15th Triennial World Congress, Barcelona, Spain

ROBUST LEARNING CONTROL FOR NONLINEAR UNCERTAIN SYSTEMS BASED ON COMPOSITE ENERGY FUNCTION

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Abstract: In this paper the learnability of Iterative Learning Control (ILC) under the framework of energy function is explored. First we show that ILC is in essence a pointwise adaptation learning mechanism which can henceforth learn iteration-independent time-varying uncertainties. Next we propose a new robust ILC scheme to address norm-bounded uncertainties. The concept of Composite Energy Function (CEF) is introduced in the analysis of the learning convergence, consequently the proposed ILC schemes are applicable to quite general systems.

Keywords: Iterative Learning Control, Lyapunov Methods, Parametric Uncertainty, Norm-bounded Uncertainty, Initial Condition

1. INTRODUCTION

Practical tracking control tasks must be accomplished in a finite time interval. Asymptotic convergence in time domain without a specified convergence speed obviously does not meet the task requirements. On the other hand, it is a much harder problem in control theory to specify the system transient performance or achieve perfect tracking over a finite time interval. Most advanced control methods developed hitherto only ensure asymptotic convergence property. ILC complements the existing control methods in the sense that it targets at perfect tracking in a finite time interval, which is possible under a repeatable control environment and achieved through asymptotic convergence in iteration domain.

The traditional ILC schemes are based on contraction mapping (CM-type ILC). In (Xu, 2002) we have shown the limitation of CM-type ILC: the requirement of global Lipschitz condition (GLC). A non-global Lipschitz nonlinear function may incur finite escape time in a simple dynamic system. It is necessary to widen the learning control framework under which ILC can handle broader classes of system nonlinearities and uncertainties, including NGLC (non-global Lipschitz continuous) dynamics, time varying parametric and norm bounded uncertainties.

There are two main streams in advanced control theories: adaptive control and robust control, both highly depending on energy function approaches. The former mainly deals with parametric uncertainties and the latter deals more with the norm-bounded perturbations. In this paper, we will first exhibit the main characteristic of energy funtion-based ILC (EF-based ILC) – pointwise adaptation when dealing with time varying parametric uncertainties. When the norm-bounded perturbation is concerned, adaptive type control methods are no longer applicable. Robust control, characterized by high gain feedback, is able to secure the uniform bound of the system states, but in general is not able to obtain asymptotic convergence because of the lack of internal model which is nonlinear in nature. By incorporating EF-based ILC, it is possible to eliminate the tracking error asymptotically. Therefore we can show another main characteristic of EF-based ILC – nonlinear internal model control realized through embedding an integrator in the control input.

2. ILC WITH COMPOSITE ENERGY FUNCTION

A. EF-based ILC – Pointwise Adaptation

As we all know, the traditional parameter adaptation mechanisms cannot deal with the time varying parameters, i.e., $\theta(t)$. On the other hand, under the repeatable control environment, $\theta(t)$ is invariant with respect to the iterations. Hence for a particular $t \in [0,\ T]$, the corresponding $\theta(t)$ is considered as a constant in iteration domain $\mathcal{N} = \{0, 1, \cdots\}$. We can use a simple integrator to do the parameter updating job along the iteration axis. This leads to a new ILC approach – the *pointwise* adaptation over the entire interval $[0,\ T]$. Let us give the control law and parameter updating law. Consider the nonlinear dynamic system in i-th iteration

$$\dot{x}_i = \theta(t)x_i^2 + u_i, \qquad x_i(0) = 0.5 \tag{1}$$

where $\theta(t) \in C^1[0,\ T]$, and $x(t)$ is to track $x_d(t) \in C^1[0,\ T]$ with $x_d(0) = 0.5$. The error dynamics is

$$\dot{e}_i = -\theta(t)x_i^2 + \dot{x}_d - u_i, \qquad e_i(0) = 0. \tag{2}$$

The control law is

$$u_i = ke_i + \dot{x}_d - \hat{\theta}_i(t)x_i^2 \tag{3}$$

and the parametric updating law is $\forall t \in [0,\ T]$

$$\hat{\theta}_i(t) = \hat{\theta}_{i-1}(t) - x_i^2(t)e_i(t) \qquad \hat{\theta}_{-1}(t) = 0. \tag{4}$$

B. Convergence with Composite Energy Function

Now let us derive the convergence property of the above ILC. For this purpose we need to find an appropriate "energy function" which plays the similar role as Lyapunov function in adaptive control. Here a composite energy function is used

$$E_i(t) = \frac{1}{2}e_i^2(t) + \frac{1}{2}\int_0^t \phi_i^2(\tau)d\tau, \tag{5}$$

where $\phi_i = \theta_i - \hat{\theta}_i$. We use E_i to distinguish it from Lyapunov functions, which can be as simple as $V_i = \dfrac{1}{2}e_i^2$. The difference of E_i is

$$\Delta E_i = \frac{1}{2}e_i^2 + \int_0^t (\phi_i^2 - \phi_{i-1}^2)d\tau - \frac{1}{2}e_{i-1}^2. \tag{6}$$

Using the initial resetting condition $e_i(0) = 0$, substituting the error dynamics (2) and the control law (3), the first term on the right hand side is

$$\frac{1}{2}e_i^2 = \int_0^t e_i\dot{e}_i d\tau = \int_0^t e_i[-\theta(\tau)x_i^2 + \dot{x}_d - u_i]d\tau$$
$$= \int_0^t [-\phi_i x_i^2 e_i - ke_i^2]d\tau. \tag{7}$$

By substituting the parameter updating law (4), the second term on the right hand side of (6) can be expressed as

$$\frac{1}{2}\int_0^t (\phi_i^2 - \phi_{i-1}^2)d\tau$$
$$= \frac{1}{2}\int_0^t (\hat{\theta}_{i-1} - \hat{\theta}_i)(2\theta - 2\hat{\theta}_i + \hat{\theta}_i - \hat{\theta}_{i-1})d\tau$$
$$= \int_0^t (\phi_i x_i^2 e_i - \frac{1}{2}x_i^4 e_i^2)d\tau. \tag{8}$$

Clearly $\phi_i x_i^2 e_i$ appears in (7) and (8) with opposite signs. The difference of the composite energy function is

$$\Delta E_i = -\int_0^t ke_i^2 d\tau - \int_0^t \frac{x_i^4 e_i^2}{2}d\tau - \frac{e_{i-1}^2}{2} < 0. \tag{9}$$

The function E_i is a monotonically decreasing sequence, hence is bounded if E_0 is bounded. The derivative of E_0 is

$$\dot{E}_0 = e_0\dot{e}_0 + \frac{\phi_0^2}{2} = -ke_0^2 - \phi_0 x_0^2 e_0 + \frac{\phi_0^2}{2}. \tag{10}$$

At iteration number $i = 0$, $\hat{\theta}_{-1}(t) = 0\ \forall[0,\ T]$, thus $\hat{\theta}_0 = -x_0^2 e_0$, and $\dot{E}_0$ becomes

$$\dot{E}_0 = -ke_0^2 + \phi_0\hat{\theta}_0 + \frac{1}{2}\phi_0^2 = -ke_0^2 - \frac{1}{2}\phi_0^2 + \phi_0\theta.$$

Using Young's inequality, we have for any $c > 0$ $\phi_0\theta \le c\phi_0^2 + \frac{1}{4c}\theta^2$. Let $0 < c < \frac{1}{2}$, $\dot{E}_0 \le -ke_0^2 - (\frac{1}{2} - c)\phi_0^2 + \frac{1}{4c}\theta^2$.

Since $\theta(t) \in C^1[0,\ T]$, there exists a finite bound $\theta_m \ge \theta(t)\ \forall t \in [0,\ T]$. Thus $\dot{E}_0$ is negative definite outside the region $(e_0, \phi_0) \in \mathcal{R}^2 \mid ke_0^2 + (\frac{1}{2} - c)\phi_0^2 \le \frac{1}{4c}\theta_m^2$, which also specifies the bound of $E_0(t)$ in the finite interval $[0,\ T]$.

Applying (9) repeatedly we have

$$E_i(t) = E_0(t) + \sum_{j=1}^{i} \Delta E_j$$

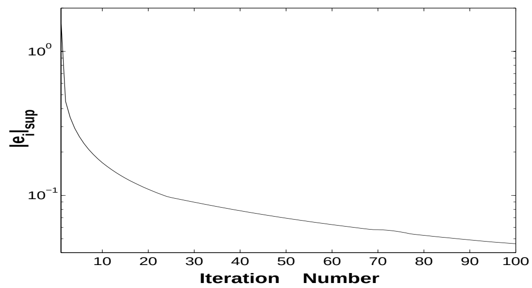

Fig. 1. Learning convergence of ILC based on CEF.

$$\lim_{i\to\infty} E_i(t) < E_0(t) - \lim_{i\to\infty}\sum_{j=1}^{i}\int_0^t ke_j^2 d\tau - \lim_{i\to\infty}\sum_{j=1}^{i-1} e_j^2(t). \quad (11)$$

Consider the positiveness of E_i and boundedness of E_0, $e_i(t)$ converges to zero pointwisely as $i \to \infty$.

Remark 1. In the pioneer work of EF-based ILC (Ham, Qu and Kaloust, 1995; Xu, 2002), a similar energy function has been used as $E_i(t) = \int_0^t \phi_i^2 d\tau$, which however is of L^2 only. By adding additional $\frac{1}{2}e_i^2$ or in general a Lyapunov function V, CEF in (5) is obvious more general with both L^2 and L^∞.

C. Illustrative Example

Consider system (1) and target trajectory is $x_d(t) = sin\pi t + 0.5$, $t \in [0, 2]$. Here the unknown time varying parameter $\theta = 3 + sin\frac{\pi}{2}t$. Applying control law (3), updating law (4) and choosing gain $k = 1$, the learning convergence is shown in Fig. 1.

3. ON THE INITIAL CONDITION

A. Relaxation – Alignment Condition

In CM-type ILC initial resetting condition is needed. However, a perfect initial resetting requires that the control system be equipped with a precise homing mechanism, which may not be possible for many practical engineering systems. In EF-based ILC we could make full use of the system knowledge especially concerning state dynamics. This opens a new avenue: replacing the initial resetting condition with a less restricted initial condition – alignment condition – and meanwhile achieving the convergent property. The alignment condition is simply $x_i(0) = x_{i-1}(T)$, i.e. the end state of preceding iteration becomes the initial state of the present iteration. In addition to this, we also need $x_d(0) = x_d(T)$.

Under the framework of CEF, let us derive the convergence property with the alignment condition. Look into the procedure in deriving $\Delta E_i(T)$ in the preceding section. Without the initial resetting equation the equation (7) is $\frac{1}{2}e_i^2(t) = \int_0^t e_i\dot{e}_i d\tau + \frac{1}{2}e_i^2(0)$. Choosing $t = T$ and using the alignment condition $e_i(0) = e_{i-1}(T)$, the relationship (9) becomes

$$\begin{aligned}\Delta E_i(T) &= -\int_0^T ke_i^2 d\tau - \frac{1}{2}\int_0^T x_i^4 e_i^2 d\tau + \frac{1}{2}e_i^2(0) - \frac{1}{2}e_{i-1}^2(T).\\ &= -\int_0^T ke_i^2 d\tau - \frac{1}{2}\int_0^T x_i^4 e_i^2 d\tau. \quad (12)\end{aligned}$$

Consequently

$$E_i(T) = E_0(T) + \sum_{j=1}^{i}\Delta E_j(T)$$

$$\lim_{i\to\infty} E_i(T) < E_0(T) - \lim_{i\to\infty}\sum_{j=1}^{i}\int_0^T ke_j^2 d\tau, \quad (13)$$

the tracking error sequence converges in L^2-norm, instead of pointwise convergence.

B. Spatial Resetting vs Temporal Resetting

The initial resetting condition in ILC usually implies both spatial resetting and temporal resetting. While time resetting is natural for a task to be finished and repeated over a finite period, the spatial resetting is however not an easy job and not so imperative. Note that it is the spatial resetting which gives rise to extra implementation difficulty and incurs criticism.

Consider a target trajectory $x_d(t) \in C^1[0,\ T]$, which forms a continously spatial path. When do we need the spatial resetting? It is necessary only when the spatial path of the target trajectory is not completely closed, i.e. $x_d(0) \neq x_d(T)$. For instance, $x_d(t) = t$, $t \in [0,\ 1]$. In such circumstance, a perfect tracking will lead to $x_i(T) = x_d(T) \neq x_d(0)$. Hence an independent control mechanism must work appropriately between two consecutive iterations so as to bring back the system state to the initial position $x_d(0)$.

For any trajectories spatially closed, i.e. $x_d(0) = x_d(T)$, we can use the alignment condition and remove the spatial resetting requirement, as discussed in the preceding subsection.

C. Extention to Repetitive Tasks

By relaxing the spatial resetting to the alignment condition, we can now extend ILC to most repetitive control tasks – either tracking a periodic

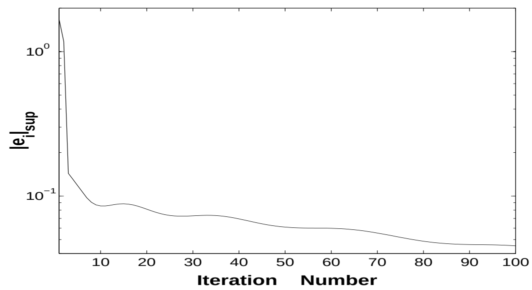

Fig. 2. Learning convergence for system with alignment condition.

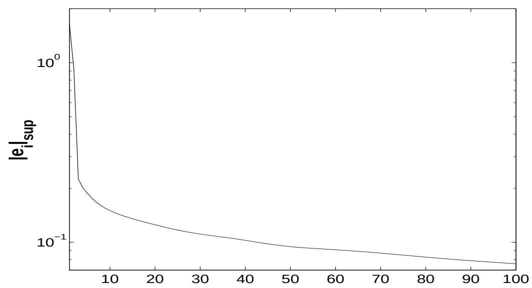

Fig. 3. Learning convergence for repetitive tracking.

trajectory or reject a periodic disturbance over $[0,\ \infty)$. Let us exhibit how to convert a repetitive control task into an ILC task. Consider the target trajectory $x_d(t) \in C^1[0,\ \infty)$ with the periodicity $x_d(t) = x_d(t-T)$. Assume that $\theta(t)$ is also periodic with the same period T. Define the state $x_i(t) = x((i-1)T+t)$, $\forall i = 1, 2, \cdots$. By virtue of the continuity, $x(iT)$ is the end point of the i-th iteration defined over $[(i-1)T,\ iT]$, and also the initial point of the $(i+1)$-th iteration defined over $[iT,\ (i+1)T]$. Note that the alignment condition is met because $x_i(T) = x_{i+1}(0)$ is in fact the same point $x(iT)$. Thus the original control problem is equivalent for $x_i(t)$ to track $x_d(t)$ over the period $[0,\ T]$, and the ILC can be directly applied.

What can we gain by converting a repetitive control problem into ILC problem? First of all, ILC is now able to handle periodic signals defined in infinite horizon, hence cover repetitive control problems. Second, ILC based on CEF is able to handle more general classes of system nonlinearities and uncertainties. Indeed, the convergence analysis of repetitive control is mainly based on small gain theorem, quite similar to the contraction mapping, consequently the application is rather limited.

Remark 2: When $e(0) = 0$, the repetitive type ILC will generate a continuous control profile. If $e(0) \neq 0$, the repetitive type ILC may generate a piecewise continuous control profile, with the discontinuities occurring at each instant $t = iT$.

B. Illustrative Example

Consider system (1) with the same parameters and target trajectory as in previous section. The initial values are $x(0) = 1 \neq x_d(0) = 0.5$. Instead of the initial resetting condition, the alignment condition $x_i(0) = x_{i-1}(T)$ is used. Applying control law (3) and updating law (4), the learning convergence is shown in Fig. 2, which is close to the case with ideal initial resetting (Fig. 1).

Next, the ILC is extended to a repetitive case where the target trajectory is $x_d = 0.5 + sin\pi t,\ t \in [0, \infty)$. The unknown time-varying parameter is $\theta = 3 + sin\frac{\pi}{2}t$. Thus the learning period should be $T = 4$. Applying the same ILC scheme with the alignment condition, the maximum error for each period is recorded in Fig. 3. The effectiveness is validated.

4. CAN WE LEARN NORM-BOUNDED UNCERTAINTIES?

A. Robust ILC

Up to now we focus on ILC with parametric uncertainties. What shall we do if the nonlinear uncertain term $\theta(t)x^2$ is only known *a priori* as a lumped disturbance with a known bounding function? Consider the following system with norm-bounded uncertainties $\eta(x,t)$ which is local Lipschitz continuous

$$\begin{aligned} \dot{x} &= \eta(x,t) + u, \qquad x(0) = 0.5 \\ or \quad \dot{e} &= \dot{x}_d - \eta - u, \qquad e(0) = 0 \end{aligned} \tag{14}$$

where $|\eta(x,t)| \leq \bar{\rho}(x,t)$ and the bounding function $\bar{\rho}$ is known *a priori*. Clearly we are unable to conduct pointwsie adaptation for η as it is x-dependent, i.e. iteration dependent. Now let us study how robust control works. A typical robust control law can be chosen as

$$\begin{aligned} u_i &= u_{r,i} = (\rho_i \kappa_i + 1)e_i \qquad (15) \\ \rho_i &= \sqrt{\dot{x}_d^2 + \varepsilon} + \bar{\rho}_i \\ \kappa_i &= \frac{\sqrt{e_i^2 + 3\varepsilon^2} + 8\varepsilon}{(\sqrt{e_i^2 + 3\varepsilon^2} + \varepsilon)^2}. \end{aligned}$$

where $\varepsilon > 0$ is a constant. Both ρ_i and κ_i are smooth functions of e_i and t.

When the control task repeats, ILC is again the appropriate candidate to improve control performance. The underlying idea is as follows. Since the system state is bounded in a set under robust control, the dynamic system is Lipschitz continuous on the set. Thus we can adopt the most widely applied CM-type ILC approach – embedding an integrator in the control input so as to learn the desired control profile directly. This leads to a new ILC strategy – robust ILC.

Choose the learning control law as below

$$u_i = proj(u_{i-1}) + u_{r,i} \qquad (16)$$

$$proj(\cdot) \triangleq \begin{cases} \cdot & |\cdot| \le u^* \\ sign(\cdot)u^* & |\cdot| > u^* \end{cases}$$

where u^* is the projection bound which is sufficiently large such that $u^* \ge \sup_{t\in[0,\ T]} |u_d(t)|$. In practice, u^* is either a physical process limitation or a virtual saturation bound which can be arbitrarily large but finite.

To analyze the convergence property of the proposed robust ILC, we define the following time weighted composite energy function

$$E_i(t) = e^{-\lambda t}e_i^2 + \int_0^t e^{-\lambda\tau}\delta u_i^2 d\tau \qquad (17)$$

where $\delta u_i = u_d - u_i$. Note the difference between two CEF (5) and (17). The former contains unknown parameters explicitely, whereas the latter contains the unknown but desired control input directly. The difference arise because the norm-bounded uncertainty is the lumped one.

The ultimate objective of learning control is to find the desired control input u_d which realizes

$$\dot{x}_d(t) = \eta(x_d, t) + u_d(t). \qquad (18)$$

According to the system dynamics (14), we can obtain

$$\delta u_i = (\dot{x}_d - \eta_d) - (\dot{x}_i - \eta_i) = \dot{e}_i + \eta_i - \eta_d \qquad (19)$$

where $\eta_d = \eta(x_d, t)$.

B. Convergence Properties

Let us show the convergence properties of the ILC scheme (16) and (15). First we will show that the boundedness of the system state is guaranteed by control laws (16) and (15). Define a Lyapunov function $V_i = \frac{1}{2}e_i^2$. Note the following fact provided that $|e_i| \ge \varepsilon$,

$$\begin{aligned} 1 - \kappa_i|e_i| &= \frac{e_i^2 + 3\varepsilon^2 + \varepsilon^2 + 2\varepsilon\sqrt{e_i^2+3\varepsilon^2}}{(\sqrt{e_i^2+3\varepsilon^2}+\varepsilon)^2} \\ &\quad - \frac{\sqrt{e_i^2+3\varepsilon^2}|e_i| + 8\varepsilon|e_i|}{(\sqrt{e_i^2+3\varepsilon^2}+\varepsilon)^2} \\ &\le \frac{e_i^2 + 4\varepsilon^2 + 2\varepsilon\sqrt{e_i^2+3e_i^2}}{(\sqrt{e_i^2+3\varepsilon^2}+\varepsilon)^2} \\ &\quad - \frac{\sqrt{e_i^2}|e_i| + 8\varepsilon|e_i|}{(\sqrt{e_i^2+3\varepsilon^2}+\varepsilon)^2} \\ &\le \frac{4\varepsilon^2 + 4\varepsilon|e_i| - 8\varepsilon|e_i|}{(\sqrt{e_i^2+3\varepsilon^2}+\varepsilon)^2} \\ &\le \frac{4\varepsilon(\varepsilon - |e_i|)}{(\sqrt{e_i^2+3\varepsilon^2}+\varepsilon)^2} < 0, \end{aligned} \qquad (20)$$

consequently we have

$$\begin{aligned} \dot{V}_i &= e_i\dot{e}_i = e_i(\dot{x}_d - \eta_i - u_i) \\ &\le |e_i|u^* - e_i^2 + (1 - \kappa_i|e_i|)(|\dot{x}_d| + \bar{\rho}_i)|e_i| \\ &\le |e_i|u^* - e_i^2 = -|e_i|(|e_i| - u^*). \end{aligned}$$

$|e_i|$ is globally uniformly bounded by $max\{\varepsilon, u^*\}$. Hence $x_i \in \mathcal{X}$ where $\mathcal{X}$ is a compact set.

Since x_i is bounded and η_i is local *Lipschitz*, there exists a *Lipschitz* constant $l_\eta \triangleq sup\left|\frac{\partial\eta_i}{\partial x_i}\right| < \infty$, $\forall i \in \mathcal{N}$ and $\forall(x_i, t) \in \mathcal{X} \times [0, T]$, such that

$$|\eta_i - \eta_d| \le l_\eta|x_i - x_d|. \qquad (21)$$

Moreover, according to the control law (15) and (16) the boundedness of x_i guarantees the finiteness of $u_{r,i}$ and u_i. Consequently, $\dot{x}_i$ and $\dot{e}_i$ are also finite on $\mathcal{X}$. From the definition of κ_i and ρ_i, it can be derived that there exists a finite constant $c_1 \triangleq \sup_{(x_i,t)\in\mathcal{X}\times[0,T]} \rho_i\kappa_i$ and a finite constant $c_2 \triangleq \sup_{(x_i,t)\in\mathcal{X}\times[0,T]} \frac{d(\rho_i\kappa_i)}{dt}$.

Next let us see the difference of $E_i(t)$.

$$\begin{aligned} \Delta E_i = e^{-\lambda t}e_i^2 + \int_0^t e^{-\lambda t}(\delta u_i^2 - \delta u_{i-1}^2)d\tau \\ - e^{-\lambda t}e_{i-1}^2. \end{aligned} \qquad (22)$$

The first term on the right hand side of (22), with the initial resetting condition, can be expressed as

$$e^{-\lambda t}e_i^2 = -\lambda\int_0^t e^{-\lambda\tau}e_i^2 d\tau + \int_0^t 2e^{-\lambda\tau}e_i\dot{e}_i d\tau. \qquad (23)$$

The second term on the right hand side of (22) can be expressed as

$$\begin{aligned} &\int_0^t e^{-\lambda\tau}(\delta u_i^2 - \delta u_{i-1}^2)d\tau \\ &\le \int_0^t e^{-\lambda\tau}[\delta u_i^2 - (u_d - proj(u_{i-1}))^2]d\tau \\ &= \int_0^t e^{-\lambda\tau}[-2(u_d - u_i)u_{r,i} - u_{r,i}^2]d\tau. \end{aligned} \qquad (24)$$

Substitute (19) into (24) and drop the $u_{r,i}^2$ term, we have

$$\int_0^t e^{-\lambda\tau}(\delta u_i^2 - \delta u_{i-1}^2)d\tau$$

$$\leq -2\int_0^t e^{-\lambda\tau}(\eta_i - \eta_d)(\rho_i\kappa_i + 1)e_i d\tau$$

$$-2\int_0^t e^{-\lambda\tau}(\rho_i\kappa_i + 1)e_i\dot{e}_i d\tau$$

$$\leq -2\int_0^t e^{-\lambda\tau}(\eta_i - \eta_d)(\rho_i\kappa_i + 1)e_i d\tau$$

$$+\int_0^t e^{-\lambda\tau}e_i^2 d(\rho_i\kappa_i) - 2\int_0^t e^{-\lambda\tau}e_i\dot{e}_i d\tau. \quad (25)$$

Substituting (23) and (25) into (22) and considering (21), yield

$$\Delta E_i \leq -\lambda\int_0^t e^{-\lambda\tau}e_i^2 d\tau + c_2\int_0^t e^{-\lambda\tau}e_i^2 d\tau$$

$$+2\int_0^t e^{-\lambda\tau}l_\eta|x_d - x_i|c_1|e_i|d\tau - e^{-\lambda t}e_{i-1}^2$$

$$\leq -\lambda\int_0^t e^{-\lambda\tau}e_i^2 d\tau + (2l_\eta c_1 + c_2)\int_0^t e^{-\lambda\tau}|e_i|^2 d\tau$$

$$-e^{-\lambda t}e_{i-1}^2$$

$$= -[\lambda - 2l_\eta c_1 - c_2]\int_0^t e^{-\lambda\tau}e_i^2 d\tau - e^{-\lambda t}e_{i-1}^2. (26)$$

There exists a sufficiently large λ such that $\lambda > 2l_\eta c_1 + c_2$ to ensure that

$$E_i(t) - E_{i-1}(t) \leq -e^{-\lambda t}e_{i-1}^2(t) \leq -e^{-\lambda T}e_{i-1}^2(t).$$

Consequently, $E_i(t) \leq E_0(t) - e^{-\lambda T}\sum_{j=0}^{i-1} e_i^2(t)$.

Since both x_0 and u_0 are bounded, $E_0(t)$ is bounded. From the positiveness of $E_i(t)$, we can derive that $\lim_{i\to\infty} e_i(t) = 0$ pointwisely. Next from (21), $\lim_{i\to\infty} |\eta_i - \eta_d| \leq \lim_{i\to\infty} l_\eta|e_i| = 0$. Thus using (19) and the boundedness of $\dot{e}_i$ we further derive

$$\lim_{i\to\infty} E_i(t) = \lim_{i\to\infty} e^{-\lambda t}e_i^2 + \lim_{i\to\infty}\int_0^t e^{-\lambda\tau}\delta u_i^2 d\tau$$

$$= \lim_{i\to\infty}\int_0^t e^{-\lambda\tau}(\dot{e}_i + \eta_i - \eta_d)^2 d\tau$$

$$= \lim_{i\to\infty}\int_0^{e_i} e^{-\lambda\tau}\dot{e}_i de_i.$$

Since $\dot{e}_i$ is bounded, $\lim_{i\to\infty} e_i = 0$ leads to $\lim_{i\to\infty} E_i(t) = 0$ pointwisely. Hence, u_i converges to u_d almost everywhere as $i \to \infty$. On the other hand the boundedness of $\dot{e}_i(t)$, which implies the uniform continuity of $e_i(t)$. Hence, $e_i(t) \to 0$ uniformly can be derived.

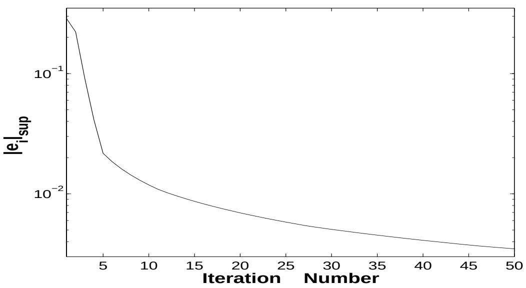

Fig. 4. Learning convergence of robust ILC for norm bounded uncertainties.

Remark 2. Similar to Section IV, alignment condition can be applied to remove the initial reseting condition, and the ILC works for repetitive tasks. In such case the CEF (5) instead of (17) should be chosen, and the Lipschitz condition will be used in the controller design.

C. Illustrative Example

Consider system (14) with $\eta = (3 + \sin t)x^2$. The target trajectory is $x_d = sin\pi t{+}0.5$, $t \in [0, 2]$. The known bounding function of η is $\bar{\rho} = (16x^4{+}1)^{1/2}$. Choose $\varepsilon = 0.3$, $u^* = 60$ and apply the robust ILC scheme. The simulation result is shown in Fig. 4. At $i = 0$, the tracking error is the result of the robust control alone. Through comparison, the learning effect is obvious.

5. CONCLUDING REMARKS

In this paper, we first demonstrate how the concept of energy function can be incorporated in ILC design and analysis. Second, synthesizing with robust control, EF-based ILC nullifies the influence from the norm-bounded disturbances. Third, with the alightment condition, we are able to remove the initial resetting condition and accomplish the repetitive control tasks.

6. REFERENCES

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