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ANALYSIS TOOLBOX STRESSING PARALLELISM OF SISO AND MIMO PROBLEMS

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Abstract:
In the paper the toolbox for dynamical system analysis is presented and illustrated for the model of real multivariable plant. Its possibilities can be observed in several different ways. The functions realized in Matlab program package can treat similarly SISO and MIMO systems in several different design phases, so they are organized in four groups: in open and closed-loop analysis and in absolute and relative validation. The access to all options is enabled also through windows oriented graphical environment. Good experiences with toolbox usage confirmed correct orientation of the discussed software. *Copyright ©2002 IFAC*

Keywords: Analysis, Validation, Single-input/single output systems, MIMO systems, Education aids

1. INTRODUCTION

Design of control systems is always related with a great number of procedures where perhaps the most often used in all design steps are different analytical operations. They are important in the modeling phase, in the phase of choosing the potentially good design algorithms, during design itself and also in the phase of result evaluation, where the solution or even a group of solutions have to be tested and compared (Atanasijević-Kunc *et al.*, 1999). These were the reasons which have motivated us to build a toolbox for analytical purposes which should fulfill the following main goals:

- it should remind and guide the student through important system properties in time and frequency domain;
- it should support open and close - loop situations;
- it should be used for single-input/single-output (SISO) as well as for multivariable (MIMO) systems;
- where possible some degree of parallelism should be established in treatment of SISO and MIMO problems;
- it should also be possible to estimate the degree of matching of design results with more or less exactly described design goals and
- help in choosing potentially the best design result;
- all this operations should be enabled in user friendly environment.

We tried to realize these ideas inside programe package Matlab with Simulink (MATLAB, 1999) as we are already using it for education purposes at several courses.

2. ORGANIZATION OF ANALYTICAL FUNCTIONS

The *Toolbox for dynamic system analysis* consists of a great number of functions which represent an extension and enlargement of the functions available in Matlab, Control System Toolbox and Multivariable Frequency Domain Toolbox (Maciejowski, 1989) in this way that several functions have been added, they are organized in graphical windows and can be called also only by pushing the button. In all functions explanation and where possible graphical representation of results is added. The amount of additional explanations can be controlled by the so called communication vector.

Mentioned functions can be divided in several different ways. Some of them can be used with SISO systems, some with MIMO and some are suitable for both kind of problems. Regarding different design situations both groups are organized into four levels:

(1) *Open-loop analysis* functions are using only the information of linear system model. They are grouped into general, time and frequency domain properties as illustrated in Fig. 1. They should give the information of the properties of the process itself and help the user to choose between different design approaches.
(2) *Closed-loop analysis* functions are organized identically however in this case also the information of used controller is needed.
(3) *Absolute validation* functions are used for evaluation of matching desired design goals. In comparison with closed-loop analysis here also design goals have to be specified. Suitable and enough general specification of design goals can of course be very difficult especially in earlier design stages where needed information is not available or in situations where the user can define contradictory design goals. To avoid somehow these problems we have introduced the possibilities with which the user can define the importance of each specified design goal. In this way design goals can be specified either very precise or in a very approximate manner where perhaps all stable results are acceptable. The result of absolute validation is in the range between 0 and 1. If the result of absolute validation is 0 design solution is unacceptable as one or more design goals are violated more than allowed. If the result is 1 this means that all design goals are completely satisfied. If the result is between 0 and 1 the solution is acceptable, but all design goals are not completely fulfilled. Better are of course solutions which are closer to 1. These results can also be used for some kind of relative validation. But it can occur that several solutions have the same validation result. In this case the next level can be used.
(4) *Relative validation* functions tend to help the user to compare the efficacy of different design solutions and to prepare him to the real situation where also different kind of non-linearities have to be expected. One which is in practice always presented is for example limitation of control signals. Relative validation functions can simultaneously compare up to five design solutions. The validation result is calculated regarding the first solution. Result greater than 1 therefore means that solution is better than the first. Higher validation values mean better or more efficient solutions.

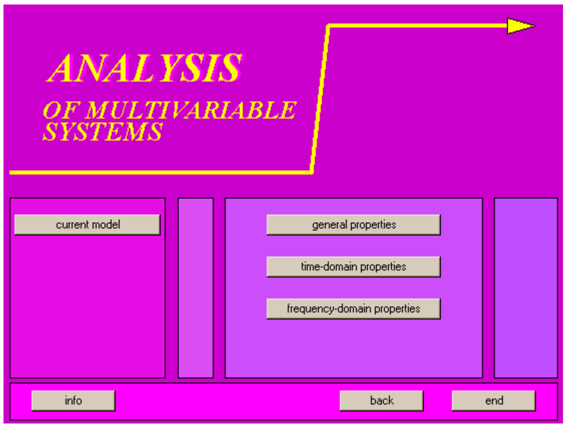

Fig. 1. Open-loop MIMO analysis

3. ILLUSTRATIVE EXAMPLE

Mentioned ideas will be illustrated for the three coupled tank system made by Amira (AMIRA, 1995) as illustrated in Fig. 2. The system consists

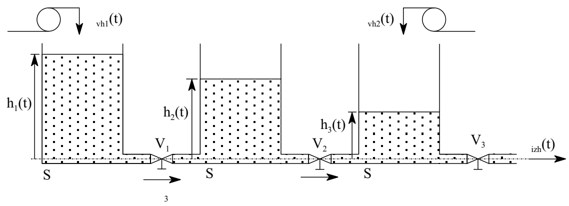

Fig. 2. Three coupled tank system

of three cylindrical coupled tanks where the level of the liquid in the first and the third tank (*outputs*) are controlled through input flows. *Input variables* are therefore voltage signals on both water pumps. There is a possibility to measure the levels (*states*) in all three tanks. All signals are in the range of ±10V. For this MIMO system the linear model of the following form was defined (Atanasijević-Kunc *et al.*, 2000):

$$\dot{x} = \begin{bmatrix} -0.0125 & 0.0126 & 0 \\ 0.0125 & -0.0246 & 0.0121 \\ 0 & 0.0120 & -0.0212 \end{bmatrix} x + \\ + \begin{bmatrix} -0.0091 & 0 \\ 0 & 0 \\ 0 & -0.0092 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} x + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} u \qquad (1)$$

By choosing to inspect the so called general properties (Fig. 3) it is possible for example to find out the values of poles and time constant of the system. As also indicated in this figure, the system has one transmission zero which can not be observed directly from transfer function matrix but represent the parallelism to SISO zeros. It is also

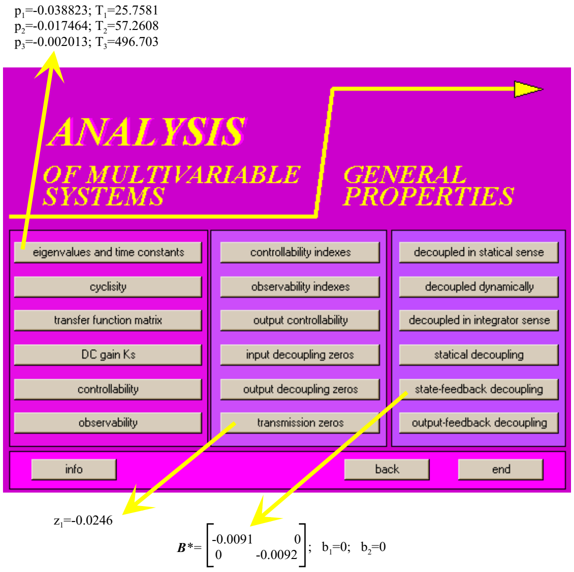

Fig. 3. Some of open-loop MIMO analysis results

possible to find out if the system can be decoupled. This information is not important only for the decoupling itself but represents the structural property of the system. The values of decoupling indexes ($b_i + 1$) give namely the information of the degree of the zeros at infinity for each input - output signal pair of MIMO - system. These structural properties can be connected with root - locus representation of MIMO systems in such way that it becomes very similar to SISO problems. The idea can be realized in connection with tuning procedures for MIMO P and PI controllers, which also represent some kind of parallelism to SISO tuning approaches. When using this type of design the controller of the following form is proposed:

$$G_{PI} = \gamma\, G_1 + \delta\, \frac{1}{s}\, G_2 \qquad (2)$$

where in the first step the so-called rough tuning matrices G_1 and G_2 are defined. They are usually chosen to be the inverse of the system at some desired frequency (Maciejowski, 1989). In the second step the so-called fine tuning scalars γ and δ are used in the same way as in SISO-cases when gains for P and I part are chosen to satisfy some close-loop properties.

By choosing the function *MV - root locus for P-tuning* from the group of frequency domain properties (Fig. 4) it is possible to observe the influence of parameter γ to the closed-loop pole-positions for the case, when rough tuning matrix of P-part is chosen to be the inverse of the system in steady state. From Fig. 5 can be concluded, that one pole (which is starting from the value -0.0020) is ending in a transmission zero, while the other two are behaving as would be the case in two uncoupled SISO systems of the first order with zero at infinity. The idea can be extended to the whole MIMO-PID structure tuning.

Suppose further that we have chosen fine tuning parameters according to (2) (Atanasijević-Kunc and Karba, 2000): $\gamma = 12$ and $\delta = 0.5$. This gives the first result:

$$G_{PI1} = \begin{bmatrix} -8.0407 & 8.1726 \\ 7.9533 & -19.9533 \end{bmatrix} + \\ + \frac{1}{s} \begin{bmatrix} -0.3350 & 0.3405 \\ 0.3314 & -0.8314 \end{bmatrix} \qquad (3)$$

which will be used later for the purposes of absolute and relative validation.

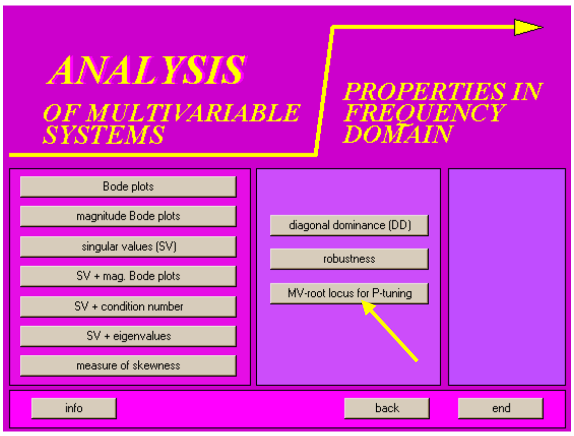

Fig. 4. Open-loop MIMO analysis - frequency domain

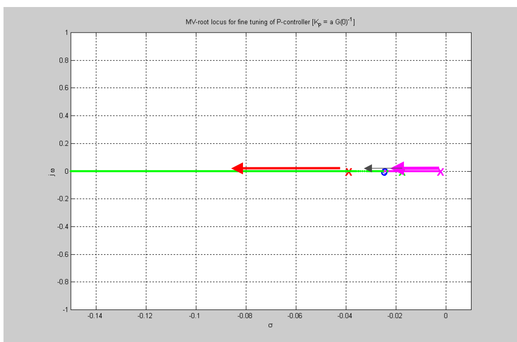

Fig. 5. MIMO-root locus for P-tuning

Parallelism to SISO - problems was realized also in time domain for well known quality indicators to step responses known as delay time, rise time, settling time and maximal overshoot, only that all these parameters are now represented by matrices (Fig. 6). The interpretation for example of settling time of decoupled system (the second closed-loop result), where the state feedback matrix F and the matrix G in direct path were chosen to be (Atanasijević-Kunc *et al.*, 1999):

$$F = \begin{bmatrix} 3.0220 & 1.3846 & 0 \\ 0 & 1.3043 & 3.1304 \end{bmatrix}$$
$$G = \begin{bmatrix} -4.3956 & 0 \\ 0 & -5.4348 \end{bmatrix} \qquad (4)$$

where further two SISO PI-controllers were added in the form:

$$G_{PI2} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \\ + \frac{1}{s} \begin{bmatrix} 0.04 & 0 \\ 0 & 0.05 \end{bmatrix} \qquad (5)$$

is illustrated in Fig. 7 for responses to the first reference signal. This property can be observed by choosing the group of closed-loop analytical functions in time-domain as represented in Fig. 6. Diagonal elements of settling time matrix are defined in the same manner as for SISO systems. For off-diagonal elements (for example for the element [2,1]) the tolerance range is shifted to the surrounding of zero as such is also the corresponding reference signal. However the width of tolerance range is defined regarding the current input (in this case 2% of the first input). From this point of view it is also obvious that cross-couplings can be interpreted in the similar way as the influence of disturbances. In the cases where the results should

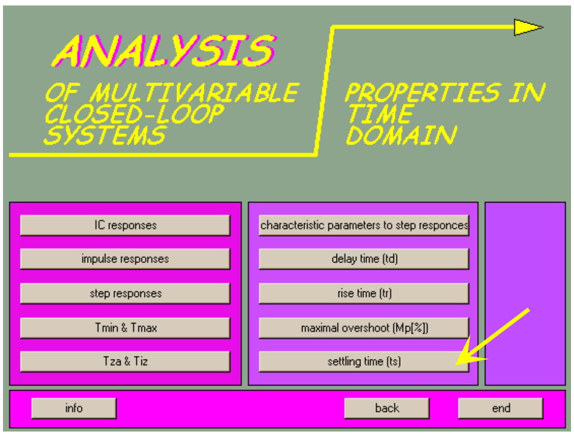

Fig. 6. Closed-loop MIMO analysis-time domain

match some desired design goals the expectations must or should be defined. In our case this is realized in the following manner. First the variable *nacin* is defined. By setting $nacin = 1$ we have chosen the situation where output signals should track corresponding references as good as possible. In the case where $nacin = 2$ good disturbance rejection should be fulfilled. When $nacin = 3$ both aspects are of the same importance. Design goals can be specified in time and frequency domain with corresponding matrices. Suppose we have chosen $nacin = 1$ and the situation where we want to specify design goals only in time domain. In this case the matrix *ciljimvc* is defined. For chosen situation it consists of six rows. Each row is connected with one of design goals: the first with stability, the second with desired minimal and maximal time constant, the third with acceptable design complexity, the fourth with desired steady-state gain, the fifth with desired settling times and the sixth with desired overshoots. The

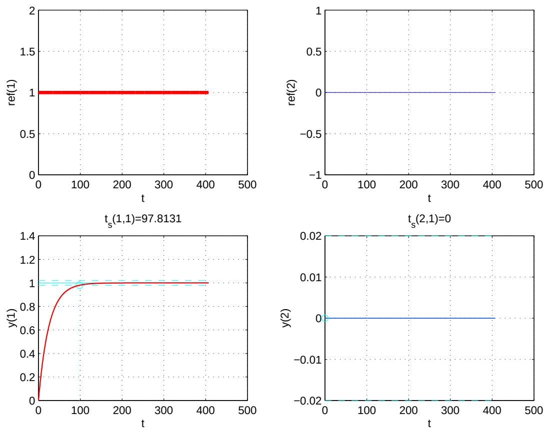

Fig. 7. Determination of settling time

elements of the first column are 1 if corresponding goal is of some importance or 0 if it is of no importance. In that case it will not influence the validation process. With the subsequent columns the so called importance and prescribed values for each goal are defined. If for example the element $ciljimvc(1,1) = 1$ and $ciljimvc(1,2) = 1$ this means that the design result is not acceptable if the closed-loop system is not stable. If $ciljimvc(2,1) = 1$ and $ciljimvc(2,2) = 0.8$ and $ciljimvc(2,3) = 100$ desired value of minimal time constant is 100 [sec]. The importance of this goal is 0.8 and the result is still acceptable if it is not violated for more then 20%. In the cases when design goal is violated more than allowed, the result of absolute validation is 0. When certain design goal is completely fulfilled the result is 1. And when it is inside the allowed tolerance the validation result value is correspondingly smaller. The overall result is calculated by multiplying all partial results. This kind of validation should stimulate to improve poorly satisfied goals while good results should not be pushed too far.

Sometimes it is of course difficult to define all these values. In such situations design goals need not to be specified explicitly. It is supposed that stable solution is desired, while all other design goals should be as good as possible and goal matrix is defined as:

$$ciljimvc = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T \qquad (6)$$

where T denotes matrix transpose. Each stable system is in this situation evaluated with 1. This is the case also for both presented results. The situation is not the same if the matrix $ciljimvc$ is defined for instance as follows:

$$ciljimvc =$$
$$= \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0.5 & 5 & 0.5 & 50 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0.8 & 500 & 2 & 0.9 & 500 & 2 & 0.9 & 500 & 2 & 0.8 & 500 & 2 \\ 1 & 0.8 & 25 & 0.8 & 25 & 0.8 & 25 & 0.8 & 25 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad (7)$$

With (7) the following design goals are specified: 1.) the system must be stable; 2.) desired minimal and maximal time constants are: $T_{min} = 5s$, $T_{max} = 50s$ but both values can be violated up to 50%; 3.)complexity of the solution is of importance; in general it can be expressed with four properties: desired controller rank, desired design freedom, with admissibility of state and output feedback; for each property the importance is first defined; so the first criterion-rank of controller is not important; the second: degree of design freedom must not be greater than 3; the third and the forth complexity criterion are of no importance and we can use state and output feedback; 4.) the closed-loop DC-gain must be the unity matrix; this is expressed with the value of importance and with desired value of certain DC-gain matrix element column-vice; 5.) settling times should all be 500s regarding 2% range; this goals can for diagonal elements be violated up to 20%, but for off diagonal only 10%; again the importance, desired value and corresponding range are defined column-vice for each matrix element; 6.) overshoots should be inside 25%, but results are still acceptable with 30% which is the upper limit for all elements.

For the controller described with (3) and design goals described with (7) the result of absolute time validation is illustrated in Fig. 8. Closer inspection shows that only the element $M_p(1,2)$ slightly exceeds the desired value and is $M_p(1,2) = -25.6611$. Therefore the final validation result is very close to 1. For illustration of the relative validation we are introducing the third solution in the following form (Atanasijević-Kunc and Karba, 2000):

$$G_{PI3} = \begin{bmatrix} -6.2848 & 0.1541 \\ 0.0244 & -7.5020 \end{bmatrix} + \\ + \frac{1}{s} \begin{bmatrix} -0.0334 & 0.0343 \\ 0.0182 & -0.0960 \end{bmatrix} \qquad (8)$$

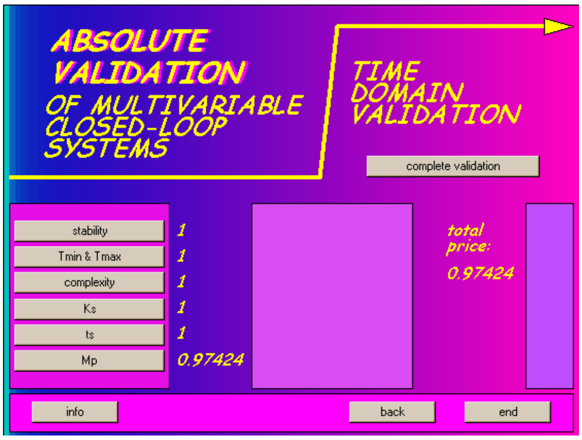

Fig. 8. Absolute validation of the first result

When relative validation on the set of solutions is performed also limitations on control signals range are taken into account. In such situations the close-loop behavior can be nonlinear if control signals are saturated. This is the reason why all the properties which are defined for linear systems are omitted from relative validation. As all design goals in time domain were defined to be of some interest, by default complexity, settling times and overshoots are proposed for relative validation. The values of the last two can differ regarding absolute validation if closed-loop system properties are nonlinear. With this information the user can also evaluate the difference between theoretical linear and more realistic nonlinear situation. In addition two criteria are proposed. The first should be regarded as the measure of achieved quality as design goal of good matching of outputs with corresponding reference signals is always presented. It is defined as:

$$J_1 = \sum \int |ref_i(t) - y_i(t)| dt \qquad (9)$$

On the other hand the measure of demanded control activity can be inspected with:

$$J_2 = \sum \int |u_i(t)| dt \qquad (10)$$

We have to mention that reference signals needed for calculation of J_1 and J_2 are generated automatically in the following manner. First the parameter ΔT is defined as $\Delta T = 4 * \overline{t_s}$, where $\overline{t_s}$ is the mean settling time of compared linear systems. Then step changes are realized in ΔT intervals for each of input signals so that transient responses of direct paths and cross-coupling can be observed. The situation is illustrated in Fig. 9 for the third solution. The fist ΔT can be used for start up in nonlinear situations.

Taken into account all presented criteria the results of relative validation are presented in Fig. 10. The first result (defined with (3)) is regarded as a norma while the validation results of other two represent the measure of improvement regarding the calculated norma. From the presented the following can be concluded. Regarding all criteria the best is the third result as it has the highest total validation result value. But if the complexity is omitted from validation as illustrated in Fig. 11, the best result is the second one. The main reason lies in elimination of cross couplings which for MIMO systems can be regarded as the source of cross-coupling disturbances. With decoupling also the shortest path for each input-output pair of signals is achieved.

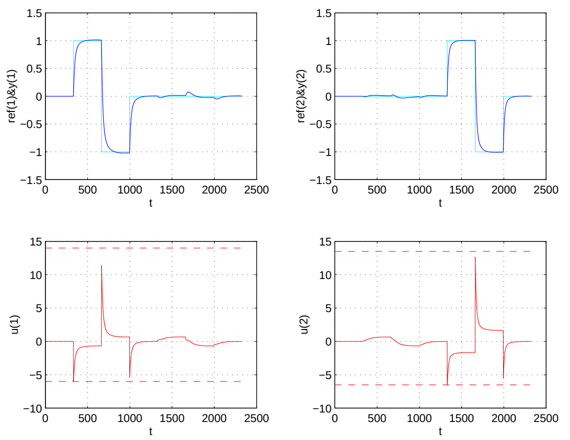

Fig. 9. Time responses of the third system used for relative validation

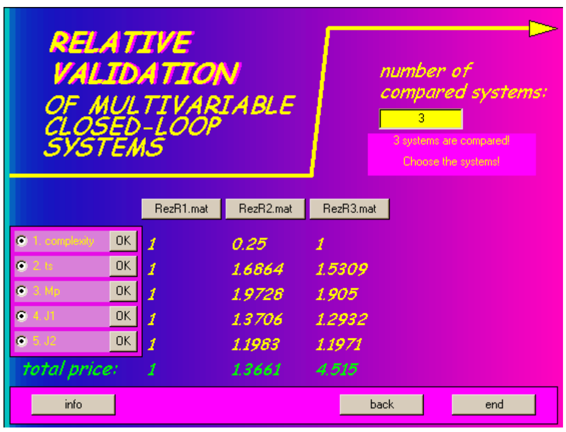

Fig. 10. Relative validation of three closed-loop systems

4. CONCLUSIONS

On the basis of presented results and our own experiences the following can be concluded:

- Presented analytical toolbox enable user friendly, simple and systematic treatment of system properties.
- The access to the functions is enabled also through the calls of the whole group of functions, which are especially suitable for certain design level.
- In our opinion the analysis of parallelism and differences between SISO and MIMO systems improves students understanding of control problems.
- The concept of design goals definition and evaluation is introduced in such a manner that it does not represent any kind of limitation but only stimulates the criticism of definition of what is good and why.
- It also enable some further development in the direction of nonlinear systems.

In this sense the presented toolbox offers easier analysis as well as enough abilities to allow the students creative and autonomous work thus increasing their motivation. The first experiences with toolbox usage confirmed correct orientation of the discussed software.

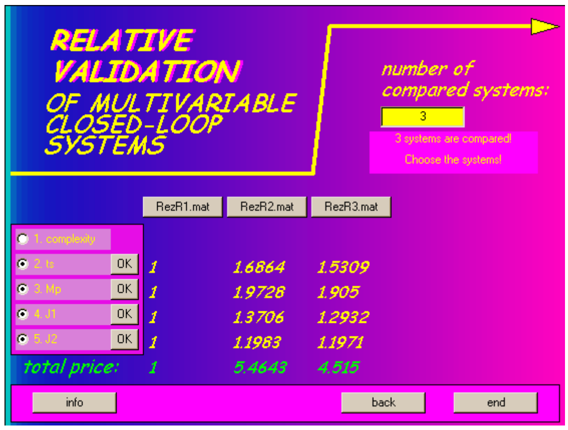

Fig. 11. Relative validation of three closed-loop systems: the complexity of the solutions is not important

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