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DESIGN AND CONTROL OF HYDRAULIC ACTUATORS: SIMULATIONS AND EXPERIMENTAL RESULTS

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Abstract: In this paper the problem of experimental control of hydraulic actuators is considered. Experimental results of a backstepping implementation are analyzed in the context of practical difficulties, mainly the measurements of the acceleration and pressure difference. A two degree-of-freedom controller is proposed to circumvent these difficulties in a framework that takes into account the measurement and the robustness problems. Simulation results illustrate the main features of the backstepping and the two degree-of-freedom controllers when applied on a hydraulic actuator.

Keywords: Hydraulic Actuators, Nonlinear Control, Linear Control, Robustness, Disturbance Rejection.

1. INTRODUCTION

Hydraulic actuators provide a high power density, therefore they are largely employed in industry. Some examples of applications are the use of these actuators in vehicle suspensions (Fialho and Balas, 1998), in construction (Chiang and Murrenhoff, 1998), in machine tools and in robot manipulators (Heintze, 1997).

However, hydraulic actuators present a highly nonlinear behavior, lightly damped dynamics (due to fluid compressibility) and parameter variations. These characteristics are an obstacle for applications that require high performance, for example, robot systems (the behavior of manipulators equipped with hydraulic actuators is dominated by the dynamics of the hydraulic actuators).

In order to overcome these obstacles, many kinds of controllers for hydraulic actuators have been proposed in literature. In (Edge, 1997), a review of research on the control of fluid power systems is presented. In (Yao *et al.*, 1998), a backstepping control technique is proposed and simulation results are present. In (Cunha *et al.*, 2000), a controller that uses feedback linearization and backstepping methodology is proposed and experimental results are presented. Nevertheless, backstepping methodology has two major drawbacks: the successive derivatives may limit the real time implementation on a physical system and also, in physical implementations, it may be difficult to measure the signals that are needed (in particular, the acceleration).

In this paper, the design and control of hydraulic actuators are presented with two main purposes: a robust control design able to compensate mechanical and hydraulic uncertainties and the problem of real time implementation. Due to these limitations a two degree-of-freedom controller structure, which was designed according to the technique described in (Wolovich, 1995), is used. The results are compared with the backstepping procedure.

In section 2, both a linear and a nonlinear model of the hydraulic actuator are presented. In section 3, the experimental setup is described. Experimental results obtained with the backstepping technique are shown in section 4. In section 5, the two degree-of-freedom controller is proposed. Simulation results are shown in section 6. In section 7, the conclusions are presented.

2. MODELLING OF A SERVO HYDRAULIC ACTUATOR

The hydraulic actuator shown in Fig. 1 consists of a cylinder controlled by a symmetrical critical-center 4-way servo valve. In this figure, P_S is the supply fluid pressure (supplied by a high-pressure fluid pump, not shown), P_0 is the return pressure (that returns to a reservoir, not shown), P_1 and P_2 are the pressures in cylinder chambers 1 and 2, respectively, V_1 and V_2 are the volumes in chambers (and lines) 1 and 2, respectively, Q_1 is the fluid flow from the servo valve to chamber 1, Q_2 is the fluid flow from chamber 2 to the servo valve, M is the mass coupled to the actuator, B is the viscous friction coefficient, A is the cylinder piston cross-sectional area, u is the control input (for example, voltage applied to the servo valve), y is the actuator piston position and F_L is an external force acting on the system.

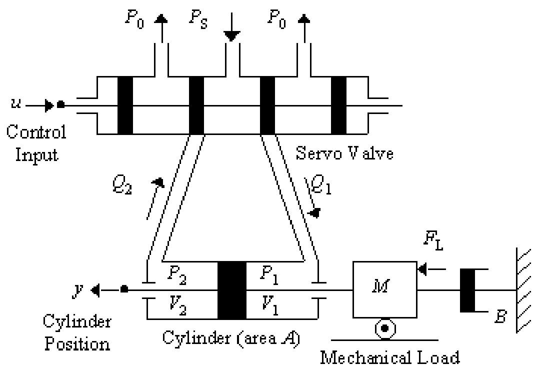

Fig. 1. Servo Hydraulic Actuator

2.1 Nonlinear Model

The nonlinear modelling of this system is developed by many authors (see (Watton, 1989; Heintze, 1997)) and the following description is obtained:

$$M\ddot{y} + B\dot{y} = AP_\Delta + F_L \quad (1)$$

$$\dot{P}_\Delta = \frac{\beta V \left(Ku\sqrt{P_S - \text{sgn}(u)P_\Delta} - A\dot{y}\right)}{\left(\frac{V}{2}\right)^2 - (Ay)^2} \quad (2)$$

where β is the fluid bulk modulus, $P_\Delta = P_1 - P_2$ is the pressure difference in the cylinder, $V = V_1 + V_2$ is the total fluid volume and K is a hydraulic constant. The bandwidth of the valve is assumed to be much higher than the bandwidth of the actuator, therefore valve dynamics are neglected.

2.2 Linearized Model

A linearization of the nonlinear model is necessary in order to apply the two DOF control design technique. The system is linearized around the origin, ($y \approx 0$), thus it is possible to make the assumption that $\frac{\beta V}{\left(\frac{V}{2}\right)^2 - (Ay)^2} \approx \frac{4\beta}{V}$.

Also, considering that in practical applications $P_\Delta < \frac{2}{3}P_S$, an approximation of the square root in Eq. 2 by a first order Taylor series is $\sqrt{P_S - \text{sgn}(u)P_\Delta} \approx \sqrt{P_S}\left(1 - \text{sgn}(u)\frac{P_\Delta}{2P_S}\right)$.

Finally, defining the constants $K_q = K\sqrt{P_S}$ (known as the flow gain) and $K_c = \frac{K|u|}{2\sqrt{P_S}}$ (known as the flow-pressure gain), the hydraulic actuator linearized model is obtained:

$$y(s) = \frac{\frac{4\beta AK_q}{VM}u(s) + \frac{1}{M}\left(s + \frac{4\beta K_c}{V}\right)F_L(s)}{s^3 + \frac{(VB + 4\beta MK_c)}{VM}s^2 + \frac{4\beta(A^2 + BK_c)}{VM}s} \quad (3)$$

3. EXPERIMENTAL SETUP

In this section, the particular system where the controller experimental implementation was carried out is described. This hydraulic system is installed on LASHIP (in Federal University of Santa Catarina, Brazil). The experimental setup consists of a double acting cylinder, a proportional valve BOSCH NG6 and its electronic card, position and pressure transducers, temperature transducers in each cylinder chamber, an acquisition and controller board DS1102 and a computerized hydraulic power and conditioning unit, which is responsible for maintaining the fluid at the required conditions.

The nominal parameters of the system are given in Tab. 1. These parameters were obtained from manufacturers and experimental data.

Table 1. Nominal Parameters

Parameter	Nominal Value
M	20.66 kg
B	316 Ns/m
A	$7.6576 * 10^{-4}$ m^2
V	$9.5583 * 10^{-4}$ m^3
β	10^9 N/m^2
P_{S}	10^7 N/m^2
K_{c}	$6.8747 * 10^{-16}$ m^6/VsN
K_{q}	$2.0624 * 10^{-4}$ m^3/Vs
K	$6.5219 * 10^{-8}$ m^4/Vs$\sqrt{\mathrm{N}}$

4. BACKSTEPPING CONTROL TECHNIQUE

In (Cunha *et al.*, 2000), a controller was designed using feedback linearization and backstepping methodologies. The main results obtained in that work are reproduced in this section.

The backstepping control law is given by:

$$u = \frac{A\dot{y} + \frac{\left(\frac{V}{2}\right)^2 - (Ay)^2}{A\beta V}\left(My_{\mathrm{d}}^{(3)} + B\ddot{y}_{\mathrm{d}} - u_{\mathrm{a}}\right)}{K\sqrt{P_{\mathrm{S}} - \mathrm{sgn}(u)P_{\Delta}}} \quad (4)$$

$$u_{\mathrm{a}} = \phi_1\dot{\tilde{y}} + \phi_2\ddot{\tilde{y}} + \frac{2}{M}\left(\dot{\tilde{y}}p_{22} + \tilde{y}p_{12}\right) + \\ +c\left(AP_{\Delta} - M\ddot{y}_{\mathrm{d}} - B\dot{y}_{\mathrm{d}} + \phi_1\tilde{y} + \phi_2\dot{\tilde{y}}\right) \quad (5)$$

where y_{d} is the desired trajectory for the output, $\tilde{y} = y - y_{\mathrm{d}}$ is the trajectory tracking error and ϕ_1, ϕ_2, p_{22}, p_{12} and c are constants to be determined according to the backstepping methodology. The control input u is responsible for linearizing Eq. 2 and u_{a} is the integrator backstepping control law.

For the particular system presented in section 3, the constants were determined to be $\phi_1 = 3 * 10^5$, $\phi_2 = 10500$, $p_{12} = -500$, $p_{22} = 1.4 * 10^4$ and $c = 500$. The experimental results are presented in Fig. 2 and Fig. 3. The system response presents good overall performance. The output tracks accurately the desired trajectory [1] and the error is not significative.

In order to implement the backstepping controller, the signals that have to be measured (directly or indirectly) are position, velocity, acceleration and pressure difference. For practical purposes the measurement of acceleration and pressure difference is not desired. In particular, the acceleration obtained from numerical differentiation of the velocity can lead to bad results due to the presence of noise.

An efficient way to circumvent this problem is to design a controller with similar performance as the backstepping employing a reduced number of measured signals. It is possible to achieve this goal with a linear control approach. As stated in (Edge, 1997) and other authors, the linearized model, Eq. 3, has a single pole in the origin and two complex conjugate poles badly damped. Classical controllers, in general simpler to practical implementation, introduce poles and zeros in the closed-loop that do not change appropriately the location of complex conjugate poles. Even when the poles can be changed appropriately as in a PID controller design, the robustness requirements are hardly achieved. Consequently, two degree-of-freedom technique is a practical way to synthesize a controller to obtain high system performance and accurately perturbation rejection.

[1] The piston moves from one position to another according to polynomial $-2t^7 + 7t^6 - 8.4t^5 + 3.5t^4$.

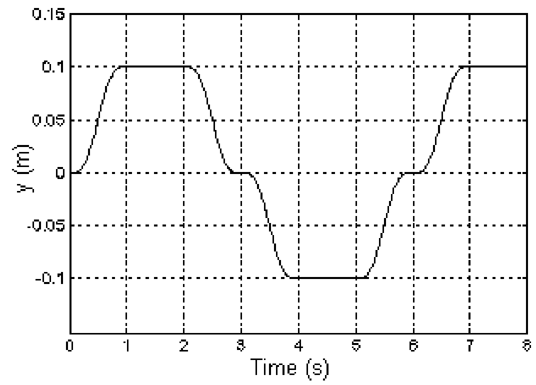

Fig. 2. Trajectory Tracking - Backstepping Controller

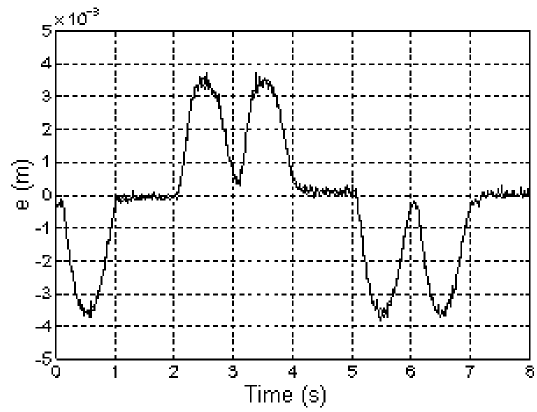

Fig. 3. Tracking Error - Backstepping Controller

5. TWO DOF CONTROLLER

The two DOF (degree-of-freedom) controller is a linear controller. It allows the closed-loop system to maintain a desired response performance while varying the loop performance (this means that it is possible to improve robust stability, disturbance rejection and noise attenuation, independent of response performance) (Wolovich, 1995).

Consider a linear system defined by:

$$y(s) = G(s)u(s) + d(s) \quad (6)$$

where $G(s) = \frac{c(s)}{a(s)}$ is a rational transfer function, which is both strictly proper and minimal[2] and $d(s)$ is some disturbance signal added to the output. The two DOF controller has the configuration shown in Fig. 4, where the $(n-1)$ degree polynomials $k(s)$, $h(s)$ and $q(s)$ are to be determined to achieve the desired closed-loop goals ($k(s)$ must be monic).

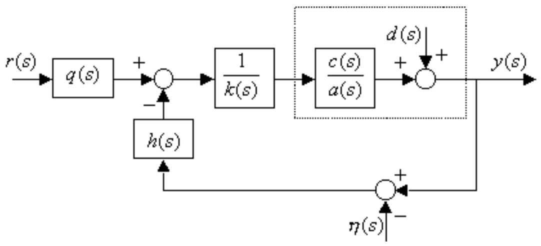

Fig. 4. Closed-Loop System with the 2 DOF Controller

The closed-loop system is given by:

$$y(s) = T(s)r(s) + S(s)d(s) + C(s)\eta(s) \qquad (7)$$

where $T(s) = \frac{c(s)q(s)}{\delta(s)}$ is the output response transfer function, $S(s) = \frac{a(s)k(s)}{\delta(s)}$ is the sensitivity transfer function, which represents the effect of the disturbance signal $d(t)$ on the output $y(t)$ and $C(s) = \frac{c(s)h(s)}{\delta(s)}$ is the complementary sensitivity transfer function, which represents the effect of the sensor noise $\eta(t)$ on the output $y(t)$. $\delta(s)$ is the characteristic polynomial, given by:

$$\delta(s) = a(s)k(s) + c(s)h(s) \qquad (8)$$

whose $(2n-1)$ roots are the poles of the closed-loop system.

The overall performance of the system depends on the ability of its output $y(t)$ to track the reference input $r(t)$ while minimizing the effect of both the disturbance signal $d(t)$ and the sensor noise $\eta(t)$ on its behavior. Thus $T(s)$ should guarantee that the output response presents some desired characteristics such as a small settling time and no overshoot, while $S(s)$ should provide disturbance rejection (in order to accomplish that, $|S(\mathrm{j}\omega)|$ should be minimized over the band of frequencies that characterize $d(t)$) and $C(s)$ should provide noise attenuation ($|C(\mathrm{j}\omega)|$ should be minimized over the band of frequencies that characterize $\eta(t)$). An additional desired characteristic for $S(s)$ is that $\|S\|_\infty = \max_\omega |S(\mathrm{j}\omega)| \leq 2 \approx 6$dB, ensuring robust stability with respect to plant parameter variations - as demonstrated in (Wolovich, 1995).

[2] I.e., the plant polynomials $a(s)$ and $c(s)$ are coprime and $deg[a(s)] = n > deg[c(s)] = m$.

A choice of arbitrary stable polynomials $\hat{q}(s)$ and $\hat{\delta}(s)$ (such that $q(s) = \alpha\hat{q}(s)$ and $\delta(s) = \hat{\delta}(s)\hat{q}(s)$) will cause zero-pole cancellations

$$T(s) = \frac{c(s)q(s)}{\delta(s)} = \frac{\alpha c(s)}{\hat{\delta}(s)} \qquad (9)$$

Therefore $\hat{\delta}(s)$ contains the n poles that define the nominal output response and $\hat{q}(s)$ contains the $(n-1)$ poles that only affect $S(s)$ and $C(s)$. Polynomials $h(s)$ and $k(s)$ will be obtained by solving Eq. 8, the so-called *Diophantine equation.*

The n poles of $\hat{\delta}(s)$ can be chosen using the Linear Quadratic Regulator (LQR) performance index, defined by

$$J_{\mathrm{LQR}} = \int_0^\infty \left(\rho y^2(t) + u^2(t)\right) \mathrm{d}t \qquad (10)$$

where ρ is an weighting factor. The minimization of J_{LQR} implies a desire to minimize both excessive output $y(t)$ excursions and the control effort $u(t)$ required to prevent such excursions. A controller that minimizes the LQR performance index implies an optimal positioning of the closed-loop poles (Wolovich, 1995).

In order to choose the n poles of $\hat{\delta}(s)$ that minimize the LQR performance index (Eq. 10), it is used the Spectral Factorization Method, described in (Chang, 1961; Wolovich, 1995). The poles will be the n negative roots of:

$$\begin{aligned}\Delta(s) &= a(s)a(-s) + \rho c(s)c(-s) \\ &= [\Delta(s)]^+[\Delta(s)]^- \end{aligned} \qquad (11)$$

for some real $\rho > 0$.

5.1 Two DOF Controller Design

The two DOF controller is designed on the linearized system. The open-loop poles of the system described in section 3 are 0 and -7.7 ± 345j.

In order to choose the closed-loop poles, a root-square locus is used. A root-square locus is an s-plane plot of all $2n$ roots of $\Delta(s)$ (Eq. 11), as ρ varies from 0 to ∞. The root-square locus of this system is shown in Fig. 5.

As it can be seen in Fig. 5, the two complex conjugate poles are far from the real axis. The system will behave approximately as a first order system, with some oscillations. The real pole is chosen to ensure that the 2% settling time t_{s} is 0.1s. For a first order system, $t_{\mathrm{s}} = 4\tau$ (where τ is the system time constant) and the only pole is given by $-\frac{1}{\tau}$. Thus, the real pole is chosen to be

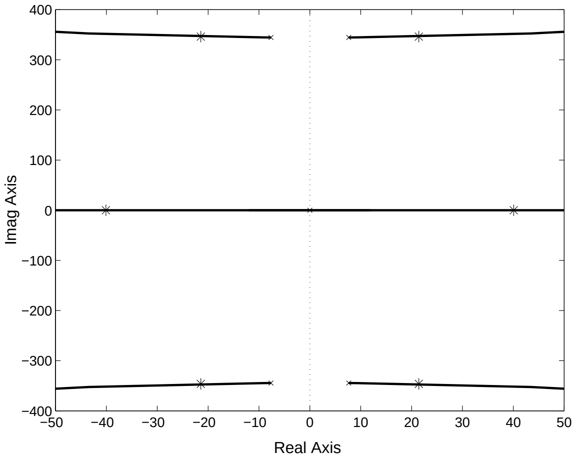

Fig. 5. Root-Square Locus of the Hydraulic System

-40. Consequently, $\rho = 2.3 * 10^4$ and the other poles are $-21.4 \pm 346\text{j}$.

Considering that the external force acting on the system is constant and its maximum value is 300N ($F_{\text{L}}(s) = \frac{300}{s}$), then $d(s)$ is given by:

$$d(s) = \frac{\frac{300}{M}\left(s + \frac{4\beta K_{\text{c}}}{V}\right)}{s^4 + \frac{(VB+4\beta M K_{\text{c}})}{VM}s^3 + \frac{4\beta(A^2+BK_{\text{c}})}{VM}s^2} \quad (12)$$

In Fig. 6, a Bode plot of $d(s)$ is shown. The disturbance signal is characterized by low frequencies. Also, the noise $\eta(t)$ in this system is characterized by high frequencies. Thus, the desired behavior for $S(s)$ and $C(s)$ considering that they are complementary and not independent is:

$$|S(\text{j}\omega)| = \begin{cases} \approx 0 \text{ at low } \omega \\ \approx 1 \text{ at high } \omega \end{cases} \quad (13)$$

$$|C(\text{j}\omega)| = \begin{cases} \approx 1 \text{ at low } \omega \\ \approx 0 \text{ at high } \omega \end{cases} \quad (14)$$

The polynomial $\hat{q}(s)$ is chosen to be $s^2 + 110s + 3000$ (the poles are -60 and -50). This choice guarantees an acceptable behavior for $S(s)$ and $C(s)$ (see Fig. 7). The other polynomials are:

$$q(s) = 150s^2 + 16000s + 450000$$
$$h(s) = -87s^2 - 19000s + 450000$$
$$k(s) = s^2 + 180s + 13000$$

6. SIMULATION RESULTS

Both the closed-loop system with the backstepping controller and the closed-loop system with the two DOF controller were simulated. The results are presented in Fig. 8 and Fig. 9, corresponding to the output response and the tracking error, respectively. Both controllers provide good

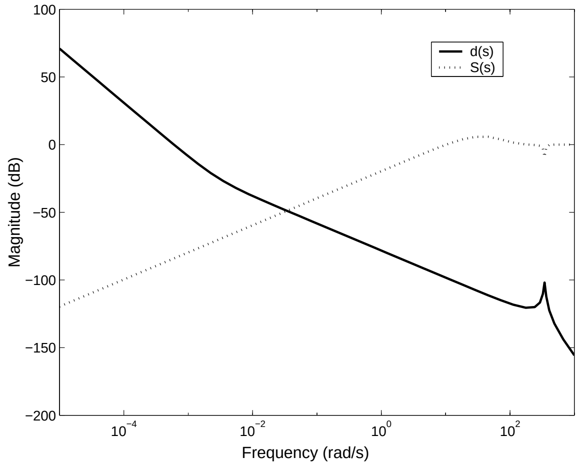

Fig. 6. Bode Diagram of $d(s)$ and $S(s)$

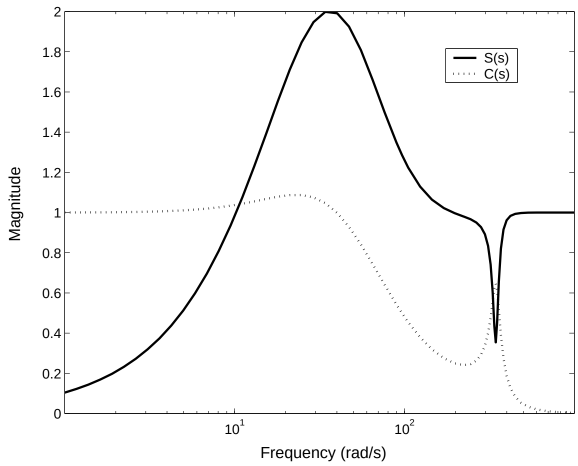

Fig. 7. Magnitude Diagram of $S(s)$ and $C(s)$

trajectory tracking, though the output response obtained with the two DOF controller presents a small delay (as expect, since this controller was designed to have $t_{\text{s}} = 0.1\text{s}$). The effect of this delay is also seen in Fig. 9, which presents the errors. The system controlled with the two DOF controller presents a bigger tracking error than the system controlled with backstepping. Nevertheless, the maximum value of this error ($\approx 6 * 10^{-3}$m) is only 6% of the maximum value of the trajectory ($= 0.1$m), which is not significative for many applications.

Both controlled systems were also simulated with other conditions than the nominal ones, in order to test their performances when parameter variations and external disturbances occur. In practical implementations, the parameters that vary the most are the fluid bulk modulus β and the supply fluid pressure P_{S}. In simulation, β was increased in 50% and P_{S} was decreased in 20%. Even with these changes, both systems presented the same behavior as the nominal ones, showing robustness to parameter variation. An external force F_{L} of 300N was considered to be acting on the system. To a step input of 0.05m of magnitude, the results

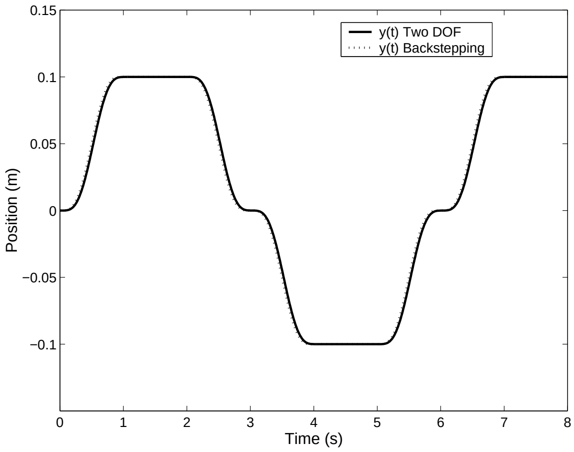

Fig. 8. Trajectory Tracking - Two DOF Controller and Backstepping

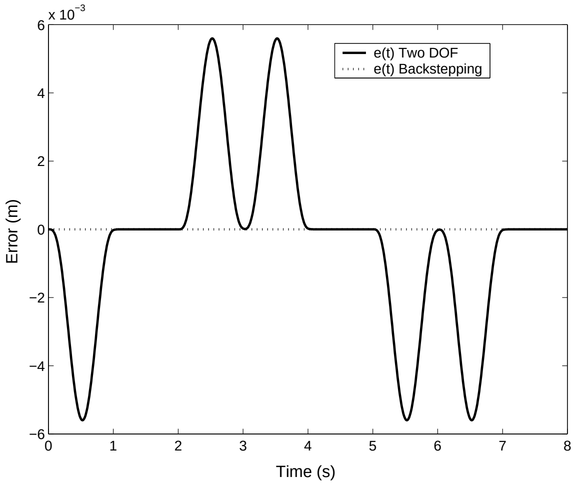

Fig. 9. Trajectory Error - Two DOF Controller and Backstepping

are shown in Fig. 10. The two DOF controller is able to reject the disturbance, while the backstepping control is not.

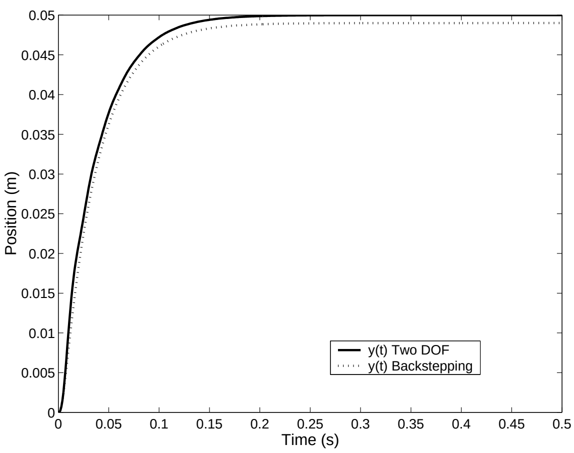

Fig. 10. Comparison of Disturbance Rejection

7. CONCLUSION

In this paper, the control of hydraulic actuators was presented. Two main aspects were considered: the robustness problem and the practical implementation problem.

The experimental results show that the backstepping technique is a good way to improve high performance and to reduce tracking errors for the considered system. However, the complexity of the controller leads to some practical restrictions: acceleration and pressure difference measurements. In comparison to classical controllers, the two degree-of-freedom controller offers an efficient way to guarantee robustness and performance, which are important requirements to practical applications. Simulations were performed to illustrate the response of both methods. Further works will be done to analyze the practical performance of two degree-of-freedom controller considering parameter variation and friction compensation.

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