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FINITE TIME CONTROL FOR ROBOT MANIPULATORS [1]

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Abstract: Finite-time control of the robot system is studied through both state feedback and dynamic output feedback control. The effectiveness of the proposed approach is illustrated by both theoretic analysis and computer simulation. In addition to offering an alternative approach for improving the design of the robot regulator, this research also extends the study of the finite-time control problem from second order systems to a large class of higher order nonlinear systems.

Keywords: Robot control, finite-time stability, state feedback, output feedback, stabilization.

1. INTRODUCTION

In this paper, we will study the problem of finite-time regulation of robots which aims at designing state or dynamic output feedback control laws such that the position of the robot can be regulated into a desired position in a finite time. Conventionally, most of the existing results on regulation of robots is achieved asymptotically. There are two common nonlinear approaches for achieving asymptotic regulation, namely, the inverse-dynamics control or computed torque control (Aubin *et al.* 1991), and the gravity-compensation control (Takegaki *et al.* 1981). Other related work can also be found in (Berghuis *et al.* 1993; Canudas de Wit *et al.* 1996; Venkataraman *et al.* 1991). The investigation on the finite-time control of robots is motivated by the following two considerations. In terms of application, finite-time controller, as the name suggested, results in a closed-loop system whose position converges in finite time as opposed to asymptotic convergence of conventional controller. Thus our approach may offer an alternative for the control of robot manipulators. In terms of theory, finite-time convergence is an important control problem on its own, and has been studied in the contexts of optimality and controllability, mostly with discontinuous or open-loop control. Recently, finite-time stabilization via continuous time-invariant feedback has been studied from different perspectives. In particular, the state feedback and the output feedback finite-time control laws for a double integrator system were given in (Bhat and Bernstein 1997,1998; Haimo 1986; Hong *et al.* 2001), respectively.

This paper will address the finite-time regulation for a class of multi-dimensional systems describing robot manipulators. We will investigate two classes of finite-time regulation controllers that correspond to the conventional inverse-dynamics control and gravity-compensation control as mentioned above. Both state feedback and output feedback control laws will be considered. We note here that while it is fairly straight-

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forward to extend the results in (Bhat and Bernstein 1997; Hong *et al.* 2001) on double integrator systems to the robot systems controlled by inverse-dynamics finite-time controller, it is technically non-trivial to establish the finite-time regulation of the robot systems controlled by the gravity-compensation controller since in the later case, the closed-loop system is not homogeneous and the results in (Bhat and Bernstein 1997; Hong *et al.* 2001) are not applicable. Thus a different technique has to be worked out to address the stability issue. For this reason, we will place our emphasis on gravity-compensation controller.

2. PRELIMINARIES

We begin with the review of the concepts of finite-time stability and stabilization of nonlinear systems following the treatment in (Bhat and Bernstein 1997, 1998).

Consider the system

$$\dot{\xi} = f(\xi), \quad f(0) = 0, \quad \xi(0) = \xi_0, \quad \xi \in R^n \qquad (1)$$

with $f : U_0 \to R^n$ continuous on an open neighborhood U_0 of the origin. Suppose that system (1) possesses unique solutions in forward time for all initial conditions.

Definition 1: The equilibrium $\xi = 0$ of system (1) is (locally) finite-time stable if it is Lyapunov stable and finite-time convergent in a neighborhood $U \subset U_0$ of the origin. The finite-time convergence means the existence of a function $T : U/\{0\} \to (0,\infty)$, such that, $\forall \xi_0 \in U \in R^n$, every solution of (1) denoted by $s_t(0,\xi_0)$ with ξ_0 as the initial condition is defined, and $s_t(0,\xi_0) \in U/\{0\}$ for $t \in [0,T(\xi_0))$, and $\lim_{t\to T(x_0)} s_t(0,\xi_0) = 0$. When $U = R^n$, we obtain the concept of global finite-time stability.

Remark 1: Clearly, global asymptotic stability and local finite-time stability imply global finite-time stability. This observation will be used in the following analysis of finite-time stability.

Definition 2: The equilibrium $x = 0$ of a nonlinear system

$$\dot{x} = f(x) + g(x)u \quad f(0) = 0, \quad x \in R^n \qquad (2)$$

is finite-time stabilizable if there is a continuous feedback law $u = \mu(x)$ with $\mu(0) = 0$ such that the origin $x = 0$ of the closed-loop system is a finite-time stable equilibrium.

Remark 2: It is easy to see that the control law $\mu(x)$ must make the closed-loop system nonsmooth. To this end, finite-time control laws are often sought from the class of homogeneous functions. Thus, let us introduce some concepts regarding homogeneous functions following the treatment of (Hermes 1991; Rosier 1992; Kawski 1995).

Let $V : R^n \to R$ be a continuous function. V is said to be homogeneous of degree $\sigma > 0$ with respect to $(r_1,\ldots,r_n)$ where $r_i > 0, i = 1,\ldots,n$, if, for any given $\varepsilon > 0$, we have

$$V(\varepsilon^{r_1}\xi_1,\ldots,\varepsilon^{r_n}\xi_n) = \varepsilon^{\sigma}V(\xi), \quad \forall \xi \in R^n \qquad (3)$$

A continuous vector field $f(\xi) = (f_1(\xi),\ldots,f_n(\xi))^T$ is said to be homogeneous of degree $k \in R$ with respect to $(r_1,\ldots,r_n)$ where $r_i > 0, i = 1,\ldots,n$, if, for any given $\varepsilon > 0$, we have

$$f_i(\varepsilon^{r_1}\xi_1,\ldots,\varepsilon^{r_n}\xi_n) = \varepsilon^{k+r_i} f_i(\xi), \quad i = 1,\ldots,n; \qquad (4)$$

for all $\xi \in R^n$. The system (1) is homogeneous if $f(\xi)$ is homogeneous.

The connection of a homogeneous system and its finite-time stability was given in (Bhat and Bernstein 1997), and is rephrased by the following two lemmas.

Lemma 1: Suppose that the equilibrium of system (1) is asymptotically stable, and the system is homogeneous of degree k. If $k < 0$, then the origin of the system is finite-time stable.

Lemma 2: The following control law

$$u = -l_1 sgn(x_1)|x_1|^{\alpha_1} + l_2 sgn(x_2)|x_2|^{\alpha_2}, \qquad (5)$$

with

$$0 < \alpha_1 < 1, \quad \alpha_2 = \frac{2\alpha_1}{1+\alpha_1},$$

finite-time stabilizes the following double integrator system

$$\dot{x}_1 = x_2, \quad \dot{x}_2 = u \qquad (6)$$

Remark 3: Clearly, (5) is not smooth, but it is a homogeneous function of degree α_2 with respect to $(\frac{2}{1+\alpha_1}, 1)$. In the limiting case where α_1 approaches 1, the control becomes the conventional PD controller, which is a smooth control law, and cannot achieved finite-time stability as described in Remark 2. For this reason, we call (5) a (homogeneous) nonsmooth PD controller.

The following result is from (Hong *et al.* 2001), which extends the result given in (Hermes 1991; Rosier 1992) from asymptotical stability to finite-time stability.

Lemma 3: Consider the following system

$$\dot{\xi} = f(\xi) + \hat{f}(\xi), \quad f(0) = 0 \quad \xi \in R^n$$

where $f(\xi)$ is a continuous homogeneous vector field of degree $k < 0$ with respect to $(r_1,\ldots,r_n)$, and $\hat{f}$ satisfies $\hat{f}(0) = 0$. Assume $\xi = 0$ is an asymptotically stable equilibrium of the system $\dot{\xi} = f(\xi)$. Then $\xi = 0$ is a locally finite-time stable equilibrium of the system if

$$\lim_{\varepsilon\to 0} \frac{\hat{f}_i(\varepsilon^{r_1}\xi_1,\ldots,\varepsilon^{r_n}\xi_n)}{\varepsilon^{k+r_i}} = 0, \; i = 1,\ldots,n; \qquad (7)$$

for all $\xi \neq 0$

In the next section, we need to extend Lemma 2 to a vector form that entails the following notation, which was used in (Haimo 1986) for simplicity.

$$sig(\xi)^{\alpha} = (|\xi_1|^{\alpha} sgn(\xi_1),\ldots,|\xi_n|^{\alpha} sgn(\xi_n))^T \in R^n,$$

for $\xi \in R^n$ and $\alpha > 0$.

3. FINITE-TIME FEEDBACK

In what follows, we will mainly study global finite-time control of robot systems described by

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau, \quad q \in R^n \tag{8}$$

where q is the vector of generalized coordinates and τ is the vector of external torque representing the control input; $M(q)$ denotes the inertia matrix, $C(q,\dot{q})\dot{q}$ is the Coriolis and centrifugal forces, and $G(q)$ represents gravitational force. Note that $M(q)$, $C(q,\dot{q})$, and $G(q)$ are all smooth. Some well-known properties of (8) are in order (Canudas de Wit *et al.* 1996, pp. 61-62):

P1. $M(q)$ is positive and bounded;

P2. $\dot{M}(q) - 2C(q,\dot{q})$ is skew-symmetric.

P3. $||G(q)|| \leq G_0$ for some bounded constant $G_0 > 0$.

Finite time regulation problem: Given a desired position $q^d \in R^n$, find a feedback control law $u = \mu(q,\dot{q})$ such that the equilibrium of the closed-loop system at $(q,\dot{q}) = (q^d, 0)$ is globally finite-time stable.

In this section, we will start with the state feedback control law of the following two basic forms:

$$\begin{aligned} \tau &= G(q) + C(q,\dot{q})\dot{q} \\ &\quad - M(q)[l_1 sig(q-q^d)^{\alpha_1} + l_2 sig(\dot{q})^{\alpha_2}], \end{aligned} \tag{9}$$

and

$$\tau = G(q) - l_1 sig(q-q^d)^{\alpha_1} - l_2 sig(\dot{q})^{\alpha_2}, \tag{10}$$

where $0 < \alpha_2 < 1, \alpha_1 = \frac{\alpha_2}{2-\alpha_2}, l_1 > 0$ and $l_2 > 0$.

Recall that the inverse dynamics *asymptotic* regulator and the gravity-compensation *asymptotic* regulator take the following forms (see Canudas de Wit *et al.* 1996):

$$\tau = G(q) + C(q,\dot{q})\dot{q} - M(q)[l_1(q-q^d) + l_2\dot{q}] \tag{11}$$

and

$$\tau = G(q) - l_1(q-q^d) - l_2\dot{q}, \tag{12}$$

respectively. Thus control laws (9) and (10) can be obtained from (11) and (12) by simply replacing the PD controllers in (11) and (12) with nonsmooth PD controllers.

Theorem 1: The equilibrium $(q,\dot{q}) = (q^d, 0)$ of the closed-loop systems composed of robot system (8) and control law (9) is globally finite-time stable.

Proof: Let $x_1 = q - q^d$, $x_2 = \dot{x}_1 = \dot{q}$, and $x = (x_1^T, x_2^T)^T$, then the closed-loop system of the robot manipulator under the control law (9) can be rewritten in the form of

$$\begin{cases} \dot{x}_1 = x_2, \quad x_i = (x_{i1}, \ldots, x_{in})^T, \quad i = 1,2; \\ \dot{x}_2 = -[l_1 sig(x_1)^{\alpha_1} + l_2 sig(x_2)^{\alpha_2}], \end{cases} \tag{13}$$

where $x = (x_1^T, x_2^T)^T \in R^{2n}$. First note that the system is a multiple version of 2-dimensional system, in a similar form studied in (Kawski 1989), and therefore, we can conclude that the solution of the system is unique in forward time by using Proposition 2.2 and its remark in (Kawski 1989).

Next, take a Lyapunov function candidate of the form $V_*(x) = \frac{l_1}{1+\alpha}(\sum_{i=1}^n |x_{1i}|^{1+\alpha_1}) + \frac{1}{2}x_2^T x_2$. Then, $\dot{V}_*(x)|_{(13)} = -l_2(\sum_{i=1}^n |x_{2i}|^{1+\alpha_2}) \leq 0$. Because $\dot{V}_*(x) \equiv 0$ together with (13) implies $(x_1, x_2) \equiv 0$, the equilibrium $x = 0$ of system (13) is globally asymptotically stable by LaSalle's invariant set theorem. Moreover, (13) is homogeneous of negative degree ($k = \alpha_2 - 1 < 0$) with respect to $(\frac{2}{1+\alpha_1}, \ldots, \frac{2}{1+\alpha_1}, 1, \ldots, 1)$. Thus, the conclusion follows from Lemma 1.

Next, we will consider the property of the closed-loop system of the robot (8) under control law (10), whose structure is simpler than that of (9). For this purpose, let $x_1 = q - q^d$, $x_2 = \dot{x}_1 = \dot{q}$, and $x = (x_1^T, x_2^T)^T$, the state equation of the closed-loop system of the robot manipulator under the control law (10) is

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -M(x_1+q^d)^{-1}[C(x_1+q^d, x_2)x_2 \\ \quad + l_1 sig(x_1)^{\alpha_1} + l_2 sig(x_2)^{\alpha_2}] \end{cases} \tag{14}$$

Clearly, $x = 0$ is the equilibrium of (14). It can be seen that the closed-loop system under this control law is no longer homogeneous. Therefore, Lemma 1 does not apply directly to this case. We will use Lemma 3 to establish the global finite-time stability for this later case. For this purpose, we rewrite (14) as follows

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -M(q^d)^{-1}[l_1 sig(x_1)^{\alpha_1} + l_2 sig(x_2)^{\alpha_2}] \\ \quad + f_{q^d}(x_1, x_2) \end{cases} \tag{15}$$

where

$$\begin{aligned} f_{q^d}(x_1,x_2) &= -M(x_1+q_d)^{-1}C(x_1+q^d, x_2)x_2 \\ &\quad - \tilde{M}(q^d, x_1)(l_1 sig(x_1)^{\alpha_1} + l_2 sig(x_2)^{\alpha_2}) \end{aligned} \tag{16}$$

with $\tilde{M}(q^d, x_1) = M(x_1+q^d)^{-1} - M(q^d)^{-1}$

It can be easily verified that the following system

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -M(q^d)^{-1}[l_1 sig(x_1)^{\alpha_1} + l_2 sig(x_2)^{\alpha_2}] \end{cases} \tag{17}$$

is homogeneous of degree $-1 < k = \alpha_2 - 1 < 0$ with respect to $(r_{11}, r_{12}, \ldots, r_{1n}, r_{21}, r_{22} \ldots, r_{2n})$ with $r_{1i} = r_1 = \frac{2}{1+\alpha_1}$ and $r_{2i} = r_2 = 1$ for $i = 1, \ldots, n$.

Theorem 2: Assume the solutions of both (14) and (17) are unique in forward time, then the origin of (14) is globally finite-time stable.

Proof: We will first show that the equilibrium of (14) is globally asymptotically stable. For this purpose, consider a Lyapunov function

$$V(x)=\frac{1}{2}x_2^TM(x_1+q^d)x_2+\frac{l_1}{1+\alpha_1}(\sum_{i=1}^{n}|x_{1i}|^{1+\alpha_1}) \quad (18)$$

Then,

$$\dot{V}(x)|_{(14)}=-x_2^T[l_1sig(x_1)^{\alpha_1}+l_2sig(x_2)^{\alpha_2}]$$
$$+l_1x_2^Tsig(x_1)^{\alpha_1}=-l_2(\sum_{i=1}^{n}|x_{2i}|^{1+\alpha_2})\le 0$$

It can be seen that $\dot{V}(x)|_{(14)\equiv 0}$ together with (14) implies $(x_1,x_2)\equiv 0$. It follows from LaSalle's theorem that the equilibrium of the closed-loop system at the origin is globally asymptotically stable.

Nevertheless, since system (14) is not homogeneous, we cannot apply Lemma 1 to conclude the global finite-time stability of (14). Thus, we need to appeal to Lemma 3.

Consider the closed-loop system (16), and take a Lyapunov function candidate of the form $V_*(x)=\frac{l_1}{1+\alpha}(\sum_{i=1}^{n}|x_{1i}|^{1+\alpha_1})+\frac{1}{2}x_2^TM(q^d)x_2$. Then, $\dot{V}_*(x)|_{(17)}=-l_2(\sum_{i=1}^{n}|x_{2i}|^{1+\alpha_2})\le 0$. With invariant set theorem and Lemma 1, the homogeneity of negative degree ($k<0$) of system (17) yields the global finite-time stability of its equilibrium $x=0$.

Next, we will conclude from Lemma 3 that the equilibrium of system (15) is locally finite-time stable by showing that $f_{q^d}(\varepsilon^{r_1}x_{11},\dots,\varepsilon^{r_2}x_{2n})$ is 'higher degree' with respect to ε^{k+r_2} in the sense of (7) where $r_1=\frac{2}{1+\alpha_1}$ and $r_2=1$. To this end, first note that, since $M(q^d+x_1)^{-1}$ and $C(x_1+q^d,x_2)$ are smooth, and $k<0$,

$$\lim_{\varepsilon\to 0}\frac{M(q^d+\varepsilon^{r_1}x_1)^{-1}C(q^d+\varepsilon^{r_1}x_1,\varepsilon^{r_2}x_2)\varepsilon^{r_2}x_2}{\varepsilon^{k+r_2}}$$
$$=M(q^d)^{-1}C(q^d,0)x_2\lim_{\varepsilon\to 0}\varepsilon^{-k}=0$$

Next applying the mean value inequality (Rudin 1976, Theorem 9.19) to each entry of $\tilde{M}(q^d,\varepsilon^{r_1}x_1)$ gives $\tilde{M}(q^d,\varepsilon^{r_1}x_1)=M(\varepsilon^{r_1}x_1+q^d)^{-1}-M(q^d)^{-1}=O(\varepsilon^{r_1})$. Thus

$$\lim_{\varepsilon\to 0}\frac{\tilde{M}(q^d,\varepsilon^{r_1}x_1)[l_1sig(\varepsilon^{r_1}x_1)^{\alpha_1}+l_2sig(\varepsilon^{r_2}x_2)^{\alpha_2}]}{\varepsilon^{k+r_2}}$$
$$=\lim_{\varepsilon\to 0}O(\varepsilon^{(r_1-k-r_2)})=\lim_{\varepsilon\to 0}O(\varepsilon^{-2k})=0$$

Thus for any fixed $x=(x_1^T,x_2^T)^T\in R^{2n}$,

$$\lim_{\varepsilon\to 0}\frac{f_{q^d}(\varepsilon^{r_1}x_{11},\dots,\varepsilon^{r_1}x_{1n},\varepsilon^{r_2}x_{21},\dots,\varepsilon^{r_2}x_{2n})}{\varepsilon^{k+r_2}}=0,$$

which shows the local finite-time stability of the equilibrium of system (15) according to Lemma 3.

Finally, invoking Remark 1 completes the proof.

Remark 4: The uniqueness of the solution in forward time of system (15) or (17) is not guaranteed in general because of the complexity of the closed-loop systems. However, for the class of robot systems with primatic joints as shown in Section 4, using the results of (Kawski 1989), it is possible to conclude the the uniqueness of the solution in forward time of system (15) or (17).

Remark 5: A variation of state feedback control law (10) is given as follows

$$\tau=G(q)-l_1sat_v(sig(q-q^d)^{\alpha_1})-l_2sat_v(sig(\dot{q})^{\alpha_2}) \quad (19)$$

with $0<\alpha_1<1,\alpha_1=\frac{\alpha_2}{2-\alpha_2},l_1>0,l_2>0$ and $sat_v(\xi)=(sat(\xi_1),\dots,sat(\xi_n))^T$, where $sat(\xi_i)$ is the saturation function, i.e., $sat(\xi_i)=\xi_i$ if $|\xi_i|\le 1$, $sat(\xi_i)=1$ if $\xi_i>1$, and $sat(\xi_i)=-1$ if $\xi_i<1$, respectively. This control law is bounded since $||G||\le G_0$, and it is desirable in case of actuator saturation.

Remark 6: In practice, it is not easy to obtain accurate measurement of the velocity $\dot{q}$. Thus it is more desirable to design a control law that relies on the measurements of the position q only. Following our work for the double integrator system in (Hong *et al.* 2001), we consider the following dynamic output feedback control law:

$$\tau=G(q)-l_1sig(\zeta_1)^{\alpha_1}-l_2sig(\zeta_2)^{\alpha_2} \quad (20)$$

$$\begin{cases}\dot{\zeta}_1=\zeta_2-k_1sig(e_1)^{\sigma_1}\\ \dot{\zeta}_2=M(q)^{-1}[-G(q)-C(q,\zeta_2)\zeta_2+\tau]\\ \quad -k_2sig(e_1)^{\sigma_2}\end{cases} \quad (21)$$

where $0<\alpha_1<1,\alpha_2=\frac{2\alpha_1}{1+\alpha_1},\sigma_1=\alpha_1,\sigma_2=2\sigma_1-1$, and $e_1=\zeta_1-x_1,e_2=\zeta_2-x_2$. Under this control law, the closed loop system can be written as follows:

$$\begin{cases}\dot{x}_1=x_2\\ \dot{x}_2=f_1(x,e)-M(q^d)^{-1}\\ \quad [l_1sig(x_1+e_1)^{\alpha_1}+l_2sig(x_2+e_2)^{\alpha_2}]\\ \dot{e}_1=e_2-k_1sig(e_1)^{\sigma_1}\\ \dot{e}_2=f_2(x,e)-k_2sig(e_1)^{\sigma_2}\end{cases} \quad (22)$$

where

$$f_1=-M(x_1+q^d)^{-1}C(x_1+q^d,x_2)x_2$$
$$-\tilde{M}(q^d,x_1)[l_1sig(x_1+e_1)^{\alpha_1}+l_2sig(x_2+e_2)^{\alpha_2}]$$

and

$$f_2=-M(x_1+q^d)^{-1}[C(x_1+q^d,x_2)x_2$$
$$-C(x_1+q^d,x_2+e_2)(x_2+e_2)]$$

and $\tilde{M}(q^d,x_1)$ is as defined in Theorem 2. Associated with (22) is the following system

$$\begin{cases}\dot{x}_1=x_2\\ \dot{x}_2=-M(q^d)^{-1}[l_1sig(x_1+e_1)^{\alpha_1}\\ \quad +l_2sig(x_2+e_2)^{\alpha_2}]\\ \dot{e}_1=e_2-k_1sig(e_1)^{\sigma_1}\\ \dot{e}_2=-k_2sig(e_1)^{\sigma_2}\end{cases} \quad (23)$$

It can be easily verified that (23) is homogeneous.

Theorem 3: Assume the solutions of of (22) and (23) are unique in forward time, then the origin of (22) is locally finite-time stable for any positive constants l_1, l_2, k_1, and k_2.

The proof is a combination of the proof of Theorem 2 here and the proof of Proposition 2 in (Hong *et al.* 2001). We omit the proof for avoiding redundancy.

4. SIMULATIONS

In the section, we will show the performance of the finite-time controllers using a two-link robotic manipulators moving in a plane by Matlab 5.2.

Consider a robot system with two prismatic joints of form (8) with

$$M(q) = \begin{bmatrix} (m_1 + m_2) & 0 \\ 0 & m_2 \end{bmatrix},$$

$C = 0$, and

$$G(q) = g \begin{bmatrix} 0 \\ m_2 q_2 \end{bmatrix}$$

where g is the acceleration of gravity, $m_1 = 18.8$, and $m_2 = 13.2$.

Our controller takes the following general form

$$\tau = G - (k_1 |q_1 - q_1^d|^{\alpha_1} sgn(q_1 - q_1^d) + k_2 |\dot{q}_1|^{\alpha_2} sgn(\dot{q}_1),$$

$$k_3 |q_2 - q_2^d|^{\alpha_1} sgn(q_2 - q_2^d) + k_4 |\dot{q}_2|^{\alpha_2} sgn(\dot{q}_2))^T \quad (24)$$

where $0 < \alpha_1 < 1$, $\alpha_2 = 2\alpha_1/(1+\alpha_1)$, and $k_i > 0, i = 1,2,3,4$. It is not difficult to obtain that the solution of the closed loop system is unique in forward time with Proposition 2.2 and its remark in (Kawski 1989).

This controller includes the conventional gravity-compensated asymptotic controller as a limiting case with $\alpha_1 = \alpha_2 = 1$. The performance of the controller is defined by two sets of parameters, namely, the PD gains (k_1, k_2, k_3, k_4), and an additional parameter α_1. For comparison, we select two different sets of PD gains and three different α_1's for simulations. First we take $k_1 = 400, k_2 = 300, k_3 = 120$, and $k_4 = 100$ with $\alpha_1 = 1/3, 1/4, 1/5, 1$, respectively. The step responses of the positions of the first link, and second link are shown in Figures 1 and 2, respectively. In both figures, the star line, solid line, dotted line, and dashed line correspond to the controller with $\alpha_1 = 1/2, 1/3, 1/4$ and 1, respectively. As expected, the curve corresponding to $\alpha_1 = 1$ (hence $\alpha_2 = 1$) is the response by an asymptotic regulator while the other three curves exhibit the behaviors of finite time convergence. Due to the finite time convergence property, responses of all the three finite time controllers are clearly faster than the asymptotic controller. The characterization of the quantitative relation between the settling time and parameter α_1 is interesting, and worth further investigation. Figures 3 and 4 repeat the experiment shown in Figures 1 and 2 with $k_1 = 220, k_2 = 200, k_3 = 80$, and $k_4 = 70$. The conclusion is consistent with the first experiment.

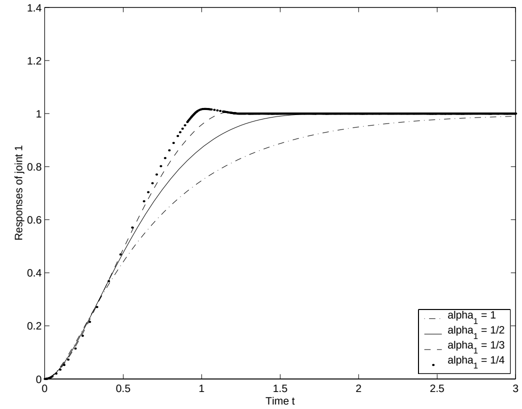

Fig. 1. Time response of q_1 under regulators

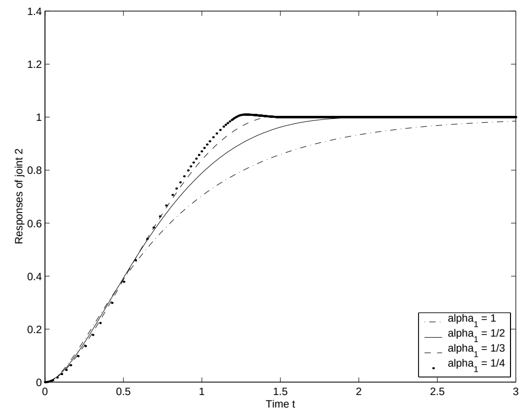

Fig. 2. Time response of q_2 under regulators

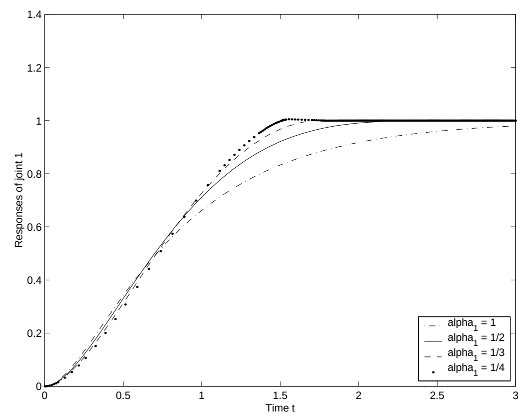

Fig. 3. Time response of q_1 under regulators

5. CONCLUSIONS

This paper has studied the problem of the finite-time regulation of robot systems. The research has offered an alternative approach for improving the design of the robot regulator. Also simulation shows that the control law can achieve faster response than conventional PD control law. This virtue may be attributed to the extra parameters in the control law.

Two issues have not been adequately addressed in this paper, namely, the relation of values of the controller parameter to the settling time, and the establishment of the uniqueness of the solution in forward time of

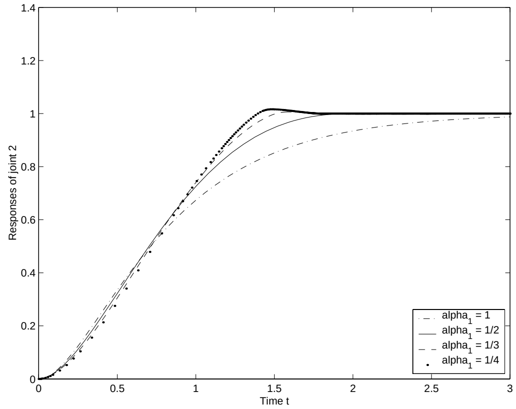

Fig. 4. Time response of q_2 under regulators

the closed-loop system (14). As mentioned in Remark 4, the answer to the first issue relies on the explicit construction of the Lyapunov function. It is possible to find an explicit Lyapunov function for the closed-loop system resulting from the first controller, but it does not seem to be easy to do so for the closed-loop system resulting from the second controller since the closed-loop system is not homogeneous. As for the second issue, we have given a complete analysis of the stability property of the closed-loop system resulting from the first controller. However, for the closed-loop system resulting from the second controller, our results rely on the assumption of the uniqueness of the solution in forward time. The uniqueness condition may be verified for robot systems with prismatic joints, but may not be guaranteed for more general robot systems. Thus this issue remains to be an unsolved and challenging one.

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