mathbf{B} = \mathbf{B}_\circ + \delta\mathbf{B} \tag{3}$$

where $\{\mathbf{A}_\circ,\ \mathbf{B}_\circ\}$ are estimates of $\{\mathbf{A},\ \mathbf{B}\}$ from training data, and $\{\delta\mathbf{A},\ \delta\mathbf{B}\}$ are discrepancies between $\{\mathbf{A},\ \mathbf{B}\}$ and their estimates, i.e., they represent the model–plant mismatch (MPM). However, we assume that $\mathbf{C}$ is exactly known, i.e. $\mathbf{C} = \mathbf{C}_\circ$, because it is the sensor gain matrix.

The combination of Eqns. 1, 2 and 3 results in

$$\begin{aligned} \mathbf{x}_{k+1} &= (\mathbf{A}_\circ + \delta\mathbf{A})\mathbf{x}_k + (\mathbf{B}_\circ + \delta\mathbf{B})\mathbf{u}_k^* + \mathbf{E}\mathbf{d}_k \\ &= \mathbf{A}_\circ\mathbf{x}_k + \mathbf{B}_\circ\mathbf{u}_k^* + [\delta\mathbf{A}\ \ \delta\mathbf{B}]\begin{bmatrix}\mathbf{x}_k \\ \mathbf{u}_k^*\end{bmatrix} + \mathbf{E}\mathbf{d}_k \\ &= \mathbf{A}_\circ\mathbf{x}_k + \mathbf{B}_\circ\mathbf{u}_k + \mathbf{e}_k - \mathbf{B}_\circ\mathbf{f}_k^u \\ \mathbf{y}_k &= \mathbf{C}_\circ\mathbf{x}_k + \mathbf{o}_k + \mathbf{f}_k^y \end{aligned} \tag{4}$$

where $\mathbf{e}_k \equiv [\delta\mathbf{A}\ \ \delta\mathbf{B}]\begin{bmatrix}\mathbf{x}_k \\ \mathbf{u}_k^*\end{bmatrix} + \mathbf{E}\mathbf{d}_k \in \Re^n$ is the process uncertainty vector due to n_e $(1 \leq n_e \leq n)$ independent sources.

2.3 *Problems of Robust Residual Generation*

After the establishment of Eqn. 4, the problem of robust residual generation can be stated briefly as follows:

(1) From a set of training data, obtain the estimates $\{\mathbf{A}_\circ,\ \mathbf{B}_\circ,\ \mathbf{C}_\circ\}$;
(2) In terms of the estimated system matrices, generate a sequence of PRV, which under certain condition is completely independent of the process uncertainty vector $\mathbf{e}_k$. Use this generated PRV for the detection and isolation of input and output sensor faults.

3. GENERATION OF ROBUST PRIMARY RESIDUAL VECTOR

3.1 *Decoupling Process Uncertainties from the PRV*

After performing algebraic manipulation on Eqn. 4, the following stacked equation (Chow and Willsky, 1984) can be obtained

$$\begin{aligned}\mathbf{y}_{s,k} = &\boldsymbol{\Gamma}_s^\circ \mathbf{x}_{k-s} + \mathbf{H}_s^\circ \mathbf{u}_{s,k} + \mathbf{f}_{s,k}^y - \mathbf{H}_s^\circ \mathbf{f}_{s,k}^u \\ &+\mathbf{G}_s^\circ \mathbf{e}_{s,k} + \mathbf{o}_{s,k} \end{aligned} \quad (5)$$

where

$$\boldsymbol{\Gamma}_s^\circ = \begin{bmatrix} \mathbf{C} \\ \mathbf{C}\mathbf{A}_\circ \\ \vdots \\ \mathbf{C}\mathbf{A}_\circ^s \end{bmatrix} \in \Re^{m_s \times n}$$

is the extended observability matrix, with s being the order of parity space;

$$\mathbf{H}_s^\circ = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{C}\mathbf{B}_\circ & \mathbf{0} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{C}\mathbf{A}_\circ^{s-1}\mathbf{B}_\circ & \mathbf{C}\mathbf{A}_\circ^{s-2}\mathbf{B}_\circ & \cdots & \mathbf{0} \end{bmatrix} \in \Re^{m_s \times l_s}$$

$$\mathbf{G}_s^\circ = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{C} & \mathbf{0} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{C}\mathbf{A}_\circ^{s-1} & \mathbf{C}\mathbf{A}_\circ^{s-2} & \cdots & \mathbf{0} \end{bmatrix} \in \Re^{m_s \times n_s}$$

are two lower triangular block Toeplitz matrices, with $m_s = m(s+1)$, $l_s = l(s+1)$ and $n_s = n(s+1)$; $\mathbf{y}_{s,k} = \begin{bmatrix} \mathbf{y}_{k-s}^T & \cdots & \mathbf{y}_k^T \end{bmatrix}^T \in \Re^{m_s}$ is the stacked output vector; $\mathbf{u}_{s,k} \in \Re^{l_s}$, $\mathbf{f}_{s,k}^y \in \Re^{m_s}$, $\mathbf{f}_{s,k}^u \in \Re^{l_s}$, $\mathbf{e}_{s,k} \in \Re^{n_s}$ and $\mathbf{o}_{s,k} \in \Re^{m_s}$ are similarly stacked as $\mathbf{y}_{s,k}$.

Note that the optimal determination of s is beyond the scope of this paper. For the sake of simplicity, we select $s = n$.

From Eqn. 5, we have

$$\begin{aligned}&\mathbf{y}_{s,k} - \mathbf{H}_s^\circ \mathbf{u}_{s,k} \\ &= [\boldsymbol{\Gamma}_s^\circ \;\; \mathbf{G}_s^\circ] \begin{bmatrix} \mathbf{x}_{k-s} \\ \mathbf{e}_{s,k} \end{bmatrix} + [\mathbf{I}_{m_s} \;\; -\mathbf{H}_s^\circ] \begin{bmatrix} \mathbf{f}_{s,k}^y \\ \mathbf{f}_{s,k}^u \end{bmatrix} + \mathbf{o}_{s,k}\end{aligned} \quad (6)$$

By selecting a transformation matrix $\mathbf{W}_\circ$ from the *null space* of $\Psi_s^\circ = [\boldsymbol{\Gamma}_s^\circ \;\; \mathbf{G}_s^\circ]$, i.e., $\mathbf{W}_\circ \Psi_s^\circ = \mathbf{0}$, and pre–multiplying both sides of Eqn. 6 by $\mathbf{W}_\circ$, the unknown state vector $\mathbf{x}_{k-s}$ and process uncertainty vector $\mathbf{e}_{s,k}$ can be removed from Eqn. 6, leading to the following PRV:

$$\begin{aligned}\boldsymbol{\varepsilon}_{s,k} &= \mathbf{W}_\circ \mathbf{P}_s^\circ \begin{bmatrix} \mathbf{y}_{s,k} \\ \mathbf{u}_{s,k} \end{bmatrix} \\ &= \mathbf{W}_\circ \mathbf{P}_s^\circ \begin{bmatrix} \mathbf{f}_{s,k}^y \\ \mathbf{f}_{s,k}^u \end{bmatrix} + \mathbf{W}_\circ \mathbf{o}_{s,k} \in \Re^{m_s - n_s}\end{aligned} \quad (7)$$

where $\mathbf{P}_s^\circ = [\mathbf{I}_{m_s} \;\; -\mathbf{H}_s^\circ]$ and $\mathbf{W}_\circ$ at least has $m_s - n_s$ independent rows. Notice that Ψ_s° contains n_s non–zero columns, hence it is at most n_s–dimensional. Note that in Eqn. 7, the first line on the right hand side (RHS) is the computational form of the PRV, and $\mathbf{W}_\circ \mathbf{P}_s^\circ$ is the PRV model. Eqn. 7 can be splited into:

$$\boldsymbol{\varepsilon}_{s,k} = \boldsymbol{\varepsilon}_{s,k}^f + \boldsymbol{\varepsilon}_{s,k}^* \quad (8)$$

where $\boldsymbol{\varepsilon}_{s,k}^f = \mathbf{W}_\circ \mathbf{P}_s^\circ \begin{bmatrix} \mathbf{f}_{s,k}^y \\ \mathbf{f}_{s,k}^u \end{bmatrix}$ and $\boldsymbol{\varepsilon}_{s,k}^* = \mathbf{W}_\circ \mathbf{o}_{s,k}$ are the fault–related and fault–free terms respectively, and both of them are completely insensitive to the process state vector $\mathbf{x}_{k-s}$ and uncertainty vector $\mathbf{e}_k$.

$\boldsymbol{\varepsilon}_{s,k}^*$ is a moving average (MA) of measurement noise $\mathbf{o}_k$. Since $\mathbf{o}_k$ has been assumed to follow a zero–mean Gaussian–distribution, $\boldsymbol{\varepsilon}_{s,k}^*$ is also a zero-mean Gaussian-distributed random vector (Johnson and Wichern, 1998) with covariance matrix

$$\mathbf{R}_{s,\varepsilon^*} = \mathbf{W}_\circ \mathbf{R}_{s,o} \mathbf{W}_\circ^T \quad (9)$$

where $\mathbf{R}_{s,o} = \mathbf{I}_{s+1} \otimes \mathbf{R}_o$ is the covariance matrix of $\mathbf{o}_{s,k}$, and $\otimes$ stands for the kronecker tension product.

In the presence of sensor and/or actuator faults, assuming that $\mathbf{W}_\circ$ is not located in the null space of $\mathbf{P}_s^\circ$, i.e., $\mathbf{W}_\circ \mathbf{P}_s^\circ \neq 0$, $\boldsymbol{\varepsilon}_{s,k}^f$ will be non–zero. As a result, the mean of $\boldsymbol{\varepsilon}_{s,k}$ will be non–zero, but the covariance of $\boldsymbol{\varepsilon}_{s,k}$ is unchanged. Therefore, fault detection can be carried out by simply checking whether the mean of $\boldsymbol{\varepsilon}_{s,k}$ has deviated from zero.

3.2 *Calculation of* $\mathbf{W}_\circ$

The transformation matrix $\mathbf{W}_\circ$ should be designed to be located in the null space of Ψ_s°, and have maximized covariance with $\mathbf{P}_s^\circ$ such that the PRV will be most sensitive to any fault.

Using the algorithm proposed by Li and Shah (2002), it can be easily shown that $\mathbf{W}_\circ^T$ consists of the largest eigenvectors associated with the non–zero eigenvalues of matrix $(\Psi_s^\circ)^\perp \mathbf{P}_s^\circ$, where $(\Psi_s^\circ)^\perp = \mathbf{I}_{m_s} - \Psi_s^\circ (\Psi_s^\circ)^\dagger$, and $\dagger$ stands for the Penrose-Moore pseudo-inverse.

3.3 *Fault Detection Index*

Instead of simply using the PRV, one uses the squared weighted residual (SWR) as an index for *fault detection*, because it has a better performance (Oxby and Shah, 1998). The SWR is calculated by:

$$\eta_{s,k} = \boldsymbol{\varepsilon}_{s,k}^T \mathbf{R}_{s,\varepsilon^*}^{-1} \boldsymbol{\varepsilon}_{s,k} \tag{10}$$

Since $\boldsymbol{\varepsilon}_{s,k}^*$ is zero–mean Gaussian–distributed random vector, $\eta_{s,k}$ will follow a central chi–square distribution with $m_s - n_s$ degrees of freedom under fault–free condition (Johnson and Wichern, 1998), i.e. $\eta_{s,k} \sim \chi^2(m_s - n_s)$, $\forall\ \mathbf{f}_k^u = \mathbf{0}$ *and* $\mathbf{f}_k^y = \mathbf{0}$. However, if any sensor is faulty, $\eta_{s,k}$ will violate the central chi–square distribution. Therefore, fault detection can be carried out by checking $\eta_{s,k}$ against a predetermined threshold $\chi_\alpha^2(m_s - n_s)$, where α is a selected level of significance, e.g. $\alpha = 5\%$. Under the situation that the measurements are noisy, an exponentially weighted moving average (EWMA) filter can be applied to $\boldsymbol{\varepsilon}_{s,k}$ first in order to remove the effect of high–frequency noises. Subsequently, we can use the filtered PRV to calculate the fault detection index.

4. FAULT ISOLATION

Although analyzing the PRV can detect fault, to isolate the faulty sensor, one has to transform $\boldsymbol{\varepsilon}_{s,k}$ into a set of structured residual vectors (SRVs). In this paper, we consider the simplest scenario: isolation of single sensor/actuator fault once at a time. Isolation of multiple sensors can also be similarly carried out if there are enough degrees of freedom in the residual model $\mathbf{W}_\circ \mathbf{P}_s^\circ$. Interested readers can refer to Li and Shah (2002) for details.

We design the i^{th} SRV $\mathbf{r}_{s,k}^i$ such that it is insensitive to the i^{th} input or output sensor fault, while having maximized sensitivity to the other sensors. Mathematically,

$$\begin{aligned}\mathbf{r}_{s,k}^i &= \mathbf{W}_i \boldsymbol{\varepsilon}_{s,k} \\ &= \mathbf{W}_i \mathbf{W}_\circ \mathbf{P}_s^\circ \begin{bmatrix} \mathbf{f}_{s,k}^y \\ \mathbf{f}_{s,k}^u \end{bmatrix} + \mathbf{W}_i \mathbf{W}_\circ \mathbf{o}_{s,k}\end{aligned} \tag{11}$$

where $\mathbf{W}_i$ is a transformation matrix with appropriate dimensions, and Eqn. 7 has been employed. For the sake of convenience, denote $\mathbf{M} = \mathbf{W}_\circ \mathbf{P}_s^\circ \in \Re^{(m_s - n_s)\times(m_s + n_s)}$.

To ensure $\mathbf{r}_{s,k}^i$ to be insensitive to the i^{th} sensor, clearly $\mathbf{W}_i$ should be orthogonal to $s+1$ columns of the $\mathbf{M}$ matrix, i.e.

$$\mathbf{W}_i[\mathbf{M}(:,i)\ \mathbf{M}(:,i+m)\dots\mathbf{M}(:,i+ms)] = \mathbf{0},\quad \forall\ \ i = [1,\ \ m]$$

$$\mathbf{W}_i[\mathbf{M}(:,ms+i)\dots\mathbf{M}(:,ms+i+ls)] = \mathbf{0},\quad \forall\ \ i = [m+1,\ \ m+l]$$

where $\mathbf{M}(:,j)$ is the j^{th} column of matrix $\mathbf{M}$.

As a result, $\mathbf{r}_{s,k}^i$ has $m_s - n_s - (s+1)$ independent rows. The calculation of $\mathbf{W}_i$ can be conducted using the algorithm developed by Li and Shah (2002).

From $\mathbf{r}_{s,k}^i$, one can similarly calculate the isolation indices $\eta_{s,k}^i = (\mathbf{r}_{s,k}^i)^T(\mathbf{R}_{s,\varepsilon^*}^i)^{-1}\mathbf{r}_{s,k}^i$, $\forall\ i = [1,\ m+l]$, where $\mathbf{R}_{s,\varepsilon^*}^i = \mathbf{W}_i \mathbf{R}_{s,\varepsilon^*} \mathbf{W}_i^T$ is the covariance matrix of $\mathbf{r}_{s,k}^i$. If

$\eta_{s,k}^i \le \chi_\alpha^2(m_s - n_s - s - 1),\ \forall\ i \in [1,\ m+l];$

$\eta_{s,k}^j \ge \chi_\alpha^2(m_s - n_s - s - 1),\ \forall\, j \in [1,\ m+l] \cap \{j \neq i\}.$

then it can be concluded that the i^{th} sensor fails.

5. CONDITIONS FOR PERFECT AND PARTIAL DECOUPLING

5.1 *Condition for Perfect Decoupling*

To completely make the PRV uncorrelated from any process uncertainties, $m - n > 0$ must be satisfied. Furthermore, to leave some degrees of freedom for the design of fault isolation, a stricter condition: $m_s - n_s - (s+1) > 0$, should be ensured for the isolation of single sensor fault, which implies $m - n - 1 > 0$.

It should be noted that $m - n - 1 > 0$ seems to be overly restrictive, but is still possible, because most processes have redundant or duplicate sensors for critical process variables.

5.2 *Condition for Partial Decoupling*

In the case that the perfect decoupling condition is not satisfied, a partial decoupling scheme is proposed in this section.

Performing singular value decomposition (SVD) on matrix $\mathbf{G}_s^\circ$ results in

$$\mathbf{G}_s^\circ = \mathbf{U}_G \mathbf{S}_G \mathbf{V}_G^T \tag{12}$$

where $\mathbf{S}_G$ is a diagonal matrix with ns singular values in decreasing order in the diagonal, $\mathbf{U}_G$ and $\mathbf{V}_G$ are two unitary matrices.

$\mathbf{U}_G \mathbf{S}_G \mathbf{V}_G^T$ can be splited into two parts:

$$\mathbf{U}_G \mathbf{S}_G \mathbf{V}_G^T = \mathbf{U}_{G,1} \mathbf{S}_{G,1} \mathbf{V}_{G,1}^T + \mathbf{U}_{G,2} \mathbf{S}_{G,2} \mathbf{V}_{G,2}^T \tag{13}$$

where $\mathbf{S}_{G,1}$ is the first main submatrix of $\mathbf{S}_G$, which contains the n_e^s largest singular values of $\mathbf{G}_s^\circ$; and $\mathbf{U}_{G,1}$ and $\mathbf{V}_{G,1}$ are the first n_e^s columns of $\mathbf{U}_G$ and $\mathbf{V}_G$, respectively. The determination of n_e^s depends on how much the n_e^s largest singular values can approximate the total singular value in

$\mathbf{S}_G$, e.g. 80%, and the availability of the degrees of freedom in the residual model.

Substituting Eqn. 13 into Eqn. 6 gives

$$\begin{aligned}\mathbf{y}_{s,k} - \mathbf{H}_s^\circ \mathbf{u}_{s,k} = & \mathbf{\Gamma}_s^\circ \mathbf{x}_{k-s} + \mathbf{U}_{G,1}\mathbf{S}_{G,1}\mathbf{V}_{G,1}^T \mathbf{e}_{s,k} \\ & + \mathbf{U}_{G,2}\mathbf{S}_{G,2}\mathbf{V}_{G,2}^T \mathbf{e}_{s,k} \\ & + \mathbf{P}_s^\circ \begin{bmatrix} \mathbf{f}_{s,k}^y \\ \mathbf{f}_{s,k}^u \end{bmatrix} + \mathbf{o}_{s,k} \end{aligned} \quad (14)$$

Using Eqn. 14, by selecting $\mathbf{\Psi}_s^\circ = [\mathbf{\Gamma}_s^\circ \quad \mathbf{U}_{G,1}]$, the transformation matrix $\mathbf{W}_\circ$ can be similarly calculated as before. Eventually, in this case, the PRV will be

$$\begin{aligned}\boldsymbol{\varepsilon}_{s,k} = & \mathbf{W}_\circ \mathbf{P}_s^\circ \begin{bmatrix} \mathbf{f}_{s,k}^y \\ \mathbf{f}_{s,k}^u \end{bmatrix} + \mathbf{W}_\circ \mathbf{U}_{G,2}\mathbf{S}_{G,2}\mathbf{V}_{G,2}^T \mathbf{e}_{s,k} \\ & + \mathbf{W}_\circ \mathbf{o}_{s,k} \end{aligned} \quad (15)$$

From Eqn. 15, the dimension of PRV is $m_s - n - n_e^s$. Further, for the isolation of single sensor fault, the dimension of SRVs will be $m_s - n - n_e^s - (s+1)$. Obviously, at least. $m_s - n - n_e^s - (s+1) > 0$ should be ensured.

6. NUMERICAL EXAMPLE

A numerical example is provided to demonstrate the correctness of the proposed scheme. The simulated process is a second order continuous-time dynamic system under closed–loop condition with four outputs, two inputs and two unmeasured disturbances. The process disturbances are simulated by integrated white noises. The outputs are assumed to be corrupted by Gaussian–distributed white noises with covariance $0.1^2\mathbf{I}_4$. The continuous-time system is sampled with a period of 0.5 second. The system matrices in the discrete–time domain are

$$\mathbf{A} = \begin{bmatrix} 0.28283 & -0.00059394 \\ 1.2584 & 0.042506 \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} 0.2844 & -0.00032506 \\ 0.68872 & 0.15287 \end{bmatrix}$$

$$\mathbf{C} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0.5 & 0.5 \\ 0.5 & -0.5 \end{bmatrix}$$

A set of training data is used for the identification of the system matrices using the *CANSTART* command in MATLAB, whose estimates are given below.

$$\mathbf{A}_\circ = \begin{bmatrix} 0.3041 & -0.0050 \\ 1.3334 & 0.0182 \end{bmatrix}$$

$$\mathbf{B}_\circ = \begin{bmatrix} 0.2801 & 0.0004 \\ 0.6880 & 0.1513 \end{bmatrix} \quad \mathbf{C}_\circ = \mathbf{C}$$

Further, based on $\mathbf{A}_\circ$, $\mathbf{B}_\circ$ and $\mathbf{C}_\circ$, $\mathbf{H}_s^\circ$ and $\mathbf{\Psi}_s^\circ$ are constructed, and $\mathbf{W}_\circ$ is calculated. Since there are 6 sensors in total (4 outputs and 2 inputs) in the simulated system, accordingly, 6 transformation matrices, $\mathbf{W}_i$, $i = [1,\ 6]$, are calculated. With $\mathbf{H}_s^\circ$, $\mathbf{W}_\circ$ and $\mathbf{W}_i$, a sequence of PRV and 6 sequences of SRVs can be produced from the training data. Further, the covariance matrices $\mathbf{R}_{s,\varepsilon^*}$ and $\mathbf{R}_{s,\varepsilon^*}^i$, $\forall\ i = [1,\ 6]$ are estimated. Note that the i^{th} SRV $\mathbf{r}_{s,k}^i$ is designed to be insensitive to the i^{th} sensor but to be most sensitive to the other 5 sensors.

Four types of faults, e.g., bias, drift, complete failure and precision degradation, are simulated in this example. A bias $f_k^i = 0.05$ is introduced to one sensor from time instant 500 to 700. As illustrated in Fig. 1, since the first fault isolation index $\eta_{s,k}^1$ is within a predetermined confidence limit all the time, while the other isolation indices $\eta_{s,k}^i$ $i = [2, 6]$ are all beyond the limit during 500 to 700, it can be inferred that the first output sensor is faulty.

The FDI results by partially decoupling the process uncertainties are displayed in Fig. 2. Since the major elements in the uncertainty vector have been removed from the residuals, correct detection and isolation result has been obtained with an acceptable performance.

For comparison, the original Chow–Willsky scheme is applied to the same data set, and the FDI results are illustrated in Fig. 3. Therein the faulty sensor is not correctly identified, because the first fault isolation index $\eta_{s,k}^1$ is beyond the limit due to the effects of process uncertainties, violating the predetermined logic of isolation. It shows clearly that the original Chow–Willsky scheme fails to isolate the faulty sensor under the same condition.

7. CONCLUSIONS

A robust scheme for the detection and isolation of sensor and actuator faults in dynamic processes has been proposed. This approach can completely decouple the effects of any process uncertainties, such as MPM and unmeasured disturbances from the PRV under certain conditions. In comparison with the existing robust FDI schemes, the simplicity of our approach is obvious.

This approach has been applied to a simulation example, where four types of sensor faults, including bias, drift, complete failure and precision degradation, are simulated. In the simulation, complete and partial decoupling of the process

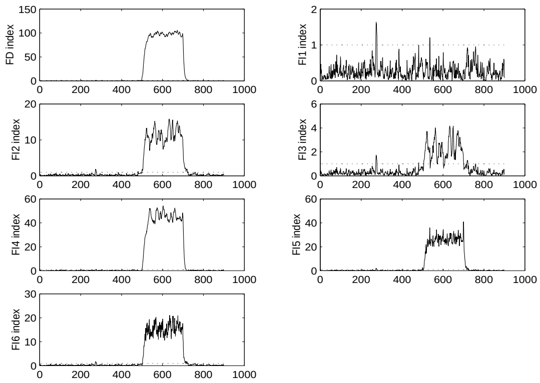

Fig. 1. *Complete robust detection and isolation indices of a bias fault in the* 1^{st} *output sensor*

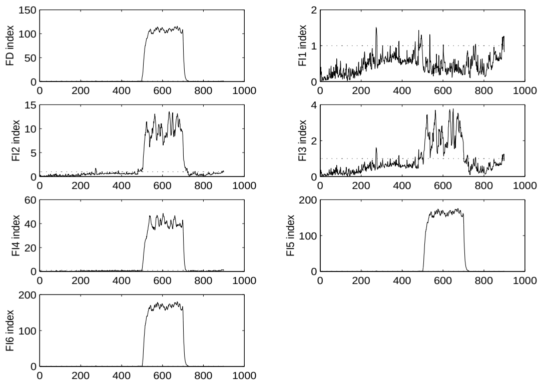

Fig. 2. *Partial robust detection and isolation indices of a bias fault in the* 1^{st} *output sensor*

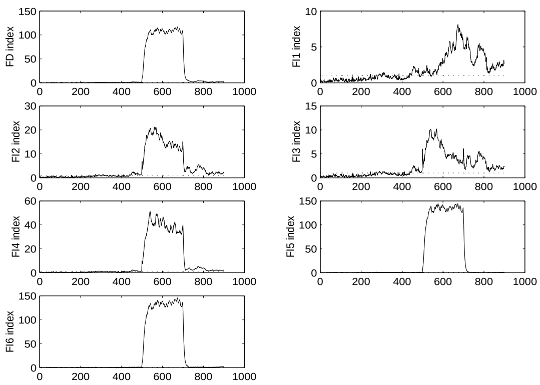

Fig. 3. *Detection and isolation indices of a bias fault in the* 1^{st} *output sensor with traditional Chow–Willsky approach*

uncertainties have been conducted. The simulation also demonstrates that our new approach is more robust with respect to process uncertainties in comparison with the original Chow–Willsky approach.

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