15th Triennial World Congress, Barcelona, Spain

DETECTION FILTER DESIGN FOR LPV SYSTEMS – A GEOMETRIC APPROACH

Z. Szabó [**] **J. Bokor** [**,1] **G. Balas** [*]

[*] *Department of Aerospace Engineering and Mechanics, University of Minnesota*
[**] *Computer and Automation Research Institute, Budapest Hungarian Academy of Sciences Kende u. 13–17. H–1111 Budapest, Hungary*
e-mail: szaboz@decst.scl.sztaki.hu

Abstract: By using the concept of parameter varying (C,A)-invariant subspace and parameter varying unobservability subspace, this paper investigates the problem of fault detection and isolation in linear parameter varying (LPV) systems. The so called detection filters approach, formulated as the fundamental problem of residual generation (FPRG) for linear time invariant (LTI) systems is extended for a class of LPV systems. The question of stability is addressed in the terms of Lyapunov quadratic stability by using an LMI technique.

Keywords: Failure detection, LPV systems,

1. PROBLEM FORMULATION

There are various approaches to residual generation, see e.g. the detection filter approach initiated by Massoumnia (1986) for LTI systems and used also by Edelmayer *et al.* (1997),Keviczky *et al.* (1993) for LTV systems and by Hammouri *et al.* (1999) for bilinear systems, the dedicated observers and the parity space approaches Gertler (1998), the multiple model and the generalized likelihood ratio approaches, just to mention a few. These approaches are used in a number of situations differing in the assumptions on noise, disturbances, robustness properties and in the specific design methods, see some important representations in the literature,(Basseville, 1988), (Frank and Ding, 1997),(Isermann, 1997), (Chen and Patton, 1999), (Mangoubi, 1998).

[1] This research was supported in part by NASA Langley, NASA Grant NCC-1-337 Dr. Christine M. Belcastro technical monitor and in part by the Hungarian National Science Foundation (OTKA) under Grant T 030182.

Throughout this paper the problem of fault detection and isolation for the class of linear parameter–varying (LPV) systems of which state matrix depends affinely on the parameter vector will be considered. This class of systems can be described as:

$$\dot{x}(t) = A(\rho)x(t) + B(\rho)u(t) + \sum_{j=1}^{m} L_j(\rho)v_j(t)$$
$$y(t) = Cx(t), \tag{1}$$

where v_j are the failures to be detected, C is right invertible,

$$A(\rho) = A_0 + \rho_1 A_1 + \cdots + \rho_N A_N, \tag{2}$$
$$B(\rho) = B_0 + \rho_1 B_1 + \cdots + \rho_N B_N, \tag{3}$$
$$L_j(\rho) = L_{j,0} + \rho_1 L_{j,1} + \cdots + \rho_N L_{j,N}, \tag{4}$$

and ρ_i are time varying parameters. It is assumed that each parameter ρ_i and its derivatives $\dot{\rho}_i$ ranges between known extremal values $\rho_i(t) \in$

$[-\overline{\rho}_i, \overline{\rho}_i]$ and $\dot{\rho}_i(t) \in [-\overline{\dot{\rho}}_i, \overline{\dot{\rho}}_i]$, respectively. Let us denote this parameter set by $\mathcal{P}$.

For the linear time invariant case – when ρ is constant – the problem of designing a (stable) filter capable of detecting the occurrence of a specific unmeasured input within a prescribed set is sometimes referred as the fundamental problem of residual generation (FPRG). In Massoumnia *et al.* (1989) it was shown that the existence of the solution of the FPRG depends on the relation between the subspace $\mathcal{L}$ determined by the direction of the failure to be detected and the minimal unobservability subspace containing the rest of the failure directions. The aim of this paper to extend this result to the LPV systems (1).

2. INVARIANT SUBSPACES

In the so called "geometrical approach" to fault detection a central role is played by the (C,A)-invariant subspaces and certain unobservability subspaces, (Massoumnia, 1986; Massoumnia *et al.*, 1989) or observability codistributions, (Persis and Isidori, 2000a). As it is well known, for LTI models, a subspace $\mathcal{W}$ is (C,A)-invariant if $A(\mathcal{W} \cap KerC) \subset \mathcal{W}$ that is equivalent with the existence of a matrix G such that $(A+GC)\mathcal{W} \subset \mathcal{W}$. A (C,A)-unobservability subspace $\mathcal{U}$ is a subspace such that there exist matrices G and H with the property that $(A+GC)\mathcal{U} \subset \mathcal{U}$, i.e., $\mathcal{U}$ is (C,A)-invariant, and $\mathcal{U} \subset KerHC$. The family of (C,A)-unobservability subspaces containing a given set $\mathcal{L}$ has a minimal element $\mathcal{U}^*$.

For the parameter varying case one can extend these notions, and introduce the *parameter varying (C,A)-invariant subspaces*, as follows:

Definition 1. Let $\mathcal{C}(\pi)$ denote $KerC(\rho)$. Then a subspace $\mathcal{W}$ is called a parameter varying (C,A)-invariant subspace if for all the parameters $\rho \in \mathcal{P}$:

$$A(\rho)(\mathcal{W} \cap \mathcal{C}(\rho)) \subset \mathcal{W}. \qquad (5)$$

As in the classical case one has the following characterization of the parameter varying (C,A)-invariant subspaces:

Proposition 2. $\mathcal{W}$ is a parameter varying (C,A)-invariant subspace if and only if for any $\rho \in \mathcal{P}$ there exists a state feedback matrix $G(\rho)$ such that

$$(A(\rho) + G(\rho)C(\rho))\mathcal{W} \subset \mathcal{W}. \qquad (6)$$

The set of all parameter varying (C,A)-invariant subspaces containing a given subspace $\mathcal{B}$, is a lower semilattice with respect to the intersection of subspaces. This semilattice admits a minimum, denoted by

$$\mathcal{W}^*_{p.v.}(\mathcal{B}) := \min \mathcal{W}(C(\rho), A(\rho), \mathcal{B}). \qquad (7)$$

In the "geometrical approach" to fault detection for LTI systems an important role play certain unobservability subspaces, too. The notion of "unobservability subspace" extends to the notion of "unobservability distribution" for the larger class of not time invariant systems. It can be shown, see e.g. (Persis and Isidori, 2000a), that for the LPV systems considered in this paper the "unobservability distribution" $\mathcal{U}$ can be described as the largest subspace such that there exist a parameter dependent gain matrix $G(\rho)$ and constant output mixing map H such that

$$(A(\rho) + G(\rho)C(\rho))\mathcal{U} \subset \mathcal{U}, \text{ for all } \rho \in \mathcal{P}, \qquad (8)$$
$$\mathcal{U} \subset KerHC. \qquad (9)$$

For the LPV systems (1) one can obtain the following algorithm for the computation of the smallest (parameter varying) unobservability subspace $\mathcal{U}^*$ containing $\mathcal{W}$:

$$\mathcal{U}_0 = \mathcal{W} + KerC$$
$$\mathcal{U}_k = \mathcal{W} + (\cap_{k=0}^N A_k^{-1}\mathcal{U}_{k-1}) \cap KerC,$$

for details see (Kabore *et al.*, 2000), in the context of bilinear systems.

Let us recall the fact, see (Massoumnia, 1986; Kabore *et al.*, 2000), that there exist matrices $H, G(\alpha)$ such that $\mathcal{U}^*$ is a parameter varying $(HC, A(\alpha) + G(\alpha)C)$-invariant subspace. Moreover, if one starts with a minimal subspace $\mathcal{W}^*$, given by one of the algorithm presented above, then $KerHC = \mathcal{W}^* + KerC$ and $G(\alpha)$ is determined by $\mathcal{W}^*$.

3. THE FUNDAMENTAL PROBLEM OF RESIDUAL GENERATION

Let us consider the following LTI system, that has two failure events:

$$\dot{x}(t) = Ax(t) + Bu(t) + L_1 m_1(t) + L_2 m_2(t) \qquad (10)$$
$$y(t) = Cx(t), \qquad (11)$$

then the task to design a residual generator that is sensitive to L_1 and insensitive to L_2 is called the FPRG.

Let us denote by $\mathcal{S}^*$ the smallest unobservability subspace containing $\mathcal{L}_2$, where $\mathcal{L}_i = ImL_i$. Then one has the following result, (Massoumnia *et al.*, 1989):

Proposition 3. A FPRG has a solution if and only if $\mathcal{S}^* \cap \mathcal{L}_1 = 0$, moreover, if the problem has a solution, the dynamics of the residual generator can be assigned arbitrary.

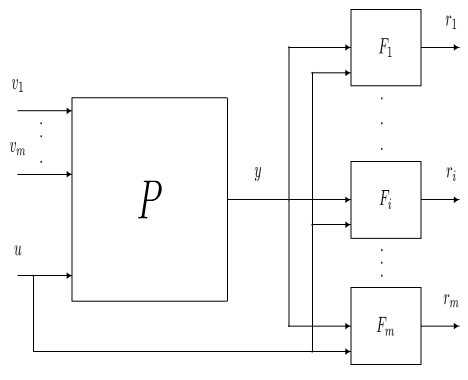

Fig. 1. Block diagram of residual generator.

Given the residual generator in the form

$$\dot{w}(t) = Nw(t) - Gy(t) + Fu(t) \tag{12}$$
$$r(t) = Mw(t) - Hy(t), \tag{13}$$

then H is a solution of $kerHC = KerC + \mathcal{S}^*$, and M is the unique solution of $MP = HC$, where P is the projection $P : \mathcal{X} \to \mathcal{X}/\mathcal{S}^*$. Let us consider a G_0 such that $(A + G_0C)\mathcal{S}^* \subset \mathcal{S}^*$, and denote by $A_0 = A + G_0C|_{\mathcal{X}/\mathcal{S}^*}$. Then there is a G_1 such that $N = A_0 + G_1M$ has prescribed eigenvalues. Then set $G = PG_0 + G_1H$ and $F = PB$.

Extending this result to the case with multiple events one has the extension of the fundamental problem of residual generation (EFPRG), that has a solution if and only if $\mathcal{S}_i^* \cap \mathcal{L}_i = 0$, where $\mathcal{S}_i^*$ is the smallest unobservability subspace containing $\overline{\mathcal{L}}_i := \sum_{j \neq i} \mathcal{L}_j$.

These ideas were also applied to nonlinear systems, see (Kabore *et al.*, 2000; Persis and Isidori, 2000b). In what follows the method presented in (Persis and Isidori, 2000b) for bilinear systems will be modified in order to fit in the LPV context. The assertion of the Proposition 3 remains valid also for the LPV systems (1), i.e.,

Proposition 4. For the LPV systems (1) one can design a – not necessarily stable – residual generator of type

$$\dot{w}(t) = N(\rho)w(t) - G(\rho)y(t) + F(\rho)u(t) \tag{14}$$
$$r(t) = Mw(t) - Hy(t), \tag{15}$$

if and only if for the smallest (parameter varying) unobservability subspace $\mathcal{U}^*$ containing $\mathcal{L}_2$ one has $\mathcal{U}^* \cap \mathcal{L}_1 = 0$, where $\mathcal{L}_i = \cup_{j=0}^{N} ImL_{i,j}$.

Proof Let H be the solution of $kerHC = KerC + \mathcal{U}^*$, and M is the unique solution of $MP = HC$, where P is the projection $P : \mathcal{X} \to \mathcal{X}/\mathcal{U}^*$. By the definition of the unobservabilitry subspaces there is a matrix $G_0(\rho)$ such that $(A(\rho) + G_0(\rho)C)\mathcal{U}^* \subset \mathcal{U}^*$ holds. Then set $A_0(\rho) = A(\rho) + G_0(\rho)C|_{\mathcal{X}/\mathcal{U}^*}$, $N(\rho) = A_0(\rho)$ and $F = PB(\rho)$.

One can compute an acceptable $G_0(\rho)$ as follows, see (Persis and Isidori, 2000a; Persis and Isidori, 2000b): let H_1 be the matrix that completes H to a nonsingular matrix and let us consider a matrix K_1 that has as raws the coordinates of the basis vectors for $\mathcal{X} \ominus \mathcal{U}^*$. Let us denote by $K = \begin{bmatrix} K_1 \\ H_1C \\ K_3 \end{bmatrix}$, where K_3 is an arbitrary matrix that makes K nonsingular.

Then

$$KA(\rho)K^{-1} = \begin{bmatrix} A_{11}(\rho) & A_{12}(\rho) & 0 \\ A_{21}(\rho) & A_{22}(\rho) & A_{23}(\rho) \\ A_{31}(\rho) & A_{32}(\rho) & A_{33}(\rho) \end{bmatrix},$$

and the matrix $G_0(\rho)$ can be chosen as

$$G_0(\rho) = K^{-1} \begin{bmatrix} 0 & -A_{12}(\rho) \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} H \\ H_1 \end{bmatrix}.$$

4. THE QUESTION OF STABILITY

An LPV systems (1) is said to be quadratically stable if there exist a matrix $P = P^T > 0$ such that

$$A(\rho)^T P + PA(\rho) < 0 \tag{16}$$

for all the parameters $\rho \in \mathcal{P}$. A necessary and sufficient condition for a system to be quadratically stable is that the condition (16) holds for all the corner points of the parameter space, i.e., one can obtain a finite system of LMI's that has to be fulfilled for $A(\rho)$ with a suitable positive definite matrix P, see (Gahinet *et al.*, 1996),(Becker and Packard, 1994),(Fen *et al.*, 1996),(Packard and Becker, 1992).

In order to obtain a quadratically stable residual generator one can set $N(\rho) = A_0(\rho) + G(\rho)M$ in (14), where $G(\rho) = G_0 + \rho_1 G_1 + \cdots \rho_N G_N$ is determined such that the LMI defined in (16),i.e.,

$$(A_0(\rho) + G(\rho)M)^T P + P(A_0(\rho) + G(\rho)M) < 0$$

holds for suitable $G(\rho)$ and $P = P^T > 0$. By introducing the auxiliary variable $K(\rho) = G(\rho)P$, one has to solve the following set of LMIs on the corner points of the parameter space:

$$A_0(\rho)^T P + PA_0(\rho) + M^T K(\rho)^T + K(\rho)M < 0.$$

Remark 5. If $KerC \subset \mathcal{U}^*$ then one can choose $G(\rho)$ such that the matrix $N(\rho)$ be parameter independent with arbitrary eigenvalues, since the equation $G(\rho)CU = UT - A(\rho)U$ has a solution for arbitrary T, where U is the insertion map of $\mathcal{X}/\mathcal{U}^*$.

5. EXAMPLE

As an illustrative example let us consider the following linearized parameter varying model of an aircraft:

$$\begin{aligned} \dot{x}(t) &= A(\rho)x(t) + Bu(t) + L_1 v_1(t) + L_2 v_2(t) \\ y(t) &= Cx(t), \end{aligned}$$

where $A(\rho) = A_0 + \rho_1 A_1 + \rho_2 A_2$. It is assumed that the parameter ρ_1 and ρ_2 vary in the intervals $[-0.3, 0.3]$ and $[-0.6, 0.6]$, respectively. The state matrices are:

$$A_0 = \begin{bmatrix} -1.05 & -2.55 & 0 & 0 & -169.66 & -0.0091 \\ 2.55 & -1.05 & 0 & 0 & 57.09 & 0.0017 \\ 0 & 0 & -77.53 & 39.57 & 0 & 0 \\ 0 & 0 & 0 & -20.20 & 0 & 0 \\ 0 & 0 & -8.80 & 0 & -20.20 & 0 \\ 0 & 0 & 0 & 0 & 0 & -0.1000 \end{bmatrix},$$

$$A_1 = \begin{bmatrix} 0&1&0&0&1&0 \\ 1&0&0&0&1&0 \\ 0&0&0&0&0&0 \\ 0&0&0&0&0&0 \\ 0&0&0&0&0&0 \\ 0&0&0&0&0&0 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 1&0&0&0&0&0 \\ 0&1&0&0&0&0 \\ 0&0&0&0&0&0 \\ 0&0&0&0&0&0 \\ 0&0&0&0&0&0 \\ 0&0&0&0&0&0 \end{bmatrix},$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -4.4944 \\ 0 \\ 0 \end{bmatrix}, \quad L = \begin{bmatrix} 0 & 3.55 & 2.41 \\ 0 & -0.55 & 8.04 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1.00 & -0.02 & 0.56 \\ 0 & 0 & 0 \end{bmatrix},$$

$$C = \begin{bmatrix} -0.01 & 0.09 & 0.07 & 0 & 0.00 & -0.0000 \\ -0.48 & -0.59 & 0.00 & 0 & -49.51 & -0.0026 \\ 0.03 & 0.09 & -0.06 & 0 & -0.00 & 0.0000 \\ 0.26 & -0.07 & 0.01 & 0 & 0.00 & -0.0000 \end{bmatrix}.$$

The actuator fault of the elevator is modelled by the first column of L, the rest is a model of a sensor fault. After performing the proposed algorithm and solving the LMIs one can get the following state matrices for the filter that detect the actuator fault:

$$N_{10} = \begin{bmatrix} -0.50 & -10.06 & 0 & -0.0007 \\ 11.98 & 18.70 & 39.57 & -0.0027 \\ -5.89 & -58.69 & -20.20 & 0.0016 \\ 0.0012 & 0.07 & 0 & -0.1000 \end{bmatrix},$$

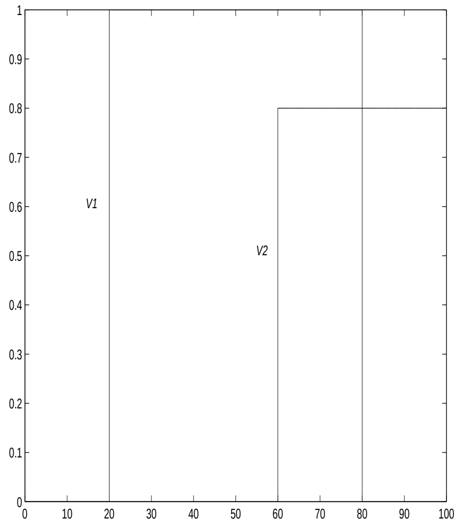

Fig. 2. Actuator fault input v_1. Sensor fault input v_2.

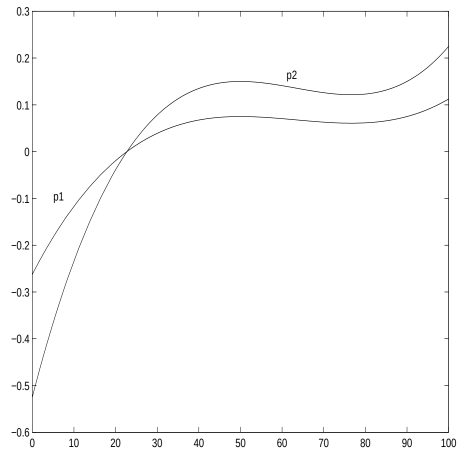

Fig. 3. Parameter variation for the simulation: p1 stands for ρ_1 and p2 for ρ_2.

$$N_{11} = \begin{bmatrix} -0.0000 & 9.62 & 0 & -0.0002 \\ -11.4713 & 0.00 & 0 & 0.0006 \\ 5.6373 & 0.00 & 0 & -0.0003 \\ 0.0004 & -0.01 & 0 & 0.0000 \end{bmatrix},$$

$$N_{12} = \begin{bmatrix} 0.00 & -0.1274 & 0 & 0 \\ 0.15 & -0.0000 & 0 & 0 \\ -0.07 & -0.0000 & 0 & 0 \\ 0.00 & 0.0002 & 0 & 0 \end{bmatrix},$$

$$G_{10} = \begin{bmatrix} -139.29 & 0.46 & 124.87 & -22.81 \\ 723.63 & 0.24 & -623.68 & 116.53 \\ -440.79 & -0.11 & 380.99 & -71.07 \\ 0.58 & 0.00 & -0.51 & 0.09 \end{bmatrix},$$

$$G_{11} = \begin{bmatrix} 71.84 & 0.0015 & -63.04 & 11.4145 \\ -4.36 & -0.2311 & -4.99 & 0.0000 \\ 2.14 & 0.1136 & 2.45 & 0.0001 \\ -0.12 & 0.0000 & 0.10 & -0.0202 \end{bmatrix},$$

$$G_{12} = \begin{bmatrix} -1.2686 & 0.0000 & 0.4758 & -0.1264 \\ 0.0578 & 0.0031 & 0.0661 & -0.0000 \\ -0.0284 & -0.0015 & -0.0325 & -0.0000 \\ 0.0017 & -0.0000 & -0.0014 & 0.0003 \end{bmatrix},$$

$$M_1 = \begin{bmatrix} -1.00 & 0.00 & 0 & 0.0001 \\ 0.00 & -1.00 & 0 & 0.0000 \end{bmatrix},$$

$$H_1 = \begin{bmatrix} -0.38 & -0.02 & -0.43 & 0.00 \\ -7.47 & -0.00 & 6.53 & -1.21 \end{bmatrix},$$

and the state matrices for the filter that detect the sensor fault:

$$N_{20} = \begin{bmatrix} -1.76 & -1.69 & -2.26 & 0.00 & 0.00 \\ 2.79 & -4.15 & -14.09 & 0.00 & 0.00 \\ -0.15 & -2.04 & 14.48 & -39.57 & 0.00 \\ -1.18 & -3.08 & 61.03 & -20.20 & 0.00 \\ 3357.00 & 4178.00 & 37997.00 & 0.00 & -1.00 \end{bmatrix},$$

$$N_{21} = \begin{bmatrix} -0.01 & -0.06 & 0.00 & 0.00 & 0.00 \\ -0.07 & -0.17 & 0.01 & 0.00 & 0.00 \\ -0.01 & -0.01 & 0.00 & 0.00 & 0.00 \\ 0.04 & 0.07 & 0.00 & 0.00 & 0.00 \\ 223.00 & 373.00 & -9.00 & 0.00 & 0.00 \end{bmatrix},$$

$$N_{22} = \begin{bmatrix} 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & -0.01 & 0.01 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ -0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 2.00 & -17.00 & 0.00 & 0.00 \end{bmatrix},$$

$$G_{20} = \begin{bmatrix} -24.62 & -3.42 & 4.50 & -10.49 \\ -115.45 & 1.15 & 81.31 & -12.88 \\ 678.06 & 0.00 & -611.98 & 111.99 \\ 440.84 & 0.00 & -416.64 & 71.39 \\ 299410.00 & 0.00 & -233220.00 & 31778.00 \end{bmatrix},$$

$$G_{21} = \begin{bmatrix} -4.73 & 0.02 & -5.40 & 0.56 \\ -1.77 & 0.02 & -2.78 & -3.66 \\ -0.06 & 0.00 & -0.08 & -0.03 \\ 0.39 & -0.00 & 0.49 & 0.12 \\ 1847.00 & 0.00 & 2360.00 & 570.00 \end{bmatrix},$$

$$G_{22} = \begin{bmatrix} -1.00 & 0.00 & -1.84 & -3.5 \\ -4.44 & 0.00 & -5.17 & 0.56 \\ -0.04 & 0.00 & -0.05 & 0.00 \\ -0.04 & -0.00 & -0.00 & -0.01 \\ -120.00 & 0.00 & 120.00 & 20.00 \end{bmatrix},$$

$$M_2 = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \end{bmatrix},$$

$$H_2 = \begin{bmatrix} -1.01 & 0.00 & -1.82 & -3.55 \\ -4.52 & 0.00 & -5.07 & 0.55 \\ -7.47 & 0.00 & 6.53 & -1.21 \end{bmatrix}.$$

The simulation result is depicted on Figure 4., when at the fault inputs step signals from Figure 2. were applied. The parameter variation in the simulation is depicted on Figure 3.

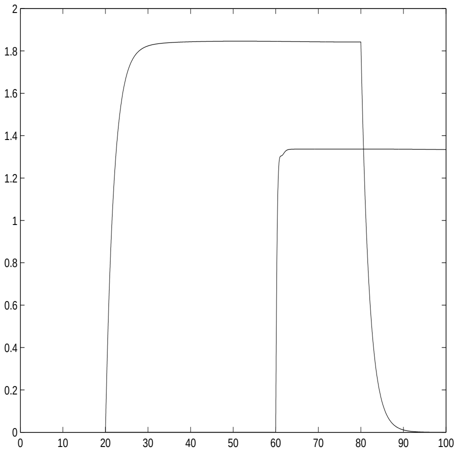

Fig. 4. Simulation result for the residual generator.

6. CONCLUSION

The detection filter approach elaborated for fault detection and isolation in LTI systems was extended for a class of LPV systems. The parameter dependence in the state matrix of these LPV systems was assumed in affine form. It was shown that the filter gain will be parameter dependent, and a procedure was derived to obtain them by computing a suitable system of invariant subspaces. The approach proposed can be extended to include disturbance decoupling or to the design of detection filters being robust against modeling uncertainties. The question of stability was addressed in the terms of Lyapunov quadratic stability by using a LMI technique.

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