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ON FEEDBACKS FOR POSITIVE DISCRETE-TIME SINGULAR SYSTEMS

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Abstract: In this work, N-periodic linear singular s stems are considered. Feedbacks for obtaining positi e N-periodic singular s stems are anal ed. Some properties on the existence of these feedbacks are established. The concept of holdabilit of the set $\mathbb{R}^n_+$ is used for getting a positi e trajector of the closed-loop singular s stems. This paper examines how feedbacks can be used to modif the trajector of the s stem.

Ke words: Singular control s stem,N-periodic s stem, nonnegati e matrix, positi e singular s stem, feedbacks, holdabilit .

1. INTRODUCTION

Problems in ol ing linear time- ariable singular s stems ha e recei ed attention in the last ears. Structural properties ha e been studied in (Campbell and Terrell 1991) and different feedback problems ha e been anal ed in (Wang 1994) and (Ling and Kucera 1997). A special case of time- ariable singular s stem is the N-periodic singular s stem. This kind of s stem can be rewritten as two subs stems. Related topics about N-periodic singular s stems ha e been studied in (Estruch *et al.* 1998), (Tornambe 1996) and (Sreedhar and Van Dooren 1999).

Periodic models are useful for multirate control where the measurement samples and control calculations must be performed at different frequencies. Examples in engineering applications such as electrical circuit networks, aerospace engineering and chemical processing, can be found in (De la Sen *et al.* 1987), (Sun *et al.* 1996) and (Zhu and Ling 1994). These problems can be modeled b a standard N-periodic s stem such as

$$x(k+1) = A(k)x(k) + B(k)u(k) \qquad (1)$$

where the period $N \in \mathbb{N}$, $A(k) = A(k+N) \in \mathbb{R}^{n\times n}$, $B(k) = B(k+N) \in \mathbb{R}^{n\times m}$. The s stem is called *positive* if the trajector of the s stem is nonnegati e, that is, the *state* ector $x(k) \in \mathbb{R}^n_+$ is nonnegati e when nonnegati e *control* ector $u(k) \in \mathbb{R}^m_+$ and nonnegati e initial state are considered. In this case the s stem is denoted b $(A(\cdot), B(\cdot))_N \geq 0$.

In (Bru and Hernánde 1989) it was shown that the $N-$periodic s stem is *positive,* if and onl if, the matrices $A(\cdot)$, and $B(\cdot)$ ha e nonnegati e entries, that is, $A(k) \in \mathbb{R}^{n\times n}_+$, $B(k) \in \mathbb{R}^{n\times m}_+$. In the case where the period $N = 1$, then the s stem is a *positive standard invariant system* and the abo e characteri ation is $A \geq 0$, and $B \geq 0$. This was pre iousl used for stud ing structural properties of positi e standard s stems in (Coxson and Shapiro 1987) and (Rumche and James 1989).

In the last ten ears, authors as (Valcher 1996), (Bru *et al.* 2000b), (Farina and Rinaldi 2000), and (Caccetta and Rumche 2000) ha e studied different topics of positi e standard s stems.

Positi e singular in ariant s stems ha e been studied recentl . A characteri ation for this kind of s stem is gi en in (Bru *et al.* 2000). For s s-tems without restrictions, an interesting problem is to obtain feedbacks such that the trajector of the closed-loop holds nonnegati e. This problem has been studied for standard in ariant s stem in (Berman and Stern 1987), for singular in ariant s stem in (Cantó *et al.* 2001) and for singular N-periodic s stem in (Coll *et al.* 1992) The positi it of these s stems is related with the holdabilit of $\mathbb{R}^n_+$ problem.

The process modeli ation is offen not standard (see for example the circuit network and Leontief economic model gi en in (Dai 1989)). In addition, if the process is periodic, it is gi en b a singular N-periodic s stem

$$E(k)x(k+1) = A(k)x(k) + B(k)u(k), \quad (2)$$

where for all $k \in \mathbb{Z}_+$, $E(k) = E(k+N) \in \mathbb{R}^{n\times n}$ can be singular and the matrices $A(k)$, $B(k)$, and $C(k)$ are also N-periodic as in the s stem (1).

This s stem is denoted b $(E(\cdot), A(\cdot), B(\cdot))_N \geq 0$. In the in ariant case, the s stem is gi en b ,

$$Ex(k+1) = Ax(k) + Bu(k), \quad k \in \mathbb{Z}_+, \quad (3)$$

In this paper, feedbacks for obtaining positi e singular s stems are gi en. The holdabilit of $\mathbb{R}^n_+$ problem for singular s stems is used in this wa . Some issues concerning the solution of singular s stems are studied and the relation between the existence of proportional feedbacks and the holdabilit propert is anali ed.

The pair $(E(\cdot), A(\cdot))$ is said to be a regular pair if there exists $\lambda \in \mathbb{C}$ such that $\det(\lambda E(\cdot) - A(\cdot)) \neq 0$. The set of admissible initial conditions is denoted b $\mathcal{X}_0$. The ector s stem $\{e_1, \ldots, e_n\}$ denotes the standard unit basis of $\mathbb{R}^n$.

2. POSITIVE $N-$PERIODIC SINGULAR SYSTEMS

Consider a discrete-time linear N-periodic singular s stem $(E(\cdot), A(\cdot), B(\cdot))_N$ gi en b the expression (2). In the particular case, $E(k) = I$, $k \in \mathbb{Z}_+$, the s stem $(A(\cdot), B(\cdot))_N$ is gi en b (1) and it is called standard. And, when $N = 1$, the s stem (E, A, B) is gi en b (3) and it is called in ariant. In this last case, it is well-known (see (Dai 1989) and (Kac orek 1992)) that the s stem has a solution when the pair $(E(\cdot), A(\cdot))$ is regular.

A special kind of N-periodic singular s stem $(E(\cdot), A(\cdot), B(\cdot))_N$ is gi en b

$$E(k)x(k+1) = A(k)x(k) + B(k)u(k), \quad (4)$$

where

$$E(k) = \begin{bmatrix} I & 0 \\ 0 & N(k) \end{bmatrix}, \quad A(k) = \begin{bmatrix} A_1(k) & 0 \\ 0 & I \end{bmatrix}$$
$$B(k) = \begin{bmatrix} B_1(k) \\ B_2(k) \end{bmatrix},$$

$A_1(k+N) = A_1(k) \in \mathbb{R}^{n_1\times n_1}$, $N(k+N) = N(k) \in \mathbb{R}^{n_2\times n_2}$, $B_1(k+N) = B_1(k) \in \mathbb{R}^{n_1\times m}$, $B_2(k+N) = B_2(k) \in \mathbb{R}^{n_2\times m}$, $k \in \mathbb{Z}_+$, $N \in \mathbb{Z}_+$. This s stem is said to be an *N-periodic forward-backward system* and it is an interesting control model because it can be separated in two parts: the *forward subsystem*,

$$x_1(k+1) = A_1(k)\,x_1(k) + B_1(k)u(k), \quad (5)$$

and in the *backward subsystem,*

$$N(k)x_2(k+1) = x_2(k) + B_2(k)u(k). \quad (6)$$

This fact means in (Estruch *et al.* 1998) periodic reali ations of a periodic collection of nonproper rational matrices can be obtained.

Note that, b periodicit of the coefficients matrices, some properties of the $N-$periodic s s-tems are onl need be studied at time s, $s = 0, 1, \ldots, N-1$.

The solution of this kind of s stem in ol es products of matrices. Firstl , the following matrices are defined $\phi_{A_1}(k, k_0) = A_1(k-1)A_1(k-2)\ldots A_1(k_0)$, $k > k_0$, $\phi_{A_1}(k_0, k_0) = I$, and $\psi_N(k, k_0) = N(k)N(k+1)\ldots N(k_0-1)$, $k < k_0$, $\psi_N(k_0, k_0) = I$. For $s \in \mathbb{Z}$, the matrices $A_{1s} = \phi_{A_1}(s+N, s)$ and $N_s = \psi_N(s, s+N)$ are called forward monodrom and backward monodrom matrices, respecti el .

Assuming that the monodrom matrices , N_s, $s \in \mathbb{Z}$ are nilpotent, there exists an integer $h \in \mathbb{Z}$ such that the general solution of the s stem (5)-(6) is gi en b the following expression

$$\begin{aligned} x(k) &= \begin{bmatrix} I_{n_1} \\ 0 \end{bmatrix} x_1(k) + \begin{bmatrix} 0 \\ I_{n_2} \end{bmatrix} x_2(k) = \qquad (7) \\ &= \begin{bmatrix} I_{n_1} \\ 0 \end{bmatrix} (\phi_{A_1}(k, s)x_1(s) \\ &\quad + \sum_{j=s}^{k-1} \phi_{A_1}(k, j+1)B_1(j)u(j)) \\ &\quad - \begin{bmatrix} 0 \\ I_{n_2} \end{bmatrix} \sum_{j=k}^{k+h-1} \psi_N(k, j)B_2(j)u(j), \; k \geq s. \end{aligned}$$

From the abo e expression, for each $s = 0, 1, \ldots, N-1$, the state at time s, that is for $k = s$

$$\begin{aligned} x(s) &= \begin{bmatrix} I_{n_1} \\ 0 \end{bmatrix} x_1(s) \\ &\quad - \begin{bmatrix} 0 \\ I_{n_2} \end{bmatrix} \sum_{j=s}^{s+h-1} \psi_N(s, j)B_2(j)u(j). \end{aligned}$$

Thus, the set of initial conditions $\mathcal{X}_0(s)$, $s = 0, 1, \ldots N-1$, for the s stem (2) is gi en b

$$\mathcal{X}_0(s) = \text{Im}\left[H(s), H_0(s), \cdots, H_{h-1}(s)\right],$$

where $H(s) = \begin{bmatrix} I_{n_1} & 0 \\ 0 & 0 \end{bmatrix}$ and

$$H_i(s) = \begin{bmatrix} 0 & 0 \\ 0 & I_{n_2} \end{bmatrix} \psi_N(s,i)B_2(i),$$
$$i = s, \ldots, s+h-1.$$

When the holdabilit problem for N-periodic singular s stems is studied, it is necessar to anal e the set of initial conditions and the structure of the solution. In this case, the set of admissible initial conditions for N-periodic forward-backward s stem is gi en b $\mathcal{X}_0(s)$.

It is well-known (see (Estruch *et al.* 1998)) that, for an $s \in \mathbb{Z}$, there exists a forward-backward in ariant linear s stem associated with periodic s stem (4)

$$E_s \begin{bmatrix} x_{1,s}(k+1) \\ x_{2,s}(k+1) \end{bmatrix} = A_s \begin{bmatrix} x_{1,s}(k) \\ x_{2,s}(k) \end{bmatrix} + B_s u_s(k), \; k \geq 0, \tag{8}$$

with $x_{1,s}(k) = x_1(s+kN)$, $x_{2,s}(k) = x_2(s+kN)$, $u_s(k) = \text{col}[u(s+kN), \ldots, u(s+kN+N-1)]$

$$E_s = \begin{bmatrix} I & 0 \\ 0 & N_s \end{bmatrix}, A_s = \begin{bmatrix} A_{1,s} & 0 \\ 0 & I_{n_2} \end{bmatrix},$$

$$A_{1,s} = \phi_{A_1}(s+N, s) \in \mathbb{R}^{n_1 \times n_1},$$
$$N_s = \psi_N(s, s+N) \in \mathbb{R}^{n_2 \times n_2},$$

$$B_s = \begin{bmatrix} B_{1,s} \\ B_{2,s} \end{bmatrix}$$

$$B_{1,s} = \text{row}\left[\phi_{A_1}(s+N, s+j+1)B_1(s+j)\right]_{j=0}^{N-1},$$
$$B_{2,s} = \text{row}\left[\psi_N(s, s+j)B_2(s+j)\right]_{j=0}^{N-1},$$

where $B_{1,s} \in \mathbb{R}^{n_1 \times mN}$ and $B_{2,s} \in \mathbb{R}^{n_2 \times mN}$.

For each $s = 0, 1, \ldots N-1$, the set of initial conditions $\mathcal{X}_{0,s}$ at time $k = 0$, for the (E_s, A_s, B_s) s stem (8) is gi en b

$$\mathcal{X}_{0,s} = \text{Im}\left[H_s, H_{0,s}, \cdots, H_{h-1,s}\right],$$

where $H_s = \begin{bmatrix} I_{n_1} & 0 \\ 0 & 0 \end{bmatrix}$ and

$$H_{i,s} = \begin{bmatrix} 0 & 0 \\ 0 & I_{n_2} \end{bmatrix} N_s^i B_{2,s}, i = 0, \ldots, q_s - 1,$$

where q_s is the index of nilpotence of N_s, that is $N_s^{q_s} = O$, for all $s = 0, 1, \ldots N-1$.

Remark 1. (i) If we consider the set of nilpotence indices of matrices N_s, that is

$$\{q_s, s = 0, 1, \ldots N-1\},$$

it is eas to show that there exists

$$q = \min\{q_s, s = 0, 1, \ldots N-1\} + 1$$

such that $(N_s)^q = 0$, $s = 0, 1, \ldots N-1$. We denote this index q b $\text{ind}(N_s)$. Futhermore, the number h in equation (7) is gi en b $h = qN$.

(ii) From construction of the matrices N_s and $B_{2,s}$ the following relation between the sets of initial conditions is obtained

$$\{\mathcal{X}_{0,s}, \; s = 0, 1, \ldots N-1, \}$$

associated with the in ariants s stems

$$\{(E_s, A_s, B_s), \; s = 0, 1, \ldots N-1, \}$$

and the sets of initial conditions

$$\{\mathcal{X}_0(s), \; s = 0, 1, \ldots N-1, \}$$

associated with the $N-$periodic s stem

$$(E(\cdot), A(\cdot), B(\cdot))_N.$$

Since

$$\begin{aligned} &\{N_s^i B_{2,s}, i = 0, \ldots, q-1\} \\ &= \Big\{ (\psi_N(s, s+N))^i \psi_N(s,s) B_2(s), \ldots \\ &\quad (\psi_N(s, s+N))^i \psi_N(s, s+N-1) B_2(s+ N-1), \; i = 0, \ldots, q-1 \Big\} \\ &= \{\psi_N(s, s+iN)\psi_N(s,s)B_2(s), \ldots \\ &\quad \psi_N(s, s+iN)\psi_N(s, s+N-1)B_2(s +N-1), \; i = 0, \ldots, q-1 \} \\ &= \{\psi_N(s,i)B_2(i), \; i = s, ..., s+qN-1\}. \end{aligned}$$

Then $\mathcal{X}_{0,s} = \mathcal{X}_0(s)$, $s = 0, 1, \ldots N-1$.

3. ON FEEDBACKS AND HOLDABILITY

This section studies the relation between the holdabilit propert associated with an $N-$periodic singular s stem, when the set $\mathbb{R}_+^n$ is considered, and feedbacks for obtaining positi e N-periodic singular s stems are used.

Firstl , the following well-known definitions and results which will be used in the rest of the section are gi en.

Definition 1. A nonempt set $\Gamma \subset \mathbb{R}^n$ will be called holdable with respect to (2) if for each $s = 0, 1, \ldots N-1$ and for all initial state $x(s) = x_{0,s} \in$

$\mathcal{X}_0(s) \cap \Gamma$ there exists a control sequence $u(j) \in \mathbb{R}^m$, $j \geq s$, such that the trajector of the s stem belongs to Γ.

Definition 2. The $N-$periodic s stem (2) is a positi e $N-$periodic singular s stem when for each $s = 0, 1, \ldots N-1$, for each initial state $x(s) = x_{0,s} \in \mathcal{X}_0(s) \cap \mathbb{R}^n_+$ and for each control sequence $u(k) \geq 0$, $k \geq s$, the state trajector belong to $\mathbb{R}^n_+$.

Note that the abo e definition is gi en in (Bru and Hernánde 1989) for $N-$periodic standard s stems.

A characteri ation of positi e singular s stem is gi en in the following proposition.

Proposition 1. Consider a N-periodic forward-backward s stem $(E(\cdot), A(\cdot), B(\cdot))_N$. The s stem is positi e if, and onl if, $A_1(k) \geq 0$, $B_1(k) \geq 0$, $\psi_N(k,j)B_2(j) \leq 0$, $k \in \mathbb{Z}$, and $j = k, \ldots, k + qN - 1$, where $q = \text{ind}(N_s)$.

Proof. If conditions on the matrices $A_1(k) \geq 0$, $B_1(k) \geq 0$, $\psi_N(k,j)B_2(j) \leq 0$, hold, the trajector (7) is nonnegati e and then the s stem is positi e.

If the s stem is positi e, as the solution is gi en b

$$\begin{aligned} x(k) = &\begin{bmatrix} I_{n_1} \\ 0 \end{bmatrix} (\phi_{A_1}(k,s)x_1(s) \\ &+ \sum_{j=s}^{k-1} \phi_{A_1}(k, j+1)B_1(j)u(j)) \\ &- \begin{bmatrix} 0 \\ I_{n_2} \end{bmatrix} \sum_{j=k}^{k+qN-1} \psi_N(k,j)B_2(j)u(j), \ k \geq s, \end{aligned}$$

taking $s = 0$, $k = 1$, $u(\cdot) = 0$, $x_1(0) = e_i$ $i = 1, \ldots, n_1$, where e_i are the n_1 canonical ectors of $\mathbb{R}^{n_1}$,

$$\begin{aligned} x(1) &= \begin{bmatrix} I_{n_1} \\ 0 \end{bmatrix} \phi_{A_1}(1,0)x_1(s) \\ &= \begin{bmatrix} I_{n_1} \\ 0 \end{bmatrix} A_1(0)e_i \geq 0, i = 1, \ldots, n_1, \end{aligned}$$

then $A_1(0) \geq 0$. It eas to see that taking $s = j$, $k = j+1$, $u(\cdot) = 0$, $x_1(0) = e_i$ $i = 1, \ldots, n_1$, the matrices $A_1(k) \geq 0$, $k = 0, 1, \ldots N-1$. Analogousl , it can be pro ed that $B_1(k) \geq 0$, $k = 0, 1, \ldots N-1$, and $\psi_N(k,j)B_2(j) \leq 0$ using ero admissible initial conditions and taking adequate controls (canonical ectors and ero ectors). □

Firstl , the holdabilit characteri ation of $\mathbb{R}^n_+$ in the autonomous case is gi en.

Proposition 2. Consider the N-periodic forward-backward singular s stem $(E(\cdot), A(\cdot))_N$. The s stem is positi e if, and onl if, the set $\mathbb{R}^n_+$ is holdable with respect to $(E(\cdot), A(\cdot))_N$.

Proof. When N-periodic s stem is gi en b the autonomous forward-backward s stem, the trajector is

$$\begin{aligned} x(k) &= \begin{bmatrix} I_{n_1} \\ 0 \end{bmatrix} x_1(k) + \begin{bmatrix} 0 \\ I_{n_2} \end{bmatrix} x_2(k) = \\ &= \begin{bmatrix} I_{n_1} \\ 0 \end{bmatrix} \phi_{A_1}(k,s)x_1(s). \end{aligned}$$

Note that, in this case, the backward subs stem does not influence in the trajector of s stem $(E(\cdot), A(\cdot))_N$. Thus, if $A_1(k) \geq 0$, then $x(k) = \begin{bmatrix} I_{n_1} \\ 0 \end{bmatrix} \phi_{A_1}(k,s)x_1(s)$ is nonnegati e for all initial states $x(s) = \begin{bmatrix} x_1(s) \\ x_2(s) \end{bmatrix} \in \mathcal{X}_0(s) \cap \mathbb{R}^n_+$. Hence, $\mathbb{R}^n_+$ is holdable with respect to $(E(\cdot), A(\cdot))_N$.

Con ersel , it is eas to see that if $\mathbb{R}^n_+$ is holdable with respect to $(E(\cdot), A(\cdot))_N$ then $A_1(k) \geq 0$, $k = 0, 1, \ldots N-1$. □

Example 1. Consider N-periodic forward-backward singular s stem $(E(\cdot), A(\cdot))_N$, with $N = 2$, gi en b

$$E(0) = E(1) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \qquad (9)$$

$$A(0) = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \ A(1) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

As $\phi_{A_1}(k,s) \geq 0$ and $x_1(s) \geq 0$, the trajector of the s stems gi en b

$$x(k) = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \phi_{A_1}(k,s)x_1(s) \geq 0, \ k \geq s.$$

In the below result, the holdabilit relation between the periodic and the in ariant s stems is gi en

Proposition 3. Consider the N-periodic forward-backward s stem $(E(\cdot), A(\cdot))_N$. If the set $\mathbb{R}^n_+$ is holdable with respect to $(E(\cdot), A(\cdot))_N$ then $\mathbb{R}^n_+$ is holdable with respect to in ariant s stem (E_s, A_s), for all $s = 0, 1, \ldots N-1$.

Proof. For all $s = 0, 1, \ldots N-1$, for each initial state $x_s(0) = \begin{bmatrix} x_1(s) \\ x_2(s) \end{bmatrix} = x(s) \in \mathcal{X}_{0,s} \cap \mathbb{R}^n_+$, we ha e $x(s) \in \mathcal{X}_0(s) \cap \mathbb{R}_+^n$, since $\mathcal{X}_0(s) = \mathcal{X}_{0,s}$. Using the abo e proposition, the trajector of the periodic s stem is nonnegati e and, from construction of the state of in ariant s stem,

$$x_s(k) = \begin{bmatrix} x_{1,s}(k) \\ x_{2,s}(k) \end{bmatrix}$$
$$= x(s+kN) = \begin{bmatrix} x_1(s+kN) \\ x_2(s+kN) \end{bmatrix}$$

it is also nonnegati e. Then the set $\mathbb{R}_+^n$ is holdable with respect to in ariant s stem (E_s, A_s), for all $s = 0, 1, \ldots N-1$. □

Now, consider the controller s stem.

Proposition 4. Consider a forward-backward s s-tem $(E(\cdot), A(\cdot), B(\cdot))_N$. If there exists a feed-back $u(k) = \begin{bmatrix} F_1(k) & 0 \end{bmatrix} x(k)$ with $F_1(k+N) = F_1(k)$ such that $A_1(k) + B_1(k)F_1(k) \geq 0$ and $B_2(k)F_1(k) = 0$, then the set $\mathbb{R}_+^n$ is holdable with respect to $(E(\cdot), A(\cdot), B(\cdot))_N$.

Proof. Suppose there exists

$$u(k) = \begin{bmatrix} F_1(k) & 0 \end{bmatrix} x(k)$$

such that $A_1(k)+B_1(k)F_1(k) \geq 0$, and $B_2(k)F_1(k) = 0$.

The forward closed-loop subs stem is gi en b

$$x_1(k+1) = A_1(k)x_1(k) + B_1(k)F_1(k)x_1(k)$$
$$= (A_1(k) + B_1(k)F_1(k))\, x_1(k),$$

and since $A_1(k) + B_1(k)F_1(k) \geq 0$, then $x_1(k) \geq 0$. The backward closed-loop subs stem is gi en b

$$N(k)\, x_2(k+1) = x_2(k) + B_2(k)u(k),$$

and the solution is gi en b

$$x_2(k) = -\sum_{j=k}^{k+h-1} \psi_N(k,j)B_2(j)u(j) =$$
$$= -\sum_{j=k}^{k+h-1} \psi_N(k,j)B_2(j)F_1(j)x_1(j), \; k \in \mathbb{Z}_+.$$

Then $x_2(k) = 0$, since $B_2(j)F_1(j) = 0$. Thus, $x(k) \geq 0$, that is the closed-loop s stem is positi e and then $\mathbb{R}_+^n$ is holdable with respect to the s stem $(E(\cdot), A(\cdot), B(\cdot))_N$. □

Example 2. Consider the forward-backward s s-tem $(E(\cdot), A(\cdot), B(\cdot))_N$ with $N = 2$, where ma-trices $E(\cdot)$ and $A(\cdot)$ are gi en in (9) and $B(\cdot)$ are gi en b

$$B(0) = \begin{bmatrix} -1 & 2 \\ 0 & 2 \\ 0 & 0 \end{bmatrix}, \; B(1) = \begin{bmatrix} 3 & -1 \\ 1 & -1 \\ -2 & 2 \end{bmatrix}.$$

It eas to check that there exists a control se-quence such that the trajector of the s stem is negati e. For example, considering

$$u(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \; u(k) = 0, \; k \geq 1 \text{ and } x(s) = 0.$$

According to the abo e proposition, there exist-s a collection of 2−periodic feedback $F(k) = \begin{bmatrix} F_1(k) & 0 \end{bmatrix}$, for example

$$F(0) = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \; F(1) = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}.$$

Note that, $B_2(k)F_1(k) = 0$ and $A_1(k)+B_1(k)F_1(k) \geq 0$, $k = 0, 1$.

Proposition 5. Consider the N-periodic singu-lar s stem $(E(\cdot), A(\cdot), B(\cdot))_N$. If the set $\mathbb{R}_+^n$ is holdable with respect to $(E(\cdot), A(\cdot), B(\cdot))_N$ then there exists a sequence of periodic matrices $\{F(k)\} \subset \mathbb{R}^{m \times n}$, $F(k+N) = F(k)$, such that $A(s) + B(s)F(s) \geq 0$, for each $s \in \mathbb{Z}$.

Proof. Consider at time s,

$$x^i(s) = \begin{bmatrix} e_i \\ 0 \end{bmatrix} \in \mathcal{X}_0(s) \cap \mathbb{R}_+^n.$$

Since $\mathbb{R}_+^n$ is holdable with respect to $(E(\cdot), A(\cdot))_N$, there exits a control sequence $u^i(\cdot)$ such that the trajector of the s stem is nonnegati e. Using this control sequence, the feedback matrices

$$F(s), \; s = 0, 1, \ldots N-1,$$

are defined as $F(s)\, e_i = u^i(s)$, and the N-periodic extension $F(k+N) = F(k)$ is constructed. Appl -ing the feedback

$$u(k) = F(k)\, x(k)$$

to the s stem, the forward substate at time $s+1$ is nonnegati e and is gi en b

$$x_1(s+1) = A_1(s)\, x(s) + \phi_{A_1}(s+1, s+1)B_1(s)u(s)$$
$$= (A_1(s) + B_1(s)F(s))\, e_i \geq 0.$$

Thus, $A_1(s) + B_1(s)F(s) \geq 0$. □

4. CONCLUSIONS

In this work, feedbacks on N-periodic linear sin-gular s stems for obtaining a nonnegati e tra-jector of the new closed-loop s stem ha e been considered. This propert is associated with the holdabilit propert of the set $\mathbb{R}_+^n$. Thus, the relation between the holdabilit propert and special state-feedbacks has been anal ed. In the forward-backward case, some conditions on proportional state-feedbacks ha e been established for obtaining the holdabilit propert of $\mathbb{R}_+^n$ and in ersel , the feedback has been constructed when this propert holds.

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