15th Triennial World Congress, Barcelona, Spain

AN ALGORITHM FOR SUPERVISORY CONTROL OF DISCRETE-EVENT SYSTEMS VIA PLACE INVARIANTS

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Abstract: A technique for the synthesis of supervisory controllers for discrete-event systems, modeled as Petri Nets with uncontrollable transitions, is proposed. Based on the concept of place invariants, the supervisor is designed to enforce the respect of a set of operation constraints. Due to the uncontrollable transitions, it may become impossible to directly enforce the respect of a given constraint, which is then said to be inadmissible. It is then proposed a method to determine a set of admissible constraints which do not violate the original constraint. By obtaining a larger number of admissible constraints, the proposed method is able to derive more permissive controllers than those proposed in the literature. Its effectiveness is illustrated by an example.

Keywords: Supervisory control, discrete-event systems, Petri nets, invariance, constraints.

1. INTRODUCTION

The control of discrete-event systems (DES) often amounts to regulating or supervising them in order to satisfy given specifications, such as: AGVs collision avoidance, buffer overflow prevention in a manufacture systems, etc. The supervisors of DES's are therefore used to assure that the plant evolution will not violate a set of constraints imposed on its operation.

Several approaches for the synthesis of supervisors have been studied lately. One of the most complete theories developed to solve this problem is the so-called Supervisory Control Theory, proposed by Ramadge and Wonham (Ramadge and Wonham, 1989), which is based on formal languages and finite-state automata. The strength of Ramadge & Wonham (R&W) approach lies in the establishment of necessary and sufficient conditions for the existence of a minimally restrictive supervisor. The main drawback is the state explosion with the growing number of system components. Even though this difficulty can be alleviated by the use of a modular approach (Ramadge and Wonham,1989), the complexity of the resulting controller can make it difficult to be implemented in practice.

As an alternative to the complexity associated to R&W approach, several works have explored the simplicity and the graphic power of Petri Nets. In (Holloway and Krogh, 1990), a class of problems is solved with the introduction of the so-called Controlled Petri Nets, resulting in control laws which are functions of the net marking and need to be computed on-line. In (Li and Wonham, 1993,1994), by using the concept of Vector Discrete-Event Systems, it is shown that if the uncontrollable part of the plant has a given tree structure, then it is possible to explicitly compute the optimal controller. In (Giua and DiCesare, 1994), a transition/place structure is proposed to compute the supervisor. Based on R&W theory, but using Petri Nets, a closed-loop controller is constructed through the use of Integer Programming techniques. Supervisory controllers derived from Petri Nets models are also proposed in (Yamalidou *et al.*, 1995), where a set of constraints in the space state is represented in the form of linear inequalities. The controller which enforces the respect of such constraints is also a Petri Net and is obtained by forcing the set of constraints to become a place invariant of the controlled system.

The main difficulty of place invariants techniques arises from the existence of uncontrollable transitions (events). In this case, it may not be possible to directly enforce the respect of a given constraint, which is then called *inadmissible.* In (Moody and Antsaklis, 1998), a technique is proposed to transform inadmissible constraints into admissible ones. However, such a technique has two main drawbacks: it may in many cases be unable to compute a controller; the computed controllers may not be sufficiently permissive, in the sense of unnecessarily disabling some transitions.

In this work, an algorithm which seeks to reduce these drawbacks, also based on place-invariants, is proposed. From the computation of the nonnegative kernel of a given matrix, the proposed algorithm is able to generate a larger number of admissible constraints, which implies the computation of controllers in certain cases for which the algorithm in (Moody and Antsaklis, 1998) fails. Besides, more permissive controllers can be derived. The efficiency of the proposed method is asserted through an illustrative example.

2. SUPERVISORY CONTROL BASED ON PLACE INVARIANTS

In this approach, discrete processes are modeled as Petri Nets containing n places and m transitions. If the net does not have any self-loop, then it can be represented by:

$$\begin{array}{l} \mu_p(k+1) = \mu_p(k) + D_p q(k), \\ \mu_p(0) = \mu_{p0}. \end{array} \qquad (1)$$

where $D_p \in Z^{nxm}$ is the incidence matrix, $\mu_p \in Z^n$, with $\mu_p \geq 0$, is the marking vector and $q \in Z^m$, with $q \geq 0$, is the vector of transitions firing.

The control objective is to enforce the plant to respect a set of linear constraints:

$$L\mu_p \leq b \qquad (2)$$

with $L \in Z^{n_c \times n}$ and $b \in Z^{n_c}$.

This inequality can be transformed into an equality through the addition of a nonnegative variable:

$$L\mu_p + \mu_c = b \qquad (3)$$

with $\mu_c \in Z^{n_c}$, $\mu_c \geq 0$.

The controller is also a Petri Net with n_c places connected to the net transitions. The controlled system will then be a Petri Net composed by the original plant plus the controller, forming a closed-loop system whose incidence matrix $D \in Z^{(n+n_c)\times m}$ and the marking vector $\mu \in Z^{n+n_c}$ are given by: $D = \begin{bmatrix} D_p \\ D_c \end{bmatrix}$, $\mu = \begin{bmatrix} \mu_p \\ \mu_c \end{bmatrix}$.

Since the constraint (3) must be satisfied for any event occurrence k, then (3) defines a *place invariant*, which is characterized by (Murata, 1989; Yamalidou *et al.*, 1995; Moody and Antsaklis, 1998):

$$\begin{bmatrix} L & I \end{bmatrix} \begin{bmatrix} D_p \\ D_c \end{bmatrix} = 0$$

The controller and its initial marking are then obtained by:

$$\begin{array}{l} D_c = -LD_p \\ \mu_{c_o} = b - L\mu_{p_o} \end{array} \qquad (4)$$

The matrix D_c indicates the arcs which connect the controller places to the plant transitions and μ_{c_o} is the initial marking of these places.

The supervisor described above does not avoid *a priori* a deadlock situation. Nevertheless, as shown in (Moody and Antsaklis, 1998), several situations of deadlock avoidance can be casted in the form of linear constraints such as (2) and, consequently, be solved through the use of supervisors like (4). A method for the automatic generation of linear inequalities which guarantee that the system is deadlock-free was recently proposed in (Iordache, *et al.*, 2000).

If all transitions are controllable and observable, then the controller obtained through (4) only acts to disable transitions whose firing leads the system to a state which violates the constraint (2). Such a controller is therefore, in this case, maximally permissive (Moody and Antsaklis, 1998).

3. CONSTRAINT TRANSFORMATION

In many cases, certain transitions cannot be disabled by the action of the controller. Such transitions are then called *uncontrollable transitions* and correspond to events such as sensor faults, end of machine operations, etc. It is therefore illegal to assign control actions to these transitions.

A given constraint $l^T \mu_p \leq b$, with $l \in Z^n$ is admissible if it satisfies the following conditions:

- the initial marking of the plant satisfies the constraint;
- there exists a maximally permissible controller which enforces the respect of the constraint and does not disable uncontrollable transitions.

Given a plant containing uncontrollable transitions, in view of (4), the second condition can be written in the following form:

$$l^T D_{uc} \leq 0, \tag{5}$$

where D_{uc} is the matrix formed by the columns of the incidence matrix D_p associated to uncontrollable transitions. This condition assures that there will not be arcs from the controller places to the uncontrollable transitions of the plant.

If a given constraint is not admissible, it is still possible to control the system by determining an admissible constraint which satisfies the original constraint for all reachable marking of the net.

(Moody and Antsaklis, 1998) propose constraints in the following form:

$$L_n \mu_p \leq b_n, \quad \text{with} \begin{cases} L_n = R_1 + R_2 L \\ b_n = R_2(b + \mathbf{1}) - 1 \end{cases} \tag{6}$$

where: $R_1 \in Z^{n_c \times n}$, $R_1 \mu_p \geq 0$ for all $\mu_p \geq 0$; $R_2 \in Z^{n_c \times n_c}$ is a diagonal positive definite matrix; $\mathbf{1}$ is a vector of 1's with dimension n_c.

It is shown in (Moody and Antsaklis, 1998) that any controller which enforces (6) also enforces (2). It is then proposed an algorithm for the computation of a new set of constraints, based on elementary operations on the rows of a given matrix. However, as pointed out by the authors, this algorithm may fail in some cases, being unable to give an admissible controller.

Besides, the controller obtained through this approach is not in general maximally permissible. In the aim of improving the permissiveness, Moody and Antsaklis (1998) propose to transform each constraint into a linear disjunction of the associated admissible constraints. In words, given a constraint and a set of associated admissible constraints, one tries to enforce, at each time, at least one of these constraints, instead of all the constraints at the same time. It is shown in (Li and Wonham, 1993,1994) that if the uncontrolled part of the plant has a certain tree structure, then this strategy can generate a maximally permissible controller.

A linear disjunction is represented by:

$$\bigvee_{i=1}^{n_c} l_i^T \mu_p \leq b_i. \tag{7}$$

The rule for transition firing is then changed to:

$$\max_{i \in 1,\ldots,n_c} (\mu_{ci}(k) + D_{ci}(k) q(k)) \geq 0. \tag{8}$$

Therefore, at each time, only one of the controller places needs to have nonnegative marking (according to the standard rules of Petri Nets). The other controller places can have negative marking and the firing of some transitions may not be disabled by the controller places.

Now, with the introduction of the new constraint defined by (6), the inequality (5) becomes:

$$(R_1 + R_2 l^T) D_{uc} \leq 0.$$

By transforming the inequality above into an equality through the addition of a nonnegative variable Δ, one has: $(R_1 + R_2 l^T) D_{uc} + \Delta = 0$ and, in matrix form:

$$\begin{bmatrix} R_1 & R_2 & \Delta \end{bmatrix} \begin{bmatrix} D_{uc} \\ l^T D_{uc} \\ I \end{bmatrix} = 0. \tag{9}$$

In (Moody and Antsaklis, 1998), an algorithm is proposed to compute a set of admissible constraints which satisfy a given inadmissible constraint. It is based on the computation of the left kernel of the matrix $\begin{bmatrix} D_{uc} \\ l^T D_{uc} \\ I \end{bmatrix}$. In order to obtain only nonnegative elements, elementary operations are then performed on the rows of the matrix formed by the vectors which define the kernel. In the end, if the algorithm does not fail, a set of admissible constraints which respect the original constraint is obtained. The final controller is then computed as the disjunction of the constraints corresponding to the rows of this transformed matrix.

Due to the heuristic nature of the matrix operations, this algorithm can generate a reduced number of new constraints, restricting thereby the permissiveness of the controller. In some cases, the algorithm can even result in the null matrix and no controller is then obtained.

4. PROPOSED METHOD

4.1 *Nonnegative Kernel*

Let $A = \begin{bmatrix} D_{uc} \\ l^T D_{uc} \\ I \end{bmatrix}$, with $A \in \Re^{(n+1+n_{uc}) \times n_{uc}}$ and consider the following set which represents the nonnegative left kernel o matrix A:

$$\Gamma = \{w \in \Re^{n_{uc}} \mid w \geq 0; \quad w^T A = \mathbf{0}\}. \quad (10)$$

The set Γ is a polyhedral cone (Schrijver, 1987), which can be completely characterized by a set of *generators*. A set of vectors $\{m_1, m_2, \cdots, m_{ng}\}$ is a *generating set* of a polyhedral cone Γ if, $\forall w \in \Gamma$, there exist $\xi_i \geq 0$ such that $w = \sum_{i=1}^{ng} \xi_i m_i$. The vectors $m_1, m_2, \cdots, m_{ng}$ are the called *generators* of Γ. The generating set is minimal if it is formed by the least possible number of generators.

There are several methods for the computation of the minimal generating set. In the case of matrix A, which only contains integer elements, the Fourier-Motzkin elimination method (Schrijver, 1987) is particularly suitable. Even though this method is not conceived to this end, it can be easily adapted to the computation of a matrix $M \geq 0$ such that $MA = \mathbf{0}$ (Dórea and Hennet, 1999).

Let then A^1 be the first column of matrix A and define the following sets: $I^0 = \{i \mid A_i^1 = 0\}$, $I^+ = \{i \mid A_i^1 > 0\}$ e $I^- = \{i \mid A_i^1 < 0\}$, where A_i^1 represents the i-th element of A^1.

The objective is to find a matrix M_1 such that $M_1 A^1 = \mathbf{0}$, with all the elements M_{ij}^1 of M^1 such that $M_{ij}^1 \geq 0$.

For all index $j \in I^0$, a row m^T is added to the matrix M_1 so that: $\begin{cases} m_i = 1 & \text{for } i = j \\ m_i = 0 & \text{for } i \neq j \end{cases}$.

For all index $j \in I^+$, all the indices $k \in I^-$ are scanned and, for each k, a row m^T is added to the matrix M_1 so that: $\begin{cases} m_j = |A_k^1| \\ m_k = A_j^1 \\ m_i = 0 & \text{for } i \neq j \text{ and } i \neq k \end{cases}$.

Since $A = \begin{bmatrix} A^1 & A^2 & \cdots & A^{n_{uc}} \end{bmatrix}$, one can easily see that: $M_1 A = \begin{bmatrix} 0 & M_1 A^2 & \cdots & M_1 A^{n_{uc}} \end{bmatrix}$.

The algorithm proceeds by performing the procedure described above on matrix $M_1 A^2$. The nonnegative matrix M_2 such that $M_2(M_1 A^2) = 0$ is then obtained, and so on. After n_{uc} steps, the matrix $M = M_{n_{uc}} M_2 \cdots M_1$ such that $MA = 0$ is finally obtained.

At the end, matrix M can have redundant rows. Such rows can however be deleted by applying the following property originated from Linear Programming Theory: any basic solution $w^T \geq 0$ of equation $w^T A = 0$ has at most $n_{uc} + 1$ nonnull elements (Schrijver, 1987).

If the procedure described above is used, only nonnegative R_1, R_2 e Δ are obtained and no further row operation on M is required.

4.2 *Constraints Disjunction*

The goal now is to compute the largest number of admissible constraints such that a given inequality $l^T \mu_p \leq b$ is respected. By enforcing the disjunction of such constraints, it will be therefore possible to obtain a more permissible controller.

Admissible R_1 and R_2 are directly taken from the rows of the matrix M obtained from the computation of the nonnegative left kernel of matrix A.

In some rows of M, the element corresponding to R_2 may be null. That amounts to places which would be added to the controller, but which do not depend on the original constraint $l^T \mu_c \leq b$, being therefore unnecessary. In this case, one could try to compute another constraint transformation by adding this row to another row of M containing nonnull elements in different positions of R_1. However, it has been noticed that such a calculation only generates redundant constraints. Therefore, such rows can be deleted without any loss of permissiveness.

Compared to the method proposed in (Moody and Antsaklis, 1998) to enforce the disjunction of a set of constraints, the proposed method simplifies considerably the design of controllers, for, now, any admissible controller must correspond to a nonnegative linear combination of the rows of matrix M. Any other vector $w \geq 0$ which satisfies $wA = \mathbf{0}$ is necessarily a linear combination of the rows of M, and that would result in a redundant constraint. The heuristic approach in (Moody and Antsaklis, 1998) may not compute all the necessary elements of the set Γ (10).

In the procedure proposed here, the controller can therefore be characterized as the disjunction of all admissible constraints originated from the final M.

4.3 *Constraints Transformation*

The goal now is to transform the original set of constraints, supposed to be inadmissible, $L\mu_p \leq b$, into a new set, $L_n \mu_p \leq b_n$, which is admissible and such that the original constraints are respected for any μ_p for which $L_n \mu_p \leq b_n$.

From the matrix M obtained from the computation of the nonnegative left kernel of matrix A, the new constraints can be directly computed. One only has to compute, by the method described in the previous section, the matrix M_i corresponding to each constraint $l_i^T \mu_p \leq b_i$. A set of admissible constraints $L_n \mu_c \leq b_n$ can therefore be computed by choosing a row (or a combination of rows) from each matrix M_i.

Compared to the method proposed in (Moody and Antsaklis, 1998), the algorithm presented above has the following advantages:
- It offers more options to the choice of the

transformed constraints;
- It is able to characterize all the possible matrices R_1 and R_2. This guarantees that a controller is always found, if it can be obtained through R_1 and R_2. On the contrary, the method proposed in (Moody and Antsaklis, 1998) may not result in any controller in some instances.

5. EXAMPLE

As an illustration of the application of the proposed algorithms, consider a plant modeled as the Petri Net of Figure 1.

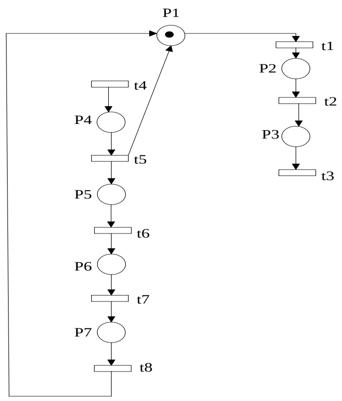

Fig. 1. Petri Net model for the Plant.

The incidence matrix and initial marking are given by:

$$Dp = \begin{bmatrix} -1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix},$$

$$\mu_{p0} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T.$$

Transitions t_6, t_7 and t_8 are considered uncontrollable. The controllable transition t_4 represents an external event. It is assumed that it can fire at any time.

The constraint on the number of tokens in place p_6 is: $\mu_6 \leq 1$.

The admissibility of the constraints is tested through (5), giving $LD_{uc} = \begin{bmatrix} 1 & -1 & 0 \end{bmatrix}$. Therefore, the constraint is not admissible and it has to be transformed.

From the application of the algorithm proposed in (Moody and Antsaklis, 1998), the following transformed admissible constraint is obtained:

$$L_n = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix}, b_n = 1.$$

Therefore, only one admissible constraint has been obtained by the aforementioned algorithm. It will be shown in the sequel that this solution is very restrictive.

Now, applying the proposed algorithm, a set of new admissible constraints can be obtained. The matrix M, whose rows form the nonnegative left kernel of matrix A (9 is then given by:

$$M = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

The matrix R_2 (9) corresponds to column 8 of M. Taking only the rows for which the element corresponding to R_2 is positive (rows 6, 8 and 10), and applying (6), 3 admissible constraints $L_n\mu \leq b_n$ are obtained:

$$L_n = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix}, \quad b_n = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}.$$

The computation of the controller which enforces the disjunction of the constraints defined by the rows of L_n and b_n gives:

$$D_c = \begin{bmatrix} 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & -2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 1 \end{bmatrix}, \quad \mu_{co} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}.$$

The controllers corresponding to each row of D_c can therefore be implemented following the disjunction scheme described previously.

Compared to the results obtained by the algorithm of (Moody and Antsaklis, 1998), the proposed method generates a controller which is composed of a larger number of places (3 versus 1). One can notice that the constraint obtained by (Moody and Antsaklis, 1998) is included in the matrix L_n computed from the proposed method. If only this constraint is enforced, transition t_5 can never fire, whereas with the disjunction of the constraints computed by the proposed method, t_5 can always fire after t_7 fires, as long as there is at least one token in p_4.

The action of the controller is illustrated in figure 2. The marking of place P_6 is supervised by the controller. If a transition connected to it fires, then, at least one of its places, according to the ordinary Petri Nets firing rules, regulates this marking. The other places can in this case have negative marking.

Suppose, for example, that t_4 fires. In this case, C_1 and C_3 enable t_5, even though C_2 "disables" it. After t_5 fires, the marking of C_1 and C_3 become null and that of C_2 becomes negative (-2).

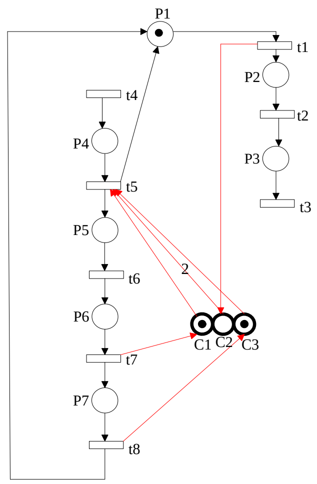

Fig. 2. Controlled Plant.

The enforcement of the disjunction of constraints can make it difficult to implement the supervisor in practice if the number of constraints is large. If it is desired to use the analytical and visual power of Petri Nets, one can choose, following some criterion, only one constraint from the set associated to each specifications, as described in section 4.3 (see e.g. (Basile, *et al.*, 2000) where a cost is associated to the transition firing and the supervisor is computed through the solution of an optimization problem). Such a solution would result however in a more restrictive controller.

6. CONCLUSIONS

The design of supervisors modeled as Petri Nets, based on place invariants, has shown to be a simple and elegant alternative which makes it easy to analyze and implement controllers which enforce DES's to respect operation constraints. The difficulties of this technique appear when the system has uncontrollable events. In this case, it is necessary to find constraints which are admissible, in the sense of not trying to disable uncontrollable transitions, and which respect the original constraints.

A new algorithm has then been proposed to compute a set of admissible constraints which is wider than those found in the literature. This algorithm has shown to be more efficient in the computation of this set, implying the possibility of designing much less restrictive controllers. The efficiency of the method has been illustrated through a numerical example. This method increases the possibility of using Petri Nets techniques in the synthesis of supervisors for DES's.

Even though it has not been raised in this work, the existence of unobservable events in the DES can be easily casted in the proposed framework.

7. REFERENCES

Basile, F., P. Chiacchio and A. Giua (2000). Petri Net Monitor Design with Control and Observation Costs. *Proc. 39th IEEE Conf. Decision Contr.* Sydney, Australia.

Dórea, C.E.T. and J.-C. Hennet (1999). (A, B)-invariant Polyhedral Sets of Linear Discrete-time Systems. *J. Optimiz. Theory and Applications*, **103**(3), pp. 521-542.

Giua, A. and F. DiCesare (1994). Petri Net Structural Analysis for Supervisory Control. *IEEE Trans. Robotics and Automation*, **10**(2), pp. 185-195.

Holloway, L.E. and B. H. Krogh (1990). Synthesis of Feedback Control Logic for a Class of Controlled Petri Nets. *IEEE Trans. Automat. Contr.*, **35**(5), pp. 514-523.

Iordache, M.V., J.O. Moody and P.J. Antsaklis (2000). A Method for the Synthesis of Deadlock Prevention Controllers in Systems Modeled by Petri Nets. *Proc. 2000 American Contr. Conf.* Chicago, USA, pp. 3167-3171.

Li, Y. and W.M. Wonham (1993,1994). Control of Vector Discrete Event Systems. *IEEE Trans. Automat. Contr.* Part I - The Base Model. **38**(8), pp. 1214-1227, 1993. Part II - Controller Synthesis. **39**(3), pp. 512-531, 1994.

Moody, J.O. and P.J. Antsaklis (1998). *Supervisory Control of Discrete Event Systems Using Petri Nets.* Kluwer Academic Publishers, Boston.

Murata, T. (1989). Petri Nets: Properties, Analysis and Applications. *Proceedings of the IEEE*, **77**(4), pp. 541-580.

Ramadge, P.J.G. and W. M. Wonham (1988). The Control of Discrete Event Systems. *Proceedings of the IEEE*, **77**(1), pp. 13-30.

Schrijver, A. (1987). *Theory of Linear and Integer Programming.* John Wiley and Sons, Chichester.

Yamalidou, K., J. Moody, M. Lemmon and P. Antsaklis (1995). Feedback Control of Petri Nets Based on Place Invariants. *Automatica*, **32**(1), pp. 15-28.