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NONLINEAR MPC VERSUS MPC USING ON-LINE LINEARISATION — A COMPARATIVE STUDY

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Abstract: One of the main drawbacks of NMPC schemes is the enormous computational effort these controllers require. On the other hand, linear MPC methods can be implemented solving just Quadratic Programming (QP) or Linear Programming (LP) problems. In this paper, an alternative implementation of NMPC suggested by De Keyser (1998) is implemented to reduce the computational effort. This methodology is based on on-line linearisation and solves and iterative procedure which, if convergent, provides with the solution of the NMPC problem. This controller is tested and compared with the purely nonlinear MPC schemes. Copyright © 2002 IFAC

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1. INTRODUCTION

In real life problems to be faced by the control engineer, *constraints* and *nonlinearities* stand out as the most important difficulties. Constraints limit the performance which can be achieved by the closed-loop system, and must be handled in an efficient way if performance is a priority. Several strategies have been suggested to design and analyse control systems under constraints, among which Model Predictive Control (MPC) has gained widespread acceptance during the last two decades. The reasons for MPC success are the simplicity of the ideas behind the control strategy and the possibility to handle constraints and nonlinearities in a rigorous way.

The term MPC encloses a wide family of control algorithms which make predictions of the future behaviour of the plant by using an explicit model of the process. Although the industrial processes are inherently nonlinear, the vast majority of MPC applications are based on linear models (Qin and Badgwell, 2000). In the linear MPC case, the optimisation problem can be written as a Linear or Quadratic Programming (LP or QP) problem for which there exist very efficient software tools. However, sometimes nonlinearities are significant enough to justify the application of Nonlinear Model Predictive Control (NMPC) strategies (Qin and Badgwell, 2000).

A very important issue of NMPC, which was overlooked in the first few formulations, is the real-time requirement of industrial control systems. This requirement is essential to translate the academical results into the industrial world. Due to the use of a nonlinear model, the NMPC strategy is based on solving a nonconvex optimisation problem on-line. It is well known that the available optimisation algorithms for non-

convex problems cannot guarantee that the solution obtained (if any) is a global optimum. In fact, these tools do not even guarantee that a feasible solution exists. Even when a suboptimal feasible solution is found, the computational effort is enormous compared to that of solving QP and LP counterparts, which makes it difficult to apply NMPC to industrial processes. The following quote (Morari and Lee, 1999) clarifies this point:

> "*Though the theoretical purists tend to stay away from linearisation approaches, linearisation is the only method which has found any wider use in industry beyond demonstration projects.*"

Some recent results present efficient implementations of NMPC (Morari and Lee, 1999; Qin and Badgwell, 2000), but the computational burden they require is still much larger than that involved in linear MPC applications. In order to overcome this drawback, several alternatives to purely non-linear MPC have been suggested. These alternative methods try to obtain an optimisation problem for which the computational burden and the implementation difficulties are similar to those of linear MPC. The optimisation problem associated to NMPC is substituted by a QP counterpar which is generated applying different approximate or iterative algorithms. Several approaches which follow these guidelines have been suggested (Oliveira *et al.*, 1995; Nevistić, 1997; Megías *et al.*, 1999), and an overview of these can be found in (Morari and Lee, 1999).

In this paper, the application of a modification (De Keyser, 1998) to the Extended Prediction Self-Adaptive Control (EPSAC) suggested by De Keyser and Cauwenberghe (1985) is investigated. The EPSAC formulation is similar to that of the better known Generalised Predictive Control (GPC) (Clarke *et al.*, 1987), but a nonlinear version of EPSAC has been recently proposed to handle nonlinear systems (De Keyser, 1998). At each sampling instant, the nonlinear EPSAC uses a local linearisation of the process to compute an "optimised response", analogous to the "forced response" of linear MPC methods. This scheme is applied within an iterative algorithm which, if convergent, provides with an exact solution to the optimisation problem. The controller is expected to be identical to a purely NMPC strategy, unless the optimisation yields some other local minimum. However, simulation results are not provided in (De Keyser, 1998) to show the properties of the nonlinear EPSAC. In addition, it must be pointed out that no strict theoretical results are so far available for the nonlinear EPSAC, but this paper focuses only on practical considerations.

In this paper, the nonlinear EPSAC and a standard NMPC algorithm are compared taking into account both performance and computational issues. The paper is organised as follows. In Section 2 the formulation of NMPC is shortly reviewed. Section 3 presents the formulation of the nonlinear EPSAC. In Section 4 some simulation results obtained with both the nonlinear EPSAC and a standard NMPC implementation are shown, and a comparison between these strategies is provided. Finally, conclusions are drawn in Section 5.

2. A BRIEF OVERVIEW OF NMPC

Given a Multiple-Input Multiple-Output (MIMO) process, described by the following autonomous nonlinear ODE system:

$$\begin{cases} \dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}), \\ \boldsymbol{y} = \boldsymbol{g}(\boldsymbol{x}), \end{cases} \tag{1}$$

with n states ($\boldsymbol{x}$), m inputs ($\boldsymbol{u}$) and p outputs ($\boldsymbol{y}$), the control aim is to find a control profile to minimise a cost function:

$$J(t) = \sum_{k=N_1}^{N_2} \sum_{i=1}^{p} \gamma_i \left[w_i(t+k|t) - y_i(t+k|t) \right]^2 + \sum_{k=1}^{N_u} \sum_{i=1}^{m} \lambda_i \Delta u_i^2(t+k-1|t). \tag{2}$$

where $w_i(t+j|t)$ are the future values of the setpoint for the i-th output, which are known at time t or assumed to be equal to the current value $w_i(t|t)$, and $y_i(t+k|t)$ are future predictions of the system of Equation (1). The tuning knobs N_2 and N_u are called, respectively, the prediction horizon and the control horizon, for obvious reasons, whereas N_1 is the lower prediction horizon. Finally, γ_i and λ_i are nonnegative weights, and $\Delta u_i(t+k|t)$ are future control increments (or moves). In this paper, N_1 and all the weights γ_i have been set to 1 for all the simulated experiments. The control increments from $\Delta u_i(t+N_u|t)$ and beyond are taken to be zero so that the optimisation problem depends on a finite number ($N_u \times m$) of decision variables.

In the NMPC literature, the cost function is often defined to take into account the states instead of the outputs, and the aim is to lead the state vector to the origin (regulatory problem). Note that the cost function of Equation (2) can be applied for both the setpoint tracking and the regulatory problems. In the latter case, the outputs $\boldsymbol{y}$ should be identical to the states $\boldsymbol{x}$, and the setpoints should be zero. Note, also, that state measurements or estimations are required in order to solve the model equations forward in time to compute the predictions, irrespective of which kind of cost function is used.

The optimisation problem is often solved subject to input (amplitude and moves) and output constraints:

$$\begin{aligned} u_{i\min} &\le u_i \le u_{i\max}, \quad i = 1, \ldots, m, \\ \Delta u_{i\min} &\le \Delta u_i \le \Delta u_{i\max}, \quad i = 1, \ldots, m, \\ y_{i\min} &\le y_i \le y_{i\max}, \quad i = 1, \ldots, p. \end{aligned}$$

The complexity of the optimisation problem to be carried on in NMPC controllers is very remarkable. Two

different strategies can be distinguished to solve this problem, namely the *sequential* approach and the *simultaneous* approach. In the latter, the objective function and the model equations converge at the same time, whereas in the former, the model equations are satisfied at every sampling instant and the implementation is simpler. In this work, the controllers have been implemented using the sequential approach, for which the intermediate solutions are feasible with respect to the model equations. This is a relevant advantage in the case of on-line applications.

For on-line implementations of NMPC, the use of gradient (first-order) information is essential in order to reduce the computational burden. Gradients are computed using either the adjoint system method (Rosen and Luus, 1991), the finite difference method, or the trajectory sensitivity equations method. Although the trajectory sensitivity method can be more stable compared to the adjoint method, it requires a larger number of equations and increases the computation time. For this reason, the adjoint system approach has been used throughout this work.

3. NONLINEAR EPSAC

This section presents the formulation of the nonlinear[1] EPSAC suggested in (De Keyser, 1998). For simplicity of notation, the SISO version is formulated here, but the controller has been implemented for the MIMO case. The key idea of this formulation is that the sequence of manipulated variables can be thought of as the sum of a base control sequence $u_{\rm base}(t+k|t)$ plus a sequence of increments of the manipulated variables $\delta u(t+k|t)$. That is:

$$u(t+k|t) = u_{\rm base}(t+k|t) + \delta u(t+k|t).$$

The j-step ahead output prediction is computed as the sum of the response of the process $y_{\rm base}(t+k|t)$ due to the base input sequence plus the response of the process $y_{\rm optimise}(t+k|t)$ due to the future control increments with respect to the base input sequence:

$$y(t+k|t) \approx y_{\rm base}(t+k|t) + y_{\rm optimise}(t+k|t).$$

The nonlinear model is used to compute $y_{\rm base}(t+k|t)$ while $y_{\rm optimise}(t+k|t)$ is computed from a linear model of the plant, obtained linearising the nonlinear model about the current state at each sampling instant, hence the use of the "$\approx$" sign. The cost function is quadratic in the decision variables $\delta u(t+k|t)$ and the optimisation problem can be solved using a standard QP algorithm as in linear MPC. The component $y_{\rm optimise}(t+k|t)$ is the cumulative effect of a series of impulse and step inputs:

$$\begin{aligned} y_{\rm optimise}(t+k|t) = {} & h_k \delta u(t|t) + h_{k-1}\delta u(t+1|t) \\ & + \cdots + g_{k-N_u+1}\delta u(t+N_u-1|t), \end{aligned}$$

where the parameters $h_1, h_2, \ldots, h_k, \cdots, h_{N_2}$ are the coefficients of the *unit impulse response* of the system, whereas the values g_k refer to the *unit step response* coefficients. Using matrix notation, the prediction equation becomes

$$\boldsymbol{Y} = \overline{\boldsymbol{Y}} + \boldsymbol{G}\boldsymbol{U}$$

where

$$\begin{aligned} \overline{\boldsymbol{Y}} &= \left[y_{\rm base}(t+N_1|t) \ \cdots \ y_{\rm base}(t+N_2|t) \right]^{\rm T}, \\ \boldsymbol{U} &= \left[\delta u(t|t) \ \cdots \ \delta u(t+N_u-1|t) \right]^{\rm T}, \end{aligned} \tag{3}$$

and

$$\boldsymbol{G} = \begin{bmatrix} h_{N_1} & h_{N_1-1} & h_{N_1-2} & \cdots & g_{N_1-N_u+1} \\ h_{N_1+1} & h_{N_1} & h_{N_1-1} & \cdots & g_{N_1-N_u+2} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ h_{N_2} & h_{N_2-1} & h_{N_2-2} & \cdots & g_{N_2-N_u+1} \end{bmatrix}.$$

In this description, the coefficients of the matrix $\boldsymbol{G}$ are computed using the linearised model about the current state. It is also possible to use different linear models for different time instants in the future, *i.e.* linearising about a suboptimal trajectory, thus obtaining a Linear Time Variant (LTV) version of this methodology, which has been used in the second example reported in Section 4.

The superposition principle does not hold for nonlinear processes and the output predictions generated this way will only coincide with the output prediction generated by a NMPC controller when the sequence of future control moves ($\boldsymbol{U}$) is zero. If this is not the case, the base control sequence is made equal to the last base control sequence plus the optimal control increments found by the QP algorithm. The procedure is repeated until the sequence of future controls is driven close enough to zero (the prediction equation becomes exact) in an iterative scheme. To reduce the number of iterations, the initial value of $u_{\rm base}(t+k|t)$ is critical. A simple but effective choice (De Keyser, 1998) is to start with the optimal control policy derived at the previous sample $u^*(t+k|t-1)$ with the corresponding time shift. In this paper, this strategy has been used. Another strategy which has been tried is the use of the steady-state value required to lead the output to the setpoint as $u_{\rm base}(t+k|t)$ for $k \geq N_u$, in the first iteration. However, this alternative does not reduce the number of iterations in the simulation examples.

The convergence conditions of the algorithm are very difficult to obtain, since they depend on the severity of the nonlinear characteristics of the process, on past inputs and outputs, on the future reference sequence and on perturbations.

A simple relationship exists between the control actions Δu and δu:

$$\begin{bmatrix} \Delta u(t|t) \\ \Delta u(t+1|t) \\ \cdots \\ \Delta u(t+N_u-1|t) \end{bmatrix} = \boldsymbol{A} \begin{bmatrix} \delta u(t|t) \\ \delta u(t+1|t) \\ \cdots \\ \delta u(t+N_u-1|t) \end{bmatrix} + \boldsymbol{b},$$

with the matrix $\boldsymbol{A}$ and the vector $\boldsymbol{b}$ given by:

[1] Apart from the use of a nonlinear model, the nonlinear EPSAC formulated in (De Keyser, 1998) differs from the linear counterpart in the disturbance model.

$$\boldsymbol{A} = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ -1 & 1 & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & -1 & 1 \end{bmatrix},$$

$$\boldsymbol{b} = \begin{bmatrix} u_{\text{base}}(t|t) - u(t-1) \\ u_{\text{base}}(t+1|t) - u_{\text{base}}(t|t) \\ \cdots \\ u_{\text{base}}(t+N_u-1|t) - u_{\text{base}}(t+N_u-2|t) \end{bmatrix}.$$

The cost function is a quadratic form in $\boldsymbol{U}$ defined in Equation (3), and then QP can be applied to solve the optimisation problem.

4. TEST EXAMPLES

Several simulation tests have been carried out to investigate the performance and stability properties, as well as the efficiency, of the nonlinear EPSAC approach compared to the standard NMPC. The first experiment has been carried out for a benchmark robot presented in (Nevistić, 1997), where an MPC law with linear time-varying models is shown successful. This is simple two link (1R-1P) manipulator with a robot arm and a cart that moves radially along the arm. The model of the robot dynamics is given by:

$$(J + mr^2)\ddot{\varphi} + 2m\dot{r}\dot{\varphi} = T_1$$
$$\rho m\ddot{r} - \rho m r\dot{\varphi}^2 = T_2$$

where:

$m = 1$ kg is mass of the cart,
$J = 6.4314$ kgm^2 is the joint moment of inertia,
$\rho = 1$ m is the length (radius) of the arm,
$\varphi(t) \in [0^0, 270^0]$ is the position of the robot arm,
$r(t) \in [0.27 \text{ m}, 1 \text{ m}]$ is the position of the cart, and
$T_1(t)$, $T_2(t)$ are the torques of the arm and the cart, respectively.

The outputs $\boldsymbol{y} = [\varphi, r]^{\text{T}}$ represent the position of the robot arm and of the cart, given in radians and metres, respectively. The inputs, $\boldsymbol{u} = [T_1, T_2]^{\text{T}}$ are limited by lower and upper bounds, $[T_{1\min}, T_{1\max}, T_{2\min}, T_{2\max}] = [-20, 20, -10, 10]$ Nm. The problem considered here is a setpoint change to $\varphi_{\text{ref}} = \pi/2$ rad and $r_{\text{ref}} = 0.8$ m.

In Figure 1 the results obtained with both the standard NMPC and the nonlinear EPSAC are presented. It must be pointed out that there is little difference between the results (performance) provided by both controllers, and some of the signals are almost identical, *e.g.* see $\varphi(t)$. The parameters are the same as those used in (Nevistić, 1997), which are chosen as $N_2 = 10, N_u = 2$, and the sampling time is 0.1 s. On the other hand, the associated computational efforts indicate a clear advantage of the EPSAC. For the standard NMPC, the computation time (of the whole simulated experiment) is 39.2 minutes, whereas the EPSAC takes just 2.1 minutes. The simulation has been carried out using (interpreted) MATLAB in a 800 MHz computer with 256 MBytes of RAM. This

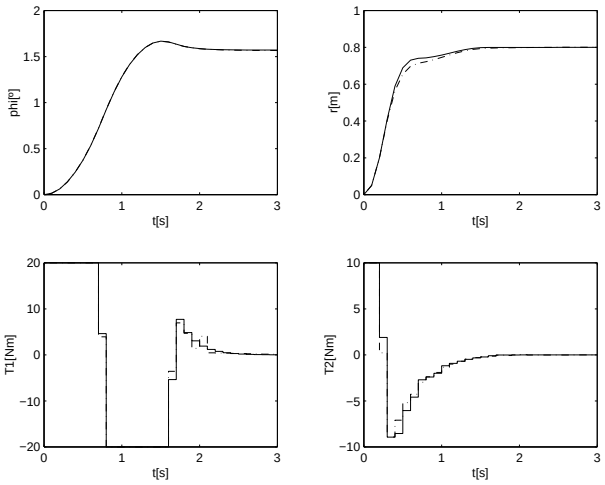

Fig. 1. Robot: nonlinear EPSAC (solid) vs. NMPC (dash-dotted), $N_2 = 10$, $N_u = 2$

advantage arises from the use of linear models, which avoids the computationally demanding prediction integrations, resulting in a significant reduction of the computational burden (by a factor of 5–100). In other experiments, which are not shown here for brevity, the difference between NMPC and the EPSAC becomes greater when the parameters are tuned to result in more aggressive control action. Compared to the results MPC+LTV presented in (Nevistić, 1997), the difference in performance is very small, and no computation times are reported. In (Blet, 2001) some examples are shown where the nonlinear EPSAC is implemented on a LTV basis, which reduces the number of iterations, although each iteration requires a somewhat larger computational burden.

Parameter	Value
k_{10}	$(1.287 \pm 0.040) \cdot 10^{12}$ h^{-1}
k_{20}	$(1.287 \pm 0.040) \cdot 10^{12}$ h^{-1}
k_{30}	$(0.043 \pm 0.270) \cdot 10^{9}$ mol^{-1} h^{-1}
ΔH_{RAB}	4.2 ± 2.36 kJ mol^{-1}
ΔH_{RBC}	-11.00 ± 1.92 kJ mol^{-1}
ΔH_{RAD}	-41.85 ± 1.41 kJ mol^{-1}

Table 1. Parameter uncertainty

For the following example, a comparative analysis between the nonlinear EPSAC and the standard NMPC is not an objective. The main aim is to to test the nonlinear EPSAC in a difficult framework in which linear MPC cannot be successfully applied. This case study considers a continuously fed stirred tank reactor (CSTR) with a cooling jacket in which cyclopentenol is produced from cyclopentadiene by acid-catalysed electrophilic hydration in aqueous solution. This process is often referred to as the van de Vusse reactor. Further details on the derivation of this benchmark process and the chemical background can be found in (Chen *et al.*, 1995). The highly nonlinear behaviour of this reactor at the operating point chosen here, which is rated 0.92 on a 0–1 nonlinearity scale suggested by Helbig *et al.* (1998), is a result of the so-called van de Vusse reaction that exhibits interesting properties, like a change of steady-state gain at the operating point. A

"reference" solution to this benchmark problem based on a NMPC scheme with an extended Kalman filter (EKF) is presented in (Chen *et al.*, 1995).

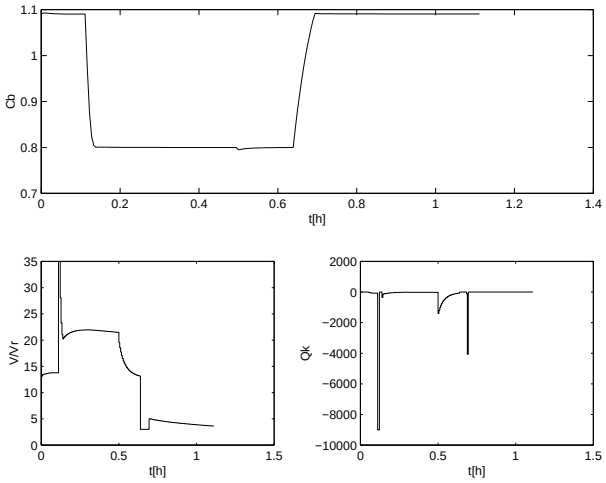

Fig. 2. Robustness to parameter uncertainties of the CSTR controlled with the nonlinear EPSAC

The reactor is fed with diluted cyclopentadiene (substance A) with concentration c_{A0} (unmeasured disturbance) and temperature ϑ_0. In a thermal reaction, cyclopentenol (substance B) with concentration c_B (the controlled variable) is produced and reacts further on in an unwanted reaction to C. In parallel, the initial reactant A reacts to D, which is not wanted either. As the process is exogenous, an external heat exchanger (energy flow $\dot{Q}_K$) is used to cool down the reactor. The manipulated variables are the volumetric flow rate $\dot{V}$ and the energy flow rate $\dot{Q}_K$, for which constraints are considered: $3\ \mathrm{h}^{-1} \leq \dot{V}/V_R \leq 35\ \mathrm{h}^{-1}$, $-9000\ \mathrm{kJ\ h}^{-1} \leq \dot{Q}_K \leq 0\ \mathrm{kJ\ h}^{-1}$.

The dynamics of the reactor can be described by the following nonlinear differential equations that are derived from component balances for substances A and B and from energy balances for the reactor and the cooling jacket:

$$\begin{aligned}
\dot{c}_A =& \frac{\dot{V}}{V_R}(c_{A0} - c_A) - k_1(\vartheta)c_A - k_3(\vartheta)c_A^2 \\
\dot{c}_B =& -\frac{\dot{V}}{V_R}c_B + k_1(\vartheta)c_A - k_2(\vartheta)c_B \\
\dot{\vartheta} =& \frac{\dot{V}}{V_R}(\vartheta_0 - \vartheta) - \frac{1}{\rho C_p}\Big[k_1(\vartheta)c_A\Delta H_{R_{AB}} \\
& + k_2(\vartheta)c_B\Delta H_{R_{BC}} + k_3(\vartheta)c_A^2\Delta H_{R_{AD}}\Big] \\
& + \frac{k_w A_R}{\rho C_p V_R}(\vartheta_K - \vartheta) \\
\dot{\vartheta_K} =& \frac{1}{m_k C_{PK}}[\dot{Q}_K + k_w A_R(\vartheta - \vartheta_K)],
\end{aligned}$$

The concentrations of A and B, the temperature in the reactor and the temperature in the cooling jacket are denoted by c_A, c_B, ϑ and ϑ_K, respectively. The reaction velocities k_i are assumed to depend on the temperature via the Arrhenius law

$$k_i(\vartheta) = k_{i0}\exp\Big(\frac{E_i}{\vartheta^0 C + 273.15}\Big),\ i = 1, 2, 3.$$

The nominal and the simulation values of the different parameters can be found in (Chen *et al.*, 1995).

The reactor is considered at an operating point where optimal yield with respect to a desired product is achieved. However, in practice, chemical reactors are often not operated at the point of maximal yield, since these conditions are very difficult to achieve due to some unfavourable properties at this point. In addition, the physico-chemical parameters of the benchmark problem are only known within bounds (see Table 1). The controller has to compensate the effects of changes in the setpoint value $c_{B\mathrm{ref}}$ and of a disturbance in ϑ_0 simultaneously. The maximal steady-state offset should no exceed 0.02 mol/l (control tolerance). In (Chen *et al.*, 1995), a solution based on NMPC+EKF with $N_u = 3$, $N_2 = 200$, $\lambda_1 = 0$, $\lambda_2 = 0$ is presented. These settings are chosen not only for stability, but also for the feasibility of the nonlinear programming problem. The sampling period is 20 s. In those results, the manipulated variables are "blocked" to remain constant for every two sampling periods. For the nonlinear EPSAC, the same sampling time has been chosen, whereas the other parameters are $N_2 = 3$, $N_u = 1$, $\lambda_1 = 0$, $\lambda_2 = 0$. The nonlinear EPSAC has been used with a LTV model at each sampling instant.

Fig. 2 shows how the nonlinear EPSAC tries to make c_B to track the setpoint changes from maximum (1.09 mol/l) to minimum (0.8 mol/l) at $t = 0.111$ hours (400 seconds) and back to maximum value at time $t = 0.639$ hours (2300 seconds), against model-plant mismatch and disturbances. There is model-plant mismatch in the first 0.5 hours (1800 seconds) according to the extreme case which results from taking the "$+$" sign for all the parameters in Table 1 and, after that, according to the other extreme case ("$-$" sign), as discussed in (Chen *et al.*, 1995). The nominal (internal) model uses the central values provided in Table 1. At time $t = 0.306$ h (1100 s) and $t = 0.833$ h (3000 s) the feed temperature ϑ_0 changes from 104.9^0C down to 100^0C and up to 115^0C, respectively. This indeed constitutes quite a difficult framework to test the solution provided by the nonlinear EPSAC. It can be observed that the controller is able to satisfy the control requirements even under these quite hard conditions.

5. CONCLUDING REMARKS

In this paper, the performance of the nonlinear EPSAC suggested by De Keyser (1998) is tested against that provided by a purely NMPC scheme. The simulation results presented for two benchmark models, commonly found in NMPC literature, are quite satisfying as performance, stability and computational burden are concerned. The nonlinear EPSAC leads to a convenient solution which is comparable to that obtained

with NMPC, but the computational burden of the latter is reduced by a factor of 5–100. The EPSAC is shown to produce a convenient closed-loop behaviour even for a van de Vusse CSTR working at an optimal yield operating point.

The idea exploited by the nonlinear EPSAC is to apply the "superposition" principle in which a "base" prediction is performed with the nonlinear model whereas an "optimise" prediction is calculated from a local linearisation at each sampling instant. This procedure is embedded within an iterative scheme until the "optimise" prediction is lead to (close to) zero. This provides with an "exact" solution of the optimisation scheme which has been shown almost identical to that obtained with a standard NMPC controller.

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