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PREDICTIVE ATTITUDE CONTROL TECHNIQUES FOR SATELLITES WITH MAGNETIC ACTUATORS

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Abstract: The problem of attitude stabilization for a small spacecraft using only magnetic coils as actuators is considered and a novel approach to the problem is given. A solution to the magnetic attitude control problem in terms of model-based predictive control is proposed and analyzed. Simulation results are also given, which show the feasibility of the approach.

Keywords: Attitude control, constraints, predictive control, satellite control applications, spacecraft vehicles, time-varying systems.

1. INTRODUCTION

Attitude control plays a fundamental role in the operation of spacecraft as it constitutes a mandatory feature both for the survival of a satellite and for the satisfactory achievement of mission goals. While a number of possible approaches to the control of attitude dynamics has been developed through the years, a particularly effective and reliable one is constituted by the use of electromagnetic actuators, which turn out to be specially suitable in practice for low Earth orbit (LEO) satellites. Such actuators operate on the basis of the interaction between a set of three orthogonal, current-driven magnetic coils and the magnetic field of the Earth and therefore provide a very simple solution to the problem of generating torques on board a satellite. Unfortunately, magnetic torquers suffer from a significant drawback from the control design viewpoint, as the torques which can be generated in this way are instantaneously constrained to lie in the plane orthogonal to the local magnetic field vector. Controllability is ensured for most orbit thanks to the variability of the geomagnetic field, however the control designer is faced with the task of working out a suitable time-varying control law to deal with such effects. In recent years a considerable effort has been devoted to the analysis of this control problem (see, e.g., (Stickler and Alfriend, 1976; Arduini and Baiocco, 1997; Wisniewski and Blanke, 1999)); in particular, as the variability of the geomagnetic field is *almost* time periodic, most of the recent work on the linear attitude control problem has focused on the use of optimal and robust periodic control theory for the design of state and output feedback regulators (see e.g., (Wisniewski and Markley, 1999; Psiaki, 2000; Lovera, 2001*a*) and (Lovera, 2001*b*) for a recent survey on this subject). Periodic control provides a good *nominal* solution to this problem, but cannot take into account the fact that the magnetic field can be accurately measured, simulated and predicted on board and this information could and should be retained in the control law.

In the light of the above considerations, the aim of this paper is to propose a novel approach to the magnetic attitude control problem, based on a predictive approach. The novel idea is to consider the magnetically controlled spacecraft as a *time invariant* system and to incorporate the time-varying actuators in the problem formulation via an appropriate set of constraints. It is possible to show that the state feedback problem can be given a simple closed form solution, and that actuator saturation constraints can also be dealt with in the same framework. The proposed control design methodology has been tested in a simulation study for a spacecraft of the MITA class (see (Della Torre *et al.*, 1999)) and the (satisfactory) results obtained so far have been also reported.

The paper is organized as follows: in Section 2 the model for a magnetically controlled rigid spacecraft is presented; Section 3 provides some background on the state space predictive control problem and the detailed presentation of the proposed approach, while the simulation results are given in Section 4.

2. SPACECRAFT ATTITUDE DYNAMICS AND KINEMATICS

2.1 *Dynamic and kinematic equations*

The equations of angular dynamics (Wertz, 1978; Sidi, 1997) can be expressed in vector form as

$$\frac{dh(t)}{dt} = T(t)$$

where h is the overall angular momentum of the spacecraft and T is the sum of the external torques (disturbance and control ones) acting on the satellite. The derivative of h is here expressed in an inertial reference frame; considering instead a body reference frame, rotating with angular rate ω, the Euler's equations become

$$\dot{h}(t) = -\omega(t) \wedge h(t) + T(t).$$

In this formulation the vector h shall include also the contribution of rotating parts of the satellite such as the momentum wheel. Concerning the attitude, a parametric expression can be obtained with different methods: the quaternion has been chosen here because of its numerical advantages and avoidance of singularities. As is well known (see, e.g., (Wertz, 1978)) the attitude matrix can be expressed as a function of the quaternion vector $q \in \mathbb{R}^4$; the time evolution of the attitude parameters as a function of the body angular rate can be represented in the following way (kinematic equations):

$$\dot{q}(t) = \frac{1}{2} W(\omega(t)) q(t)$$

where W is the skew-symmetric matrix function of ω defined as:

$$W(\omega) = \begin{bmatrix} 0 & \omega_z & -\omega_y & \omega_x \\ -\omega_z & 0 & \omega_x & \omega_y \\ \omega_y & -\omega_x & 0 & \omega_z \\ -\omega_x & -\omega_y & -\omega_z & 0 \end{bmatrix}$$

2.2 *Linearized discrete dynamics*

The spacecraft is assumed to have a momentum bias configuration (i.e., one momentum wheel, aligned with the body z axis, with moment of inertia J and angular velocity ν). The overall external torque T is split in three components, namely the gravity gradient torque T_{gg}, which will be included in the linearized dynamics, the control torque T_{contr} and the disturbance torque T_{dist}. Introducing now the state vector $x(t) = [q(t)' \ \omega(t)']'$, which is supposed to be fully accessible, and considering small displacements from the nominal values of the vector part of the attitude quaternion $q_1 = q_2 = q_3 = 0$, and small deviations of the body rates from the nominal ones $\omega_x = \omega_y = 0, \omega_z = -\Omega$ (Ω being the angular frequency associated with the orbit period), the attitude dynamics can be linearized and sampled (with sample time Δ), and the local linear dynamics for the system can be defined as

$$\delta x(k+1) = (A\Delta - I_{6\times 6})\, \delta x(k) + \\ + B\Delta \left[T_{contr}(k) + T_{dist}(k)\right] \quad (1)$$

where $I_{6\times 6}$ is the 6×6-identity matrix, $B = \begin{bmatrix} 0 \\ \mathbf{I}^{-1} \end{bmatrix}$, $\mathbf{I} = \text{diag}\{I_{xx}, I_{yy}, I_{zz}\}$ is the spacecraft inertia matrix,

$$A = \begin{bmatrix} 0 & -\Omega & 0 & 0.5 & 0 & 0 \\ \Omega & 0 & 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.5 \\ 0 & 0 & 0 & 0 & W_x & 0 \\ 0 & -6k_y\Omega^2 & 0 & W_y & 0 & 0 \\ 0 & 0 & +6k_z\Omega^2 & 0 & 0 & 0 \end{bmatrix}$$

and $k_x = \frac{I_{yy}-I_{zz}}{I_{xx}}, k_y = \frac{I_{zz}-I_{xx}}{I_{yy}}, k_z = \frac{I_{xx}-I_{yy}}{I_{zz}}$, $W_x = -k_x\Omega - K_x\nu$, $W_y = -k_y\Omega + K_y\nu$, $K_x = \frac{J}{I_{xx}}, K_y = \frac{J}{I_{yy}}$.

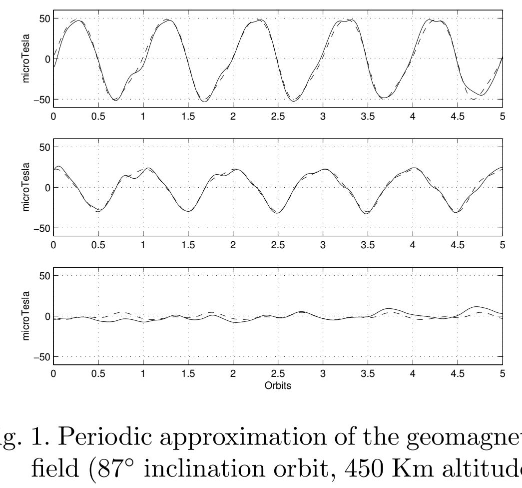

Fig. 1. Periodic approximation of the geomagnetic field (87° inclination orbit, 450 Km altitude).

2.3 *Periodic approximation of the geomagnetic field*

The geomagnetic field is essentially that of a magnetic dipole, with certain deviations from the dipole model called anomalies. A periodic approximation of the geomagnetic field can be derived, by least square fitting of the output of the

International Geomagnetic Field (IGRF) model (Wertz, 1978) to a simplified periodic structure such as $b(t) = b_0 + b_{1c}cos(\Omega t) + b_{1s}sin(\Omega t) + b_{2c}cos(2\Omega t) + b_{2s}sin(2\Omega t)$. A time history of the IGRF model for the Earth's magnetic field together with its least square approximation along five orbits for a spacecraft in polar orbit (87° inclination) are shown in Figure 1.

3. MODEL PREDICTIVE ATTITUDE CONTROL

3.1 *MPC: state space formulation*

The generic model-based predictive control design is based on a *receding-horizon* strategy in which at each sample instant k:

- the model is used to predict the output response to a certain set of future control signals;
- a function including the cost of future control actions and future deviations from the reference is optimized to obtain the 'best future control sequence';
- only the first control of the sequence is asserted, and the entire operations repeated at time $k+1$.

Let us consider a multi-variable process with n outputs and m inputs, described by the following state-space model:

$$\begin{cases} x(k+1) = Ax(k) + Bu(k) + Pv(k) \\ y(k) = Cx(k) + w(k) \end{cases}$$

where $x(k)$ is the state vector, $y(k)$ is the output vector, $v(k)$ and $w(k)$ are the noise affecting the process and the output respectively.
The optimal j-ahead prediction of the output is given by (Camacho and Bordons, 1998):

$$\hat{y}(k+j|k) = CA^j E[x(k)] + \sum_{i=0}^{j-1} CA^{j-i-1}Bu(k+i)$$

Let us consider a set of N_c (control horizon) j-ahead predictions:

$$\mathbf{y} = \begin{bmatrix} \hat{y}(k+1|k) \\ \hat{y}(k+2|k) \\ \vdots \\ \hat{y}(k+N_c|k) \end{bmatrix} =$$

$$= \begin{bmatrix} CAE[x(k)] + CBu(k) \\ CA^2E[x(k)] + \sum_{i=0}^{1} CA^{1-i}Bu(k+i) \\ \vdots \\ CA^{N_c}E[x(k)] + \sum_{i=0}^{N_c-1} CA^{N_c-1-i}Bu(k+i) \end{bmatrix}$$

which can be expressed as,

$$\mathbf{y} = \mathbf{F}\hat{x}(k) + \mathbf{Hu}$$

where $\hat{x}(k) = E[x(k)]$, $\mathbf{H}$ is a block-lower triangular matrix with its non-null elements defined by $(\mathbf{H})_{ij} = CA^{i-j}B$ and matrix $\mathbf{F}$ is defined as:

$$\mathbf{F} = \begin{bmatrix} CA \\ CA^2 \\ \vdots \\ CA^{N_c} \end{bmatrix}$$

and $\mathbf{u} = [u(k)' \dots u(k+N_c-1)']'$ is the vector of the N_c future control actions.

Remark 1. The prediction equation requires an unbiased estimation of the state vector $x(k)$. If the state vector is not accessible, a Kalman filter is required.

At each step, the control action $u(k)$ is obtained by minimizing with respect to the sequence $\mathbf{u}$ of future control moves the following performance index:

$$J(k) = (\mathbf{y} - \mathbf{y^o})'R(\mathbf{y} - \mathbf{y^o}) + \mathbf{u}'Q\mathbf{u} \qquad (2)$$

where $\mathbf{y^o} = [y^0(k+1)' \dots y^o(k+N_c)']'$ is the vector of the future evolution of the reference trajectory, $Q \geq 0$ and $R > 0$.

3.2 *Problem formulation*

The control torques generated by the magnetic coils are given by the expression:

$$T_{contr}(k) = m(k) \wedge b(k) = \overline{B}(b(k))m(k) \qquad (3)$$

where

$$\overline{B}(b(k)) = \begin{bmatrix} 0 & b_z(k) & -b_y(k) \\ -b_z(k) & 0 & b_x(k) \\ b_y(k) & -b_x(k) & 0 \end{bmatrix}$$

is a matrix the elements of which are constituted by instantaneous measurements of the geomagnetic field vector $b(k) \in \mathbb{R}^3$ and $m(k) \in \mathbb{R}^3$ is the vector of the coils' magnetic dipoles.
The control torque of the magnetic actuated satellite always lies perpendicular to the geomagnetic field vector and a magnetic moment generated in the direction parallel to the local geomagnetic field has no influence on the satellite's motion.
A common approach in literature (Wisniewski and Blanke, 1999; Lovera *et al.*, 2002) is to combine the linearized dynamics derived in Section (2.2) with a periodic approximation of the geomagnetic field in order to obtain a complete periodic model of the local spacecraft dynamics. Then periodic optimal control techniques are applied in order to solve the attitude control problem.
In this paper, the topic idea is to consider the system as a linear time-invariant one and to insert a

constraint on the control torque, which guarantees the orthogonality between the geomagnetic field vector $b(k)$ and the control vector $T_{contr}(k)$.
To this purpose, let us consider the overall linearized system (1) together with the performance index (2), where the control variable is $u(k) = T_{contr}(k)$, and the following constraint:

$$b'(k)u(k) = 0$$

which can be expressed in term of $\mathbf{u}$ as

$$\mathbf{G}\mathbf{u} = \left[b'(k)\ 0 \dots\ 0\right]\mathbf{u} = 0 \qquad (4)$$

By the use of Lagrange multipliers (Luenberger, 1984), the optimal solution of the constrained minimization problem defined by (2) and (4) is given by

$$\mathbf{u}_{opt} = \left[\Lambda\left(I - \mathbf{G}'\left(\mathbf{G}\Lambda\mathbf{G}'\right)^{-1}\mathbf{G}\Lambda\right)\right] \mathbf{H}'R\left(\mathbf{y^o} - \mathbf{F}\hat{x}\right) \qquad (5)$$

where

$$\Lambda = \left(\mathbf{H}'R\mathbf{H} + Q\right)^{-1}.$$

Note that matrix $(\mathbf{H}'R\mathbf{H} + Q)$ is always non singular (and so invertible), due to the choice of the weights $Q \geq 0$ and $R > 0$.
According to a receding horizon strategy, equation (5) has to be evaluated at every sampling time, while only the first element of $\mathbf{u}_{opt}$ is effectively used as control signal $u_{opt}(k)$. The optimum control $u_{opt}(k)$ has a periodic structure, due to the fact that the matrix $\mathbf{G}$ depends on the geomagnetic field vector $b(k)$ which is periodic.
The vector of the coils' magnetic dipoles $m(k)$ is then obtained by the following equation:

$$\begin{cases} u(k)'u(k) = \left[m(k) \wedge b(k)\right]' u(k) \\ \qquad\qquad = \left[b(k) \wedge u(k)\right]' m(k) \\ u(k)'m(k) = 0 \\ b(k)'m(k) = 0 \end{cases}$$

Note that the last two equation impose the perpendicularity between all the three vectors: the orthogonality between $u(k)$ and $m(k)$ comes directly from equation (3), while the orthogonality between $b(k)$ and $m(k)$ is a design degree of freedom, used to minimize the norm of the vector of the coils' magnetic dipoles.
Finally, the vector of the coils' magnetic dipoles $m_{opt}(k)$ can be expressed as:

$$m_{opt}(k) = \begin{bmatrix} (b(k) \wedge u_{opt}(k))' \\ u_{opt}(k)' \\ b(k)' \end{bmatrix}^{-1} \begin{bmatrix} |u_{opt}(k)|^2 \\ 0 \\ 0 \end{bmatrix}$$

Remark 2. The control problem is well-posed since the orthogonality between $u(k)$ and $b(k)$ is a necessary and sufficient condition for the existence of a magnetic dipole $m_{opt}(k)$ such that $u_{opt}(k) = m_{opt}(k) \wedge b(k)$.

3.3 *Stability analysis*

While it would be relatively straightforward to ensure closed loop stability by means of an appropriate terminal constraint in the cost function of the control problem, it is interesting to notice that the proposed approach leads to a control law which has a very similar (PD-like) structure to the ones which are used in the engineering practice of attitude control. Also, while a stability analysis in the most general case is a very difficult task, some results can be proven if an inertially spherical spacecraft is considered. It will be assumed in the following that the spacecraft has an inertia matrix which is proportional to the identity matrix, *i.e.*

$$\mathbf{I} = \mathcal{I}\ I_{3\times 3} \quad .$$

In this case, neglecting the effect of the momentum wheel, the overall linearized system becomes

$$\delta x(k+1) = \hat{A}\delta x(k) + \hat{B}\left[u(k) + T_{dist}(k)\right]$$

where $\hat{B} = \begin{bmatrix} 0_{3\times 3} \\ \Delta\,\mathbf{I}^{-1} \end{bmatrix}$, $\hat{A} = \begin{bmatrix} I_{3\times 3} & \frac{\Delta}{2} I_{3\times 3} \\ 0_{3\times 3} & I_{3\times 3} \end{bmatrix}$.
Let us consider the performance index (2), with weights $Q = \gamma I_{6N_c \times 6N_c}$ and $R = \rho I_{3N_c \times 3N_c}$. The optimum control action (5) can be expressed as

$$u_{opt}(k) = -\frac{\mathcal{I}}{\Delta}\left[I_{3\times 3} - \frac{b(k)b(k)'}{|b(k)|^2}\right]\left[k_p\,\delta\hat{q} + k_d\,\delta\hat{\omega}\right]$$

where both the proportional gain k_p and the derivative gain k_d are function of the sample time Δ, of the weight coefficients γ and ρ and of the prediction horizon N_c. Conditions for global attitude regulation under the above assumptions have been obtained in (Lovera and Astolfi, 2001) for the case of continuous time controller implementation, therefore future work will aim at bridging the gap between the continuous time analysis and the discrete time implementation of the controller.

4. SIMULATION RESULTS

In this section, some simulation results obtained by the application of the above described model-based predictive control techniques to the dynamic of the MITA spacecraft are presented.
MITA is a three axis stabilized satellite carrying on board NINA, a silicon spectrometer for charged particles developed by INFN (Istituto Nazionale di Fisica Nucleare). The selected orbit for the MITA mission is a circular one, with an altitude of 450 km and an inclination of 87.3^o, provided by a COSMOS launcher. The moments of inertia are $I_{xx} = 36$, $I_{yy} = 17$, $I_{zz} = 26$, I_{xy}=1.5, $I_{xz} = I_{yz} = 0$ kgm^2 and the attitude control system shall ensure a three axis stabilization with the NINA detector always pointing in the opposite direction of the Earth (Zenith). For the purpose of the present study, the attitude control system is

composed by the following sensors: 1 star sensor, 1 triaxial magnetometer (redundant), 5 coarse sun sensors (redundant). The attitude actuators are 1 momentum wheel, mounted along the z body axis, with moment of inertia $J = 0.01$ kgm^2 and angular velocity $\nu = 200$ rad/s and 3 magnetic coils (redundant).

In the numerical calculations, the orbit is divided in 102 samples ($\Delta = 55$s), the control horizon is $N_c = 40$ steps (which corresponds to a prediction horizon of about 40 % of an orbit), the weights are $Q = \text{diag}\{10^4, 10^4, 10^2\}\, I_{6N_c \times 6N_c}$ and $R = \text{diag}\{10^2, 10^2, 10^4\}\, I_{3N_c \times 3N_c}$ and the initial attitude is $q_0 = [0.1\ 0.1\ 0.1\ 0.9849]'$ (which correspond to the attitude angles $\{yaw, roll, pitch\} = [10.3\ 12.3\ 10.3]'$). The presence of an external torque due to a residual magnetic dipole of the spacecraft of an intensity of 1Am^2 along each body axis was assumed together with a secular disturbance torque along the pitch axis of 10^{-4}Nm.

A first set of simulation was carried out assuming no constraints on the control variables and the results are shown in Figures 2-5. A second set of simulations was carried out assuming an amplitude limit of ± 30Am^2 in the control signal and the results are shown in Figures 6-9. As can be seen, in the first set of simulations the control variables violate the amplitude constraint, while in the second set of simulations constraint violation is avoided. Obviously a price has to be paid in term of performance of the controlled variables.

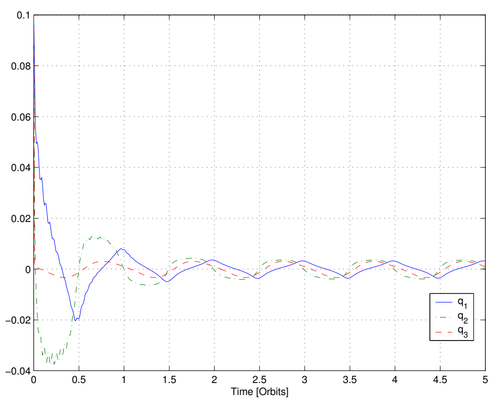

Fig. 2. Vector part of the attitude quaternion: simulation without constraints on the control variables.

5. CONCLUDING REMARKS

The attitude control problem for a small spacecraft using magnetic actuators has been considered and analyzed in the framework of model-based predictive control. An original approach to the control problem as been proposed and a solution in terms of classical model-based predictive control has been given. Simulation results show that good performance can be obtained by means of this approach.

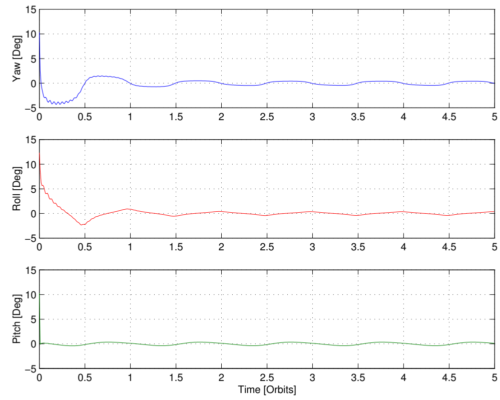

Fig. 3. Attitude angles: simulation without constraints on the control variables.

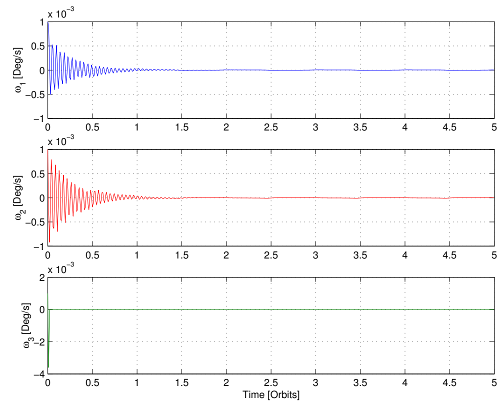

Fig. 4. Attitude angular rates: simulation without constraints on the control variables.

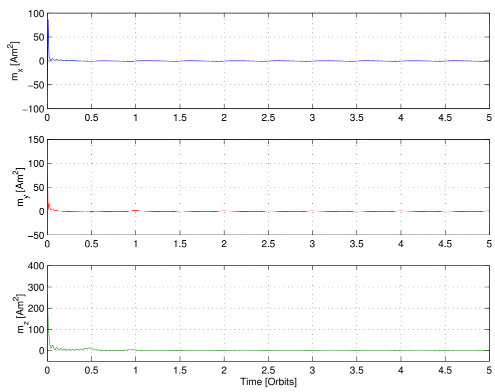

Fig. 5. Coils' magnetic dipoles: simulation without constraints on the control variables.

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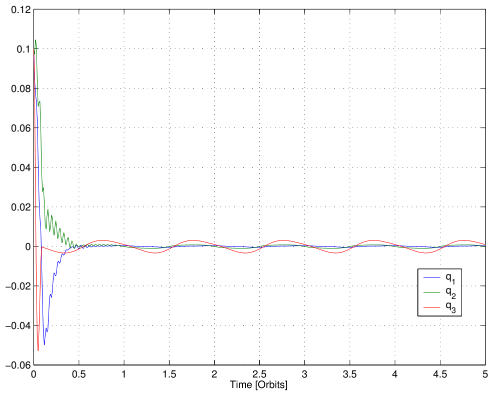

Fig. 6. Vector part of the attitude quaternion: simulation with constraints on the control variables.

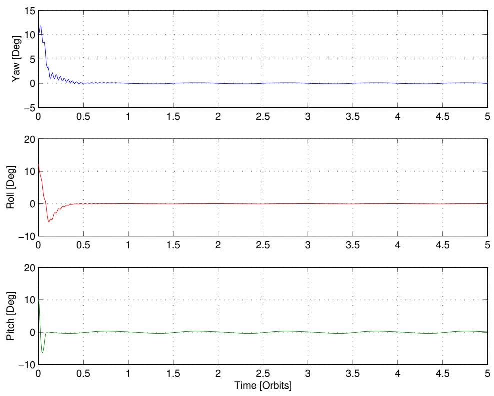

Fig. 7. Attitude angles: simulation with constraints on the control variables.

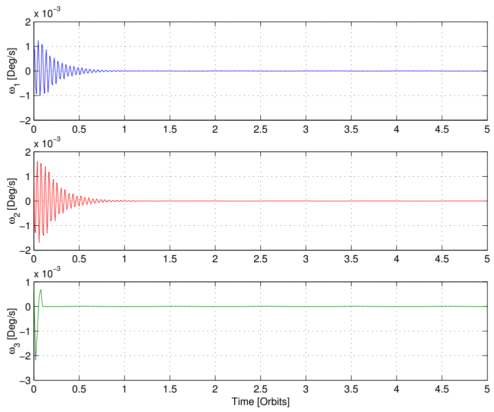

Fig. 8. Attitude angular rates: simulation with constraints on the control variables.

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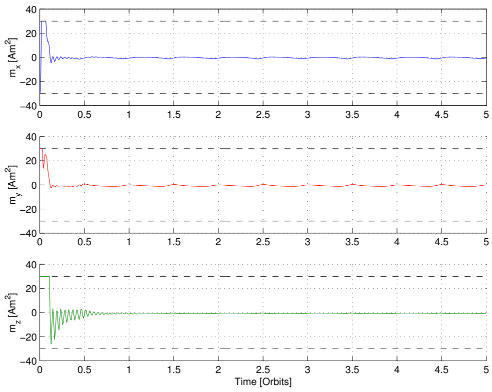

Fig. 9. Coils' magnetic dipoles: simulation with constraints on the control variables.

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