15th Triennial World Congress, Barcelona, Spain

FINITE FREQUENCY CHARACTERIZATION OF EASILY CONTROLLABLE MECHANICAL SYSTEMS UNDER CONTROL EFFORT CONSTRAINT

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Abstract: This paper is concerned with the development of a new approach for integrated design of controlled mechanical systems. The contribution is three fold. We first show through a typical design example that the closed-loop bandwidth achievable with a reasonable control effort is shown to be closely related to the frequency range for which the plant is high gain and exhibits positive-realness when high gain controllers are not allowed. Secondly, we present matrix inequality characterizations of the robust finite frequency positive-real property and the finite frequency high gain property. Thirdly, we propose a systematic method for designing mechanical systems to achieve the finite frequency properties. Then, the validity is confirmed by the smart structure design using piezo-electric films.

Keywords: structure/control design integration, finite frequency property, mechanical system, matrix inequality

1. INTRODUCTION

This paper is concerned with finite frequency characterization of easily controllable mechanical systems under control effort constraint for aiming at the development of a new approach for integrated design of controlled mechanical systems. Given a set of performance specifications, the mechanical control system design mainly consists of the following two steps: the mechanical structure design and the controller design. Conventionally, these two steps are followed *sequentially*; the structure is first designed to meet requirements involving stiffness, strength, weight, etc., which are not directly related to the closed-loop (i.e. controlled) dynamic performance, and a controller is then designed for a *given* mechanical system. Thus, the conventional method does not provide optimal solutions in general, and hence there is room for sensible design strategies to improve the overall performance.

There are many research results along this direction. Recently, the simultaneous design methods have been proposed (Kajiwara and Nagamatsu, 1990; Sultan and Skelton, 1997; Grigoriadis, Zhu and Skelton, 1996; Grigoriadis and Wu, 1997; Onoda and Haftka, 1987) via numerical optimization of the parameters of both structure and controller. Although the methods may improve the performance of a given mechanical control system used as the initial condition of the optimization algorithm, the resulting dynamical system depends heavily on the initial design and can be far from the global optimum due to the non-convexity of the problem. To have a sensible initial design, we have to go back to the conventional sequential design. Thus we need a fundamental design principle that essentially eliminates the limitations of the two-step design.

Our approach is to identify and characterize properties of mechanical systems that are critical to achieving good controlled dynamic performance when an appropriate feedback loop is closed. Once we design a mechanical system with such properties, standard optimal control methods can be applied to complete the whole design process to yield controlled mechanical systems with near optimal performance.

It is widely accepted as a fact in a mechanical design community that a good controlled performance can be expected if a mechanical system is designed such that the flexible modes are "in-phase" with respect to the rigid-body mode. There is not much theoretical justification for this claim but empirical evidence is rather convincing (Ono and Teramoto, 1992). Our research is initially motivated by this claim and we have investigated implications of the in-phase property in terms of the language of the control community. Our aim here is to characterize easily controllable mechanical systems and provide control performance limitations under control effort constraint toward a new approach for the integrated design.

The contribution of this paper is three fold. In Section 2, we investigate the limits of a servo tracking performance for a typical lightly damped flexible mechanical system. This paper shows that the closed-loop bandwidth achievable with a reasonable control effort is closely related to the frequency range for which the plant is high-gain and exhibits positive-realness. In Section 3, we present an LMI characterization of the robust finite frequency positive-real (FFPR) property and matrix characterization of the finite frequency high gain property (FFHG), where those properties hold within a finite frequency interval. Finally in Section 4, we propose a systematic method for designing mechanical systems to achieve the finite frequency properties. The method is applied to a practical applications of structure/control design integration: a smart structure design, where a piezo-electric film is used as an actuator to reduce the oscillation of a single beam. The desirable profile of piezo-electric film is systematically designed to reduce the higher mode oscillation. The validity of the proposed method is confirmed by experiments.

We use the following notation. For a matrix A, A^{T}, A^*, $\overline{\sigma}(A)$ and $\underline{\sigma}(A)$ denote its transpose, complex conjugate transpose, maximum singular value and minimum singular value, respectively. For a symmetric matrix, $A > (\geq) 0$ and $A < (\leq) 0$ denote positive (semi-)definiteness and negative (semi-)definiteness.

2. CASE STUDY: WHAT LIMITS THE CONTROL BANDWIDTH?

In this section, we consider a simple servo-tracking control design problem and answer the following basic question: What open-loop property limits the closed-loop performance, in particular, the control bandwidth?

We use the control system setup depicted in Fig. 1 where $K(s)$ is the controller, r is the unit step command input, y is the plant output which we force to follow the step command, e is the tracking error, and u is the control input. The class of target plants is lightly damped flexible systems expressed as

$$P(s) = \frac{1}{s^2} + \frac{k_1}{s^2 + 2\zeta_1\omega_1 s + \omega_1^2} + \frac{k_2}{s^2 + 2\zeta_2\omega_2 s + \omega_2^2}$$

where $(\zeta_1, \omega_1) = (0.001, 10), (\zeta_2, \omega_2) = (0.001, 100)$. For fixed values of k_1 and k_2, define the optimal $\mathcal{H}_2$ tracking performance with control level γ_u by

$$\gamma_e(k_1, k_2, \gamma_u) := \min_{K(s)} \sqrt{\int_0^\infty e(t)^2 dt}$$
$$\text{subject to} \quad \sqrt{\int_0^\infty u(t)^2 dt} < \gamma_u.$$

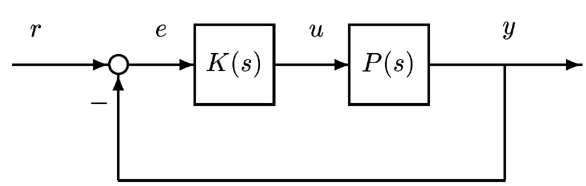

Fig. 1. Servo tracking control system setup

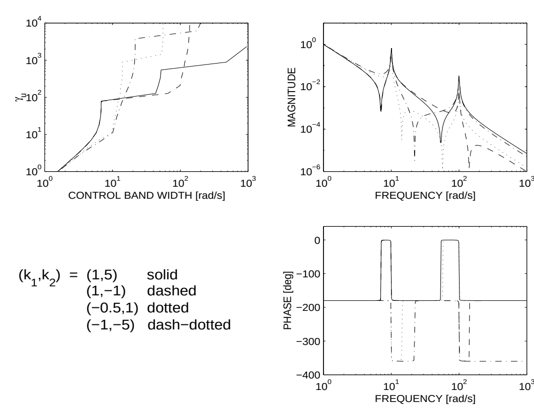

Fig. 2. What limits the control bandwidth?

Fig. 2 (upper left) shows, for the four cases of the plant parameters, the relations between γ_u and the control bandwidth of the optimal closed-loop system where the cross-over frequency of the open-loop transfer function $K(s)P(s)$ is used as an estimate of the control bandwidth. Shown to the right is the frequency response of each plant. From the figure, we see that:

- We have to put lots of control effort (γ_u) to make the control bandwidth go over the frequencies at which the phase or the gain of the plant drops, and the phase drop affect is dominant.
- If the allowable control effort is not too small nor too large, then the control bandwidth coincides roughly with the smallest frequency at which the phase of the open-loop transfer function goes below -180 [deg].

From these observations, we conclude that we should design the mechanical system such that its phase does not go below -180 [deg] and its gain is larger than a certain value within the desired control bandwidth. In other words, for $G(s) := sP(s)$, we should require the following finite frequency positive-real (FFPR) property:

$$G(j\omega) + G(j\omega)^* \geq 0, \quad {}^{\forall}\omega \ |\omega| \leq \varpi, \qquad (1)$$

where ϖ is the desired control bandwidth. For $P(s)$ itself, the following finite frequency high-gain (FFHG) property

$$\underline{\sigma}(P(j\omega)) > \gamma, \quad {}^{\forall}\omega \ |\omega| \leq \varpi \qquad (2)$$

is required. The importance of FFPR property has been already pointed out by the authors (Iwasaki and Hara, 1999), while FFHG is our new attention here.

3. CHARACTERIZATIONS OF FINITE FREQUENCY PROPERTIES

3.1 *Positive-real property*

We first give an LMI characterization of the FFPR condition(Iwasaki and Hara, 1999; Iwasaki, Meinsma and Fu, 1992).

LEMMA 1. Consider a real rational, square transfer function $G(s)$ with minimal realization $C(sI - A)^{-1}B + D$. Let a positive scalar ϖ be given. Then, $G(s)$ is FFPR with bandwidth ϖ, if and only if there exist real symmetric matrices Y and $X \geq 0$ such that

$$\begin{bmatrix} A & I \\ C & 0 \end{bmatrix} \begin{bmatrix} -X & Y \\ Y & \varpi^2 X \end{bmatrix} \begin{bmatrix} A & I \\ C & 0 \end{bmatrix}^{\mathsf{T}} \leq \begin{bmatrix} 0 & B \\ B^{\mathsf{T}} & D^{\mathsf{T}} + D \end{bmatrix}. \qquad (3)$$

The next topic is to "robustify" the finite frequency condition in Lemma 1.

Consider a linear time-invariant uncertain system

$$\begin{bmatrix} \dot{x} \\ z \\ y \end{bmatrix} = \begin{bmatrix} A & B_1 & B_2 \\ C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{bmatrix} \begin{bmatrix} x \\ w \\ u \end{bmatrix}, \qquad w = \Delta z \quad (4)$$

where all the coefficient matrices A, B_i, C_i and D_{ij} $(i = 1, 2)$ are real, and Δ is an uncertain matrix belonging to a known subset of real matrices $\mathbf{\Delta}$. Let the transfer function from u to y be denoted by $G_\Delta(s)$. We shall give a sufficient condition for the robust FFPR property:

$$G_\Delta(j\omega) + G_\Delta(j\omega)^* \geq 0, \quad \forall\, \omega \in \mathbf{\Omega}, \quad \Delta \in \mathbf{\Delta}, \quad (5)$$

which is less conservative than that in (Iwasaki, Hara and Yamauchi, 2000), where $\mathbf{\Omega}$ is defined as

$$\mathbf{\Omega} := \{\ \omega \in \mathbb{R} : \ \det(j\omega I - A) \neq 0, \ |\omega| \leq \varpi\ \}.$$

THEOREM 1. Consider the uncertain system (4) with $\Delta \in \mathbf{\Delta}$ where $\mathbf{\Delta}$ is a subset of real matrices. Assume that $D_{21} = 0$ and $\det(I - D_{11}\Delta) \neq 0$ holds for all $\Delta \in \mathbf{\Delta}$. Denote by $G_\Delta(s)$ the transfer function from u to y. Let a positive scalar ϖ be given. Then, the robust FFPR condition holds if there exist real symmetric matrices X, Y, $\Phi \in \mathbf{\Phi}$, and $\Lambda \in \mathbf{\Lambda}$ such that

$$\begin{bmatrix} \mathbf{A} & I \\ \mathbf{C} & 0 \end{bmatrix} \begin{bmatrix} -X & Y \\ Y & \varpi^2 X \end{bmatrix} \begin{bmatrix} \mathbf{A} & I \\ \mathbf{C} & 0 \end{bmatrix}^{\mathsf{T}} \leq \begin{bmatrix} \mathbf{B} & 0 \\ \mathbf{D} & \mathbf{J} \end{bmatrix} \Phi \begin{bmatrix} \mathbf{B} & 0 \\ \mathbf{D} & \mathbf{J} \end{bmatrix}^{\mathsf{T}}, \qquad (6)$$

$$X \geq \mathbf{B}_1 \Lambda \mathbf{B}_1^{\mathsf{T}} \qquad (7)$$

where

$$\mathbf{J} := [\ I \quad 0\ ]^{\mathsf{T}} \qquad (8)$$

$$\mathbf{\Phi} := \left\{ \begin{bmatrix} \Psi & 0 \\ 0 & \Pi \end{bmatrix} : \ \begin{bmatrix} \Gamma & 0 \\ 0 & \Gamma \end{bmatrix}^{\mathsf{T}} \Psi \begin{bmatrix} \Gamma & 0 \\ 0 & \Gamma \end{bmatrix} \leq 0, \ \forall\, \Gamma \in \mathbf{\Gamma} \right\}, \ \Pi := \begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix} \qquad (9)$$

$$\mathbf{\Lambda} := \{\ \Lambda : \ \Gamma^{\mathsf{T}} \Lambda \Gamma \geq 0, \ \ \forall\, \Gamma \in \mathbf{\Gamma}\ \} \qquad (10)$$

$$\mathbf{\Gamma} := \left\{ \begin{bmatrix} \Delta^{\mathsf{T}} \\ I \end{bmatrix} : \ \Delta \in \mathbf{\Delta} \right\} \qquad (11)$$

and

$$\begin{bmatrix} \mathbf{A} & \mathbf{B}_1 & \mathbf{B}_2 \\ \mathbf{C} & 0 & \mathbf{D}_2 \end{bmatrix} := \left[\begin{array}{cc|cc|c} A & 0 & 0 & B_1 & B_2 \\ C_1 & 0 & I & D_{11} & D_{12} \\ \hline C_2 & 0 & 0 & 0 & D_{22} \\ 0 & I & 0 & 0 & 0 \end{array}\right],$$

$$\begin{bmatrix} \mathbf{B} \\ \mathbf{D} \end{bmatrix} := \left[\begin{array}{cc} \mathbf{B}_1 & \mathbf{A}\mathbf{B}_1 \ \ \mathbf{B}_2 \\ \hline 0 & \mathbf{C}\mathbf{B}_1 \ \ \mathbf{D}_2 \end{array}\right]. \qquad (12)$$

Conditions (6) and (7) are both LMIs in the variables P, Q, Φ, and Λ, and hence are suitable for numerical computation. However, constraints $\Phi \in \mathbf{\Phi}$ and $\Lambda \in \mathbf{\Lambda}$ are characterized by infinitely many inequalities and make the numerical verification of robust finite frequency condition (5) difficult. To make it tractable, inner approximations of the sets $\mathbf{\Phi}$ and $\mathbf{\Lambda}$ may be used. In particular, when $\mathbf{\Delta}$ is the set of diagonal matrices with real uncertain parameters of bounded magnitude on the diagonal, such inner approximations can be given by the *D-G* scaling (Fan, Tits and Doyle, 1991) or the LFT scaling (Asai, Hara and Iwasaki, 2000). The former results in a sufficient condition for (5) that can be checked in polynomial time, while the latter improves conservatism of the former at the expense of more computation. See (Iwasaki and Shibata, 1992) for the details.

We consider the following situation in order to apply the above matrix inequality condition to the structure/control design integration. If B_2, D_{12} and D_{22}, or $\mathbf{B}_2$ and $\mathbf{D}_2$, only depend affinely on the mechanical design parameter θ, then conditions (6) and (7) are LMIs in terms of X, Y, Φ, Λ, and θ. Therefore, the design parameter θ can be found efficiently The situation includes actuator location problems (the dual situation relates to sensor location problems). This property will be used in the design of a swing-arm positioning mechanism in Section 4.1.

3.2 *High-gain property*

This subsection is devoted to matrix inequality characterization for finite frequency heigh-gain (FFHG) property. We first note that the FFHG condition (2) is equivalent to

$$\bar{\sigma}(P^{-1}(j\omega)) < 1/\gamma, \quad {}^{\forall}\omega\ |\omega| \le \varpi. \qquad (13)$$

Hence, we need to introduce the descriptor form expression for the inverse of a square transfer function

$$P(s) = C_p(sI - A_p)^{-1}B_p + D_p$$

which is given by

$$P(s)^{-1} = \hat{P}(s) := \hat{C}(s\hat{E} - \hat{A})^{-1}\hat{B} + \hat{D}$$

where

$$\hat{A} := \begin{bmatrix} A_p & B_p \\ C_p & D_p \end{bmatrix}, \quad \hat{B} := \begin{bmatrix} 0 \\ -I \end{bmatrix},$$

$$\hat{C} := \begin{bmatrix} 0 & I \end{bmatrix}, \quad \hat{D} := 0, \quad \hat{E} := \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}$$

Using the following lemma, we can get necessary and sufficient conditions for the FFHG.

LEMMA 2. A transfer function $\tilde{G}(s)$ with descriptor representation

$$\tilde{G}(s) = \tilde{C}(s\tilde{E} - \tilde{A})^{-1}\tilde{B} + \tilde{D}$$

and $\varpi > 0$ are given. Suppose

$$\det(\lambda\tilde{E} - \tilde{A}) \ne 0, \quad {}^{\forall}\lambda \in \mathbf{C}, \ \lambda + \bar{\lambda} = 0$$

holds. Then, the following two conditions are equivalent.

(i) $\overline{\sigma}(\tilde{G}(j\omega)) < \tau, \quad {}^{\forall}\omega \ \ |\omega| \le \varpi$

(ii) ${}^{\exists}P = P^{\mathsf{T}}, \ Q = Q^{\mathsf{T}} > 0 \qquad s.t.$

$$\begin{bmatrix} \tilde{A} & \tilde{E} \\ \tilde{C} & 0 \end{bmatrix} \begin{bmatrix} -Q & P \\ P & \varpi^2 Q \end{bmatrix} \begin{bmatrix} \tilde{A} & \tilde{E} \\ \tilde{C} & 0 \end{bmatrix}^{\mathsf{T}} + \begin{bmatrix} \tilde{B}\tilde{B}^{\mathsf{T}} & \tilde{B}\tilde{D}^{\mathsf{T}} \\ \tilde{D}\tilde{B}^{\mathsf{T}} & \tilde{D}\tilde{D}^{\mathsf{T}} - \tau^2 I \end{bmatrix} < 0.$$

THEOREM 2. Consider a real rational, square transfer function $P(s)$ with minimal realization $C_p(sI - A_p)^{-1}B_p$. Let a positive scalar ϖ be given. Then, $P(s)$ is FFHG with bandwidth ϖ, if and only if one of the following two conditions holds.

(i) ${}^{\exists}P_{11} = P_{11}^{\mathsf{T}}, \quad P_{12}, \quad Q := \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{12}^{\mathsf{T}} & Q_{22} \end{bmatrix} = Q^{\mathsf{T}} > 0 \qquad$ s.t.

$$\begin{bmatrix} A_p & B_p & I \\ C_p & 0 & 0 \\ 0 & I & 0 \end{bmatrix} \begin{bmatrix} -Q_{11} & -Q_{12} & P_{11} \\ -Q_{12}^{\mathsf{T}} & -Q_{22} & P_{12}^{\mathsf{T}} \\ P_{11} & P_{12} & \varpi^2 Q_{11} \end{bmatrix} \begin{bmatrix} A_p & B_p & I \\ C_p & 0 & 0 \\ 0 & I & 0 \end{bmatrix}^{\mathsf{T}} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & -(1/\gamma^2)I \end{bmatrix} < 0 \qquad (14)$$

(ii) ${}^{\exists}P_{11} = P_{11}^{\mathsf{T}}, \quad S_{23}, \quad Q := \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{12}^{\mathsf{T}} & Q_{22} \end{bmatrix} = Q^{\mathsf{T}} > 0 \qquad$ s.t.

$$\begin{bmatrix} -Q_{22} & -Q_{22}B^{\mathsf{T}} & 0 & 0 \\ * & \begin{matrix} -A_pQ_{11}A_p^{\mathsf{T}} + P_{11}A_p^{\mathsf{T}} + A_pP_{11} \\ -B_pQ_{12}^{\mathsf{T}}A_p^{\mathsf{T}} - A_pQ_{12}B_p^{\mathsf{T}} \\ +S_{23}^{\mathsf{T}}B_p^{\mathsf{T}} + BS_{23} + \varpi^2 Q_{11} \end{matrix} & \begin{matrix} -A_pQ_{11}C_p^{\mathsf{T}} + P_{11}C_p^{\mathsf{T}} \\ -B_pQ_{12}^{\mathsf{T}}C_p^{\mathsf{T}} \end{matrix} & -A_pQ_{12} + S_{23}^{\mathsf{T}} \\ * & * & -C_pQ_{11}C_p^{\mathsf{T}} + I & -C_pQ_{12} \\ * & * & * & -Q_{22} - (1/\gamma^2)I \end{bmatrix} < 0 \qquad (15)$$

The conditions (i) and (ii) are both LMI in terms of variables P_{11}, P_{12} and Q if the plant parameters A_p, B_p and C_p are given. Hence, the FFHG condition can be readily checked. More over, the maximal frequency for the plant having the FFPR property can be computed by solving a generalized eigenvalue problem. For this purpose the condition (i) is better than the condition (ii). However, if we want to apply the conditions to the integrated design, then the situation is different. Let us consider the case where B_p is an affine function of the design parameter θ and A_p and C_p are independent of θ. Neither the condition (i) nor (ii) is an LMI, but the matrix inequality (15) is a BMI while the matrix inequality (14) is not. Hence, the condition (ii) is much more suitable than the condition (i) from this point of view.

4. APPLICATION TO SMART ARM STRUCTURE DESIGN

This section is concerned with a smart structure design, where a piezo-electric film is used as an actuator to reduce the oscillation of a single beam depicted in Fig. 3. We assume that the region of piezo-electric film is restricted to a part of the arm illustrated in Fig. 4. If the film is attached uniformly, the Bode plot of the transfer function from the voltage of the piezo-electric film and the end position measured by a gap sensor is shown as the solid lines in Fig. 5. It is seen from Fig. 5 that the second mode is not in-phase, i.e., there is a phase drop around at 7.5×10^2 [rad/sec]. This implies that the second mode is not easy to control.

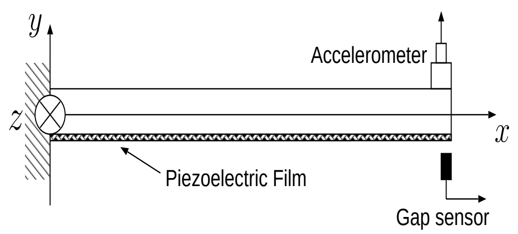

Fig. 3. Single beam arm with piezo-electric film

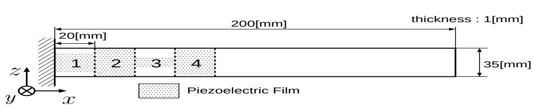

Fig. 4. Region of piezo-electric film

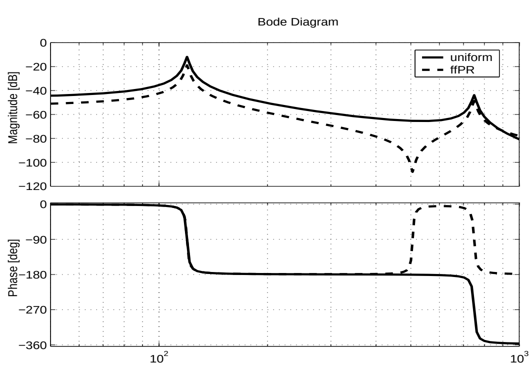

Fig. 5. Bode plots: uniform and ffPR

Consequently, our design objective is to find the desirable profile of piezo-electric film so that the related transfer function has both FFPR and FFHG properties within the higher frequency including the second mode. The design parameters are the widths of the film $\theta_1, \cdots, \theta_4$ as illustrated in Fig. 6. The design problem relates to an actuator location problem, and hence B_p only depends on the parameters affinely. Therefore, the optimal shape can be efficiently obtained by solving an LMI problem if we only consider the FFPR condition as stated in Section 3.1. The resultant profile of the film is illustrated in the left hand side of Fig. 7. We can see from the doted lines of Fig. 5 that the second mode becomes in-phase, i.e., there exists no phase drop around the second mode.

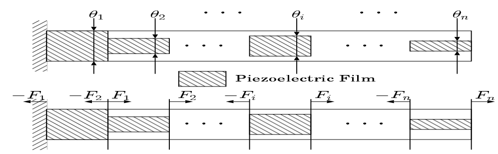

Fig. 6. Region of piezo-electric film

We then consider another design problem of which the specification includes both FFPR and FFHG conditions in order to improve the control performance. As mentioned in the remark after Theorem 2 in Section 3.2, the problem can be reduced to not an LMI but a BMI optimization problem. Therefore, the repeated process of solving LMI problems leads to a local optimal solution. A local optimal profile of the piezo-electric film for $\gamma = 10^{-3}$ is obtained as illustrated in the right hand side of Fig. 7. We can see from the Bode plot of the resultant transfer function (solid lines in Fig. 8) that our design method make the plant FFPR and the higher gain in the low frequency range can be achieved in comparison with the former design only based on the FFPR constraint shown as the dotted lines in the figure. This verifies the potential effectiveness of our approach.

The actual control performance was compared by designing H_2 optimal controllers for both ffPR and ffPR+ffHG cases. The results are shown in Fig. 9. The experimental results on the impulse responses in 10 confirmed the validity of our design method, where the dotted line corresponding to the ffPR case is worse than the solid line corresponding to the ffPR+ffHG case.

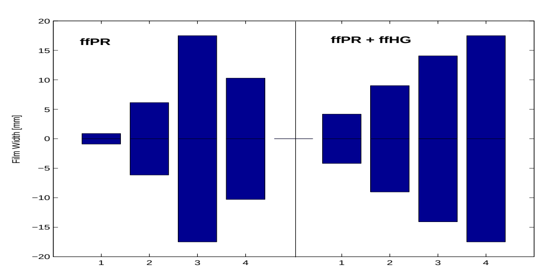

Fig. 7. Designed profiles of the film

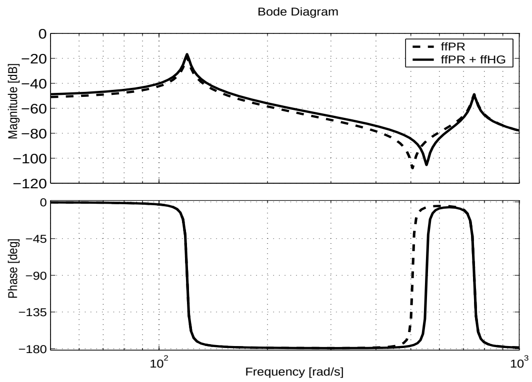

Fig. 8. Bode plots: ffPR and ffPR+ffHG

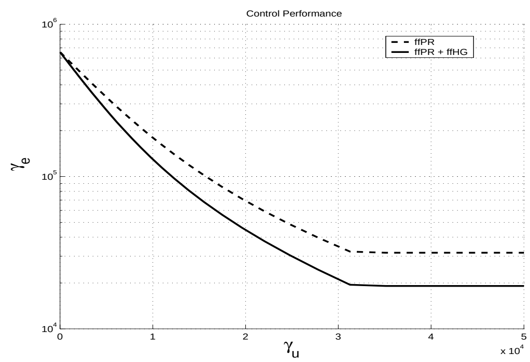

Fig. 9. Control performance

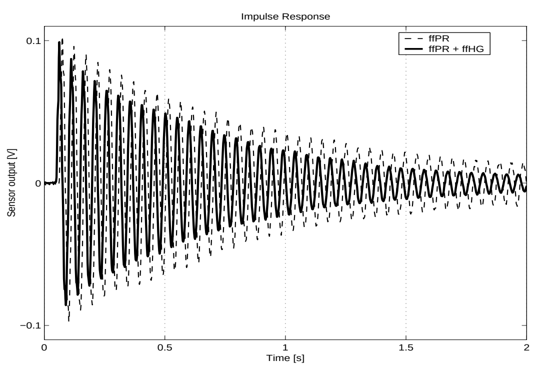

Fig. 10. Impulse responses (experiments)

5. CONCLUSION

We have shown by a servo-tracking example that certain open-loop properties – FFPR and FFHG – are critical for achieving good closed-loop performance by an optimal $\mathcal{H}_2$ controller. Then the robust FFPR property and the FFHG property have been characterized in terms of matrix inequalities. Finally, we have applied integrated design methods based on the matrix inequalities to the design of the profile of piezo-electric film for a smart structure to demonstrate applicability of our approach. The experiments confirmed the validity.

Acknowledgments: The authors gratefully acknowledge helpful discussions with H. Yamauchi and I. Kajiwara at Tokyo Institute of Technology. This research is supported in part by The Grant-in Aid for COE Research Project of Super Mechano-Systems by The Ministry of Education, Science, Sport and Culture in Japan.

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