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BEST TRACKING AND REGULATION PERFORMANCE UNDER CONTROL EFFORT CONSTRAINT: TWO-PARAMETER CONTROLLER CASE

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Abstract: This paper studies optimal tracking and regulation control problems by two-parameter controllers, in which objective functions of tracking error and regulated response, defined by integral square measures, are to be minimized jointly with the control effort, where the latter is measured by the plant input energy. The approach here is similar to our recent study for one-parameter controller case, and the problems are solved explicitly by deriving analytical expressions for the best achievable performance. Besides the plant non-minimum phase zeros, time delays, and unstable poles, the results reveal and quantify how the gain characteristics of the plant may all affect the performance. These effects are nonexistent when the control effort is not taken into account.

Keywords: performance limitation, $\mathcal{H}_2$ control, two-parameter feedback system, unstable poles and zeros

1. INTRODUCTION

In recent years there has been growing attention devoted to the studies of intrinsic performance limits achievable by feedback control (see, e.g., (Seron, Braslavsky and Goodwin, 1997; Chen, 2000) and the references therein). Two of the well-studied problems are the optimal reference tracking and optimal regulation problems (Qiu and Davison, 1993; Chen, Toker and Qiu, 2000; Seron, et.al., 1999; Qiu and Chen, 1998). It has been known that the minimal tracking error depends upon the non-minimum phase zeros and time delays in the plant, while the minimal regulation energy depends upon the plant unstable poles.

It should be recognized that the performance criteria as alluded to above are highly idealistic, and thus serve more appropriately as an ideal, theoretical bound. Indeed, for example in the optimal tracking problems, in order to attain the minimal tracking error, the input to the plant is often required to have an infinite energy. This, of course, is seldom possible in practice. The consideration leads us to study the best achievable performances when only finite input energy is available.

Our primary motivation in this work is twofold. First, not only are the problems practically more relevant and generically more meaningful, but they in fact find rather pertinent applications in the design of mechanical systems (Hara and Naito, 1998; Iwasaki, Hara and Yamauchi, 2000). Next, our investigation is also driven by a deeper goal, in hope of discovering control constraints and limitations imposed by other sources than non-minimum phase zeros, unstable poles, and time delays. We maintain that this concern may receive an answer only when more practical performance goals are taken into consideration, specifically when conflicting design objectives are to be considered jointly.

Along this direction, the authors investigated an optimal tracking control problem in which not only the step error response, but also the plant input energy, both quantified under a square-integral or an $\mathcal{H}_2$ measure, is penalized (Chen, Hara and Chen, 2001). The aim was not to give a numerical solution but to provide explicit analytical expressions for the best achievable performance. The results demonstrated in all cases how the best achievable performance may depend on the plant gain in the entire frequency range, and in particular help explain how the bandwidth, the lightly damped poles and the anti-resonant zeros of the plant may limit the achievable performance. The results thus unraveled and quantified analytically yet another source of intrinsic feedback constraints, which may not be observed in the "single-objective" control design problems, such as the standard tracking and regulation problems, the Bode and Poisson integrals, and the standard sensitivity and complementary sensitivity minimization problems.

However, the development has been restricted to tracking and regulation using a one-parameter controller, for marginally stable and minimum phase plants respectively. This paper studies more general plants which may be both unstable and non-minimum phase. It is known that a two-parameter control structure is superior when tracking or regulation constitutes the sole design objective, specifically in countering the effect of plant unstable poles on tracking, and that of plant non-minimum phase zeros on regulation (Chen, Toker and Qiu, 2000; Qiu and Chen, 1998). The purpose of this paper is to investigate whether it will also improve tracking and regulation performance when control effort is taken into consideration, and whether it may offer any advantage in circumventing the performance constraints imposed by the plant gain when we use two-parameter controllers.

Section 2 and Section 3 are respectively devoted to the tracking and regulation problems, where the same analytical expressions as in the corresponding one-parameter cases are presented for the best achievable control performance under reasonable mild assumptions of the plant. A closed form solution for a related constraint problem is also shown. The highlight of the paper is in Section 4. We will consider a joint performance objective, where tracking to reference input and regulation to disturbance input are simultaneously imposed. Surprisingly, we can get an analytical expression for the best achievable performance. All the proofs of the results in this paper are omitted due to the page limitation.

Notation: Denote the open right half plane by $\mathbb{C}_+$. For any complex number z, denote its complex conjugate by $\overline{z}$. For any signal $u(t)$, we denote its Laplace transform by $\widehat{u}(s)$. The transpose and conjugate transpose of a matrix A are denoted by A^T and A^H, its largest singular values by $\overline{\sigma}(A)$, and its smallest eigenvalue by $\underline{\lambda}(A)$. For a pair of nonzero vectors w and v, we define the principal angle $\angle(w,\ v)$ between their directions by $\cos\angle(w,\ v) := |w^H v|/(\|w\|\|v\|)$. Moreover, let $\|\cdot\|$ denote the Euclidean vector norm. and the $\mathcal{L}_2$ norm is defined as

$$\|f\|_2 := \left(\frac{1}{2\pi}\int_{-\infty}^{\infty} f^H(j\omega)g(j\omega)d\omega\right)^{\frac{1}{2}}.$$

2. OPTIMAL TRACKING PERFORMANCE

2.1 *Problem Formulation*

We consider a feedback system with two-parameter controller shown in Figure 1, where the control input u accesses the reference input r and the output y via two separately designed controllers K_1 and K_2, by

$$\widehat{u} = K_1\widehat{r} + K_2\widehat{y}.$$

For the plant transfer function matrix P, let its right and left coprime factorizations be given by

$$P = NM^{-1} = \tilde{M}^{-1}\tilde{N}, \tag{1}$$

where $N,\ M,\ \tilde{N},\ \tilde{M} \in \mathbb{R}\mathcal{H}_\infty$ and satisfy the double Bezout identity for some $X,\ Y,\ \tilde{X},\ \tilde{Y} \in \mathbb{R}\mathcal{H}_\infty$,

$$\begin{bmatrix} \tilde{X} & -\tilde{Y} \\ -\tilde{N} & \tilde{M} \end{bmatrix}\begin{bmatrix} M & Y \\ N & X \end{bmatrix} = I. \tag{2}$$

Then the set of all stabilizing two-parameter compensators is characterized by (Vidyasagar, 1985)

$$\begin{aligned}\mathcal{K}_2 := \{K:\ K = &\begin{bmatrix} K_1 & K_2 \end{bmatrix} \\ &= (\tilde{X} - R\tilde{N})^{-1}\begin{bmatrix} Q & \tilde{Y} - R\tilde{M} \end{bmatrix}, \\ &Q \in \mathbb{R}\mathcal{H}_\infty,\ \ R \in \mathbb{R}\mathcal{H}_\infty\}. \end{aligned} \tag{3}$$

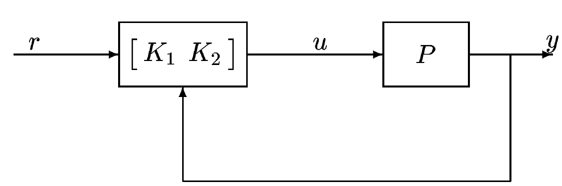

Fig. 1. Two-parameter tracker

The reference input r is assumed to be a step type function which is represented by

$$r(t) = \begin{cases} v & t \geq 0 \ ; \quad \|v\| = 1 \\ 0 & t < 0 \end{cases} , \qquad (4)$$

where the unitary vector v specifies the input direction. We adopt the integral square criterion (for $0 \leq \epsilon \leq 1$)

$$J := (1-\epsilon) \int_0^\infty \|y(t) - r(t)\|^2 dt + \epsilon \int_0^\infty \|u(t)\|^2 dt, \qquad (5)$$

as our performance measure. Here, ϵ may be used to weight the relative importance of tracking objective versus that of constraining the input energy. For any given r, we want to determine the optimal performance achievable by all stabilizing two-parameter compensators,

$$J^* := \inf_{K \in \mathcal{K}_2} J.$$

In the limiting case, when $\epsilon = 0$, J^* defines the minimal tracking error with no regard to input energy (Chen, Toker and Qiu, 2000). For $\epsilon = 1$, it reduces to an optimal energy regulation problem (Qiu and Chen, 1998).

Via a routine manipulation, it is easy to find that $\hat{y} = NQ\hat{r}$ and $\hat{u} = MQ\hat{r}$. Hence, the tracking performance measure can be written as

$$J = (1-\epsilon)\|(I - NQ)\hat{r}\|_2^2 + \epsilon\|MQ\hat{r}\|_2^2 = \left\| \begin{bmatrix} \sqrt{1-\epsilon}(I - NQ) \\ \sqrt{\epsilon}MQ \end{bmatrix} \hat{r} \right\|_2^2 .$$

Hence, the optimal performance becomes

$$J^* := \inf_{K \in \mathcal{K}_2} J = \inf_{Q \in \mathbb{R}\mathcal{H}_\infty} J.$$

It should be emphasized that our purpose here is not to give a numerical solution for the problem but to provide explicit analytical expressions for the best achievable performance J^*, because the problem is an $\mathcal{H}_2$ optimal control problem and hence it can be solved numerically using the well known Riccati or LMI solution.

2.2 *Main Results*

In order to derive the main results, the following assumptions are enforced.

Assumption 1: $P(s)$ is right-invertible and it has no zero at $s = 0$.

Assumption 2: $P(s)$ has a pole at $s = 0$.

We note that while Assumption 1 is standard in step reference tracking problems, it is clear from (5) that to maintain a finite energy cost precludes the possibility that $K_2(s)$ may have an integrator. Instead, the plant $P(s)$ may have non-minimum phase zeros. For a right-invertible $P(s)$, it is well-known (see, e.g., (Seron, Braslavsky and Goodwin, 1997)) that each of its non-minimum phase zeros is also one for $N(s)$. Let $z_i \in \mathbb{C}_+$, $i = 1, \cdots, N_z$, be the non-minimum phase zeros of $P(s)$. It is possible to factorize $N(s)$ as

$$N(s) = N_m(s) \left(\prod_{i=1}^{N_z} \left[\eta_i \ U_i \right] \cdot \begin{bmatrix} \frac{\overline{z}_i}{z_i} \frac{z_i - s}{\overline{z}_i + s} & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \eta_i^H \\ U_i^H \end{bmatrix} \right), \qquad (6)$$

where $N_m(s)$ represents the minimum phase part of $N(s)$, η_i is the direction vector associated with z_i, and the columns of U_i forms a unitary matrix together with η_i.

Note also that Assumption 2 is required to keep the second term of J finite and it can be met in many cases of interest (e.g., mechanical systems (Hara and Naito, 1998; Iwasaki, Hara and Yamauchi, 2000)), where the plant contains integrators.

We need to introduce several functions before showing the main results. We first perform an inner-outer factorization

$$\begin{bmatrix} \sqrt{1-\epsilon}N_m \\ \sqrt{\epsilon}M \end{bmatrix} = \Theta_i \Theta_o, \qquad (7)$$

where $\Theta_i, \Theta_o \in \mathbb{R}\mathcal{H}_\infty$ are inner and outer matrix functions, respectively. Then define

$$f(s) := (1-\epsilon)v^T N_m(s) \Theta_o^{-1}(s) \Theta_o^{-T}(0) N_m^T(0) v \qquad (8)$$

and factorize $f(s)$ as

$$f(s) = \left(\prod_{i=1}^{N_s} \frac{\overline{s}_i(s_i - s)}{s_i(\overline{s}_i + s)} \right) f_m(s),$$

where s_i are the non-minimum phase zeros of $f(s)$ and $f_m(s)$ is minimum phase.

Theorem 1 *Under Assumption 1-2, let $z_i \in \mathbb{C}_+$, $i = 1, \cdots, N_z$, be the zeros of $P(s)$, which admit the decomposition (6). Then, the best achievable performance is given as follows:*

1) SISO system:

$$J^* = 2(1-\epsilon) \sum_{i=1}^{N_z} \frac{1}{z_i} + \frac{1-\epsilon}{\pi} \int_0^\infty \frac{1}{\omega^2} \log\left(1 + \frac{\epsilon}{(1-\epsilon)|P(j\omega)|^2}\right) d\omega. \qquad (9)$$

2) MIMO system:

$$J^* = 2(1-\epsilon)\sum_{i=1}^{N_z}\frac{1}{z_i}\cos^2\angle(\eta_i,\ v) + 2(1-\epsilon)\sum_{i=1}^{N_s}\frac{1}{s_i} + J_o, \qquad (10)$$

where

$$J_o := -\frac{2(1-\epsilon)}{\pi}\int_0^\infty \frac{\log|f(j\omega)|}{\omega^2}d\omega. \qquad (11)$$

Furthermore,

$$J_o \geq \frac{1-\epsilon}{\pi}\int_0^\infty \frac{1}{\omega^2}\log\left(1+\frac{\epsilon}{(1-\epsilon)\overline{\sigma}^2(P(j\omega))}\right)d\omega.$$

Theorem 1 demonstrates that unlike in the standard tracking problem, the optimal performance herein also depends on the minimum phase part of the plant. This effect is captured by the the second term in (9) and the second and the third terms in (10), which are all nonnegative. The result is exactly same as that for the one-parameter controller case in (Chen, Hara and Chen, 2001), where the plant is assumed to have no unstable poles except at $s = 0$. The advantage of using two-parameter controller is to remove such an assumption. Assumptions 1 and 2 correspond to the necessary condition for the problem having a finite optimal value, and hence they are not restrictive. Consequently, Theorem 1 really provides a fundamental tracking performance limitation in practice.

Next, we will consider a constraint optimization problem instead of unconstraint optimization problem with performance index (5). The problem is defined as follows:

$$\inf_{K(s)\in\mathcal{K}_2} \|\widehat{e}\|_2^2 := \int_0^\infty \|e(t)\|^2 dt$$
$$\textit{subject to } \|\widehat{u}\|_2^2 := \int_0^\infty \|u(t)\|^2 dt \ \leq\ \gamma_u$$

and the optimal cost is denoted by $J_e^*(\gamma_u)$.

We can derive a closed form solution of the problem for SISO plants using Theorem 1.

Theorem 2 *Consider a SISO plant $P(s)$ and let a positive number $\gamma_u < \gamma_u^*$ be given, where*

$$\gamma_u^* := \frac{1}{\pi}\int_0^\infty \frac{1}{\omega^2|P(j\omega)|^2}d\omega. \qquad (12)$$

Then, under Assumption 1-2,

$$J_e^*(\gamma_u) = 2\sum_{i=1}^{N_z}\frac{1}{z_i} - \alpha^*\gamma_u + \frac{1}{\pi}\int_0^\infty \frac{1}{\omega^2}\log\left(1+\frac{\alpha^*}{|P(j\omega)|^2}\right)d\omega. \qquad (13)$$

where α^* is the unique positive solution of

$$\int_0^\infty \frac{1}{\omega^2}\cdot\frac{1}{\alpha+|P(j\omega)|^2}d\omega = \pi\gamma_u. \qquad (14)$$

Moreover,

$$J_e^*(\gamma_u) = J_e^*(\gamma_u^*) = 2\sum_{i=1}^{N_z}\frac{1}{z_i}\ ,\quad \forall\gamma_u \geq \gamma_u^*. \qquad (15)$$

There has been no analytical expression for the solution, and hence Theorem 2 is quite new. Since α^* only depends on the gain of the plant, we can see a nice separation property in the expression of $J_e^*(\gamma_u)$. The first term caused by non-minimum phase zeros of the plant is irrelevant to the choice of γ_u, while the extra two terms are completely determined by $|P(j\omega)|$ and γ_u.

3. OPTIMAL REGULATION PERFORMANCE

In this section we formulate and solve an output regulation problem under finite control effort constraint. To this end, we consider a general two-parameter control scheme as depicted in Figure 2, where the control input u is generated by processing on the output y and the disturbance d, yielding

$$\widehat{u} = K_1\widehat{d} + K_2\widehat{y}.$$

Here we take d to be the impulse signal

$$d(t) = \zeta\delta(t)\ ;\ \ \|\zeta\| = 1, \qquad (1)$$

where ζ is a constant unitary vector, which may be interpreted either as a nonzero initial condition of the system or more generally as a disturbance signal entering at the plant input.

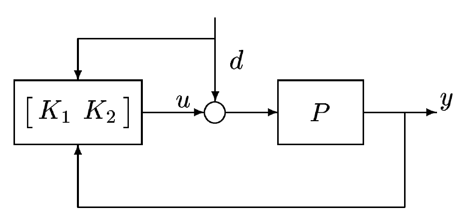

Fig. 2. Two-parameter regulator

The problem of interest is to regulate the zero-input response y to zero, by the design of the compensator $K = [K_1, K_2]$. We adopt the integral square energy criterion

$$E := (1-\epsilon)\int_0^\infty \|y(t)\|^2 dt + \epsilon\int_0^\infty \|u(t)\|^2 dt. \qquad (2)$$

With the controller parameterized by (3), the signals y and u are found respectively as

$$\widehat{y} = N\left(Q + \tilde{X} - R\tilde{N}\right)\widehat{d},$$
$$\widehat{u} = -\widehat{d} + M\left(Q + \tilde{X} - R\tilde{N}\right)\widehat{d}.$$

Let $Q_0 := Q + \tilde{X} - R\tilde{N} \in \mathbb{R}\mathcal{H}_\infty$. The regulation performance objective can then be expressed as

$$E = (1-\epsilon)\|\widehat{y}\|_2^2 + \epsilon\|\widehat{u}\|_2^2 = \left\| \begin{bmatrix} \sqrt{1-\epsilon} N Q_0 \\ \sqrt{\epsilon}(I - MQ_0) \end{bmatrix} \widehat{d} \right\|_2^2.$$

The best attainable performance then given by

$$E^* := \inf_{K\in\mathcal{K}_2} E = \inf_{Q_0\in\mathbb{R}\mathcal{H}_\infty} E.$$

In order to derive the result for the regulation problem, the following assumption is enforced.

Assumption 3: $P(s)$ is left-invertible and strictly proper.

This assumption implies that $y(0)$ is finite, a necessary condition for the output energy to be finite. We note that such an assumption is not required in the standard energy regulation problem (Qiu and Chen, 1998) ($\epsilon = 1$) and that unstable poles of the plant play a key role to derive the best achievable performance.

Let $P(s)$ have poles $p_i \in \mathbb{C}_+$, $i = 1, \cdots, N_p$. Note that it is always possible to factorize $M(s)$ as

$$M(s) = (\prod_{i=1}^{N_p} \begin{bmatrix} w_i & W_i \end{bmatrix} \begin{bmatrix} \frac{s-p_i}{s+\overline{p}_i} & 0 \\ 0 & I \end{bmatrix} \cdot \begin{bmatrix} w_i^H \\ W_i^H \end{bmatrix}) M_m(s), \quad (3)$$

where $M_m(s)$ is minimum phase, w_i is the direction vector associated with p_i and together with the columns of W_i forms a unitary matrix.

We now perform an inner and outer factorization

$$\begin{bmatrix} \sqrt{1-\epsilon} N \\ -\sqrt{\epsilon} M_m \end{bmatrix} = \Lambda_i \Lambda_o.$$

Then we can define

$$g(s) := \epsilon\zeta^T M_m(s)\Lambda_o^{-1}(s)\Lambda_o^{-T}(\infty)M_m^T(\infty)\zeta \quad (4)$$

and factorize $g(s)$ as

$$g(s) = \left(\prod_{i=1}^{N_s} \frac{s-s_i}{s+\overline{s}_i}\right) g_m(s),$$

where s_i are the non-minimum phase zeros of $g(s)$ and $g_m(s)$ is minimum phase.

Those functions lead to the result for the regulation problem, which is consistent with the result for the case where a non-minimum phase plant is controlled by a one-parameter controller.

Theorem 3 *Under Assumption 3, let $p_i \in \mathbb{C}_+$, $i = 1, \cdots, N_p$, be the poles of $P(s)$, which admit the decomposition (3). Then, the best achievable performance is given as follows:*

1) SISO system:

$$E^* = 2\epsilon \sum_{i=1}^{N_p} p_i + \frac{\epsilon}{\pi}\int_0^\infty \log\left(1 + \frac{1-\epsilon}{\epsilon}|P(j\omega)|^2\right) d\omega. \quad (5)$$

2) MIMO system:

$$E^* = 2\epsilon \sum_{i=1}^{N_p} p_i \cos^2 \angle(w_i,\ \zeta) + 2\epsilon \sum_{i=1}^{N_s} s_i + E_o, \quad (6)$$

where

$$E_o := -\frac{2\epsilon}{\pi}\int_0^\infty \log|g(j\omega)| d\omega. \quad (7)$$

Furthermore,

$$E_o \geq \frac{\epsilon}{\pi}\int_0^\infty \log\left(1 + \frac{1-\epsilon}{\epsilon}\underline{\lambda}\left[P^H(j\omega)P(j\omega)\right]\right) d\omega.$$

Theorems 3 shows that the regulation performance also hinges closely on the plant gain and bandwidth, other than its unstable poles. However, a clear distinction exists between the tracking and the regulation problems. Unlike in the tracking problem, a large plant gain is seen to be undesirable for regulation.

4. TRACKING AND REGULATION

While two-parameter controllers do offer an distinctive advantage for tracking and regulation, their effect is limited to only non-minimum phase zeros, time delays, and unstable poles in the two respective problems. Theorems 1 and 3 make it clear that the dependence of the tracking and regulation performance on the plant gain remains unchanged, and so do the constraints due to plant bandwidth, lightly damped poles, and anti-resonant zeros. Since two-parameter controllers constitute the most general linear feedback structure, these constraints are thus seen to pose a fundamental barrier to the achievable performance, and are intrinsic of the problems which both attempt to minimize conflicting design objectives.

In the final contribution of this paper, we extend the above results to a joint performance objective that addresses reference tracking and energy regulation in the presence of disturbance signals. Thus, consider the two-parameter system given in Figure 3.

We adopt the performance criterion (for $0 \leq \epsilon \leq 1$)

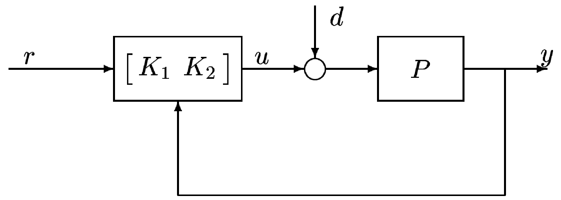

Fig. 3. Two-parameter tracker with disturbance

$$H := (1-\epsilon)\int_0^\infty \|y(t)-r(t)\|^2 dt + \epsilon \int_0^\infty \|u(t)\|^2 dt,$$

and study the optimal performance

$$H^* := \inf_{K \in \mathcal{K}_2} H.$$

The problem is a so-called "4-block $\mathcal{H}_2$ optimal control problem", where we have two exogenous inputs, r and d, and two output signals, $e = r - y$ and u, to be evaluated.

Note that H^* differs from J^* and E^*, due to the presence of r and d simultaneously. We shall assume that $P(s)$ is a square invertible transfer function matrix; this stronger assumption arises for to track the reference signal r, the plant is required to be right-invertible, while to counter the disturbance signal d, it needs to be left-invertible.

Theorem 4 *Let r and d be given by (4) and (1), respectively, and suppose that $P(s)$ satisfies Assumption 1-3. Also suppose that $P(s)$ has zeros $z_i \in \mathbb{C}_+$, $i = 1, \cdots, N_z$ and poles $p_i \in \mathbb{C}_+$, $i = 1, \cdots, N_p$, which admit the decompositions of (6) and (3). Then, we have*

$$H^* = J^* + E^* + A, \tag{8}$$

where

$$A = -2\epsilon(1-\epsilon)v^T N_m(0)\Delta_o^{-1}(0)\Delta_o^{-T}(\infty)M_m^T(\infty)\zeta$$

and Δ_o is the outer factor in the inner-outer factorization

$$\begin{bmatrix} \sqrt{1-\epsilon}N_m \\ \sqrt{\epsilon}M_m \end{bmatrix} = \Delta_i \Delta_o.$$

The theorem clearly represents the best achievable performance, which consists of J^*, E^* and a term related to the coupling affect of two different exogenous signals r and d. Note that the extra term A might be negative depending on the relative direction of v and ζ. More investigation is needed to have a nice physical implication of the term A.

5. CONCLUSION

In this paper we have studied $\mathcal{H}_2$-type optimal tracking and regulation control problems, which attempt to minimize jointly the tracking error and plant input energy, and the output response along with input energy, respectively. The results enable us to conclude that under finite control effort, the best tracking and regulation performance both depend on the plant gain in the entire frequency range, and consequently both can be significantly affected by the plant bandwidth. Furthermore, the results help clarify the roles of the gain characteristics of the plant, which have been seen to have a direct pertinence on the performance.

The first contribution of the paper is to show the results for the one-parameter controller case hold even if the plant may have unstable poles and/or zeros, The second contribution is to derive a closed form solution for a related constraint optimization problem: error minimization subject to control input effort constraint. The last contribution is to provide an analytical expression for the best achievable joint performance of tracking and regulation. These results made the fundamental control performance limitations under control effort constraint quite clear.

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