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GENERAL ATTITUDE CONTROL ALGORITHM FOR SPACECRAFT EQUIPPED WITH STAR CAMERA AND REACTION WHEELS [1]

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Abstract: A configuration consisting of a star camera, four reaction wheels and magnetorquers for momentum unloading has become standard for many spacecraft missions. This popularity has motivated numerous agencies and private companies to initiate work on the design of an imbedded attitude control system realized on an integrated circuit. This paper considers two issues: slew maneuver with a feature of avoiding direct exposure of the camera's CCD chip to the Sun and optimal control torque distribution in a reaction wheel assembly. The attitude controller is synthesized applying the energy shaping technique, where the desired potential function is carefully designed using a physical insight into the nature of the problem. The system stability is thoroughly analyzed and the control performance simulated.

Keywords: Attitude control, satellite control, energy control, quaternion feedback.

1. INTRODUCTION

A typical configuration of an attitude control system considered for many of low earth orbit spacecraft, consists of a star camera, four reaction wheels and magnetorquers for momentum unloading. The algorithms developed in this paper address two control problems: a slew maneuver and a control torque distribution. It is assumed that full state information, i.e. the angular velocity and the attitude, is available. The issues related to the attitude determination with a star camera and the momentum damping are not addressed, however the interested reader may refer to standard textbooks in the field (Sidi, 1997), (Wertz, 1990).

The algorithm presented in this paper provides an ability to perform a controlled spacecraft maneuver to the desired attitude without any restrictions on the target attitude and to keep it stabilized in all three axes. The Sun is the most dangerous point in the sky for many payloads, the controller therefore provides a built-in safety mechanism for that. The control torque is distributed among available reaction wheels such that the resultant angular momentum of each reaction wheel is kept nearest possible, in the Euclidean norm sense, to the nominal value.

The algorithm presented in this work is based on the energy shaping method. The advantage of this approach is that it provides a physical insight into the design. Stabilization by the energy shaping of a Hamiltonian system was first proposed in mid eighties, (van der Schaft, 1986). The control action was the sum of the gradient of potential energy and the dissipation force. Such a control law made the system uniformly asymptotically stable to the desired reference point - the point of minimal potential energy, (Nijmeijer and van der Schaft, 1990).Later, the concept was generalized to a coordinate-free setting on a Riemannian manifold, (Koditschek, 1989). In this paper the energy shaping method is applied to the attitude control problem.

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The paper is organized as follows. A canonical form for motion of a rigid spacecraft is derived in Section 2. The control synthesis is addressed in Section 3. An algorithm for control torque distribution in the reaction wheel assembly is provided in Section 4. A simulation case study comprises the final part of this paper.

2. CANONICAL FORM FOR A RIGID BODY

To apply the energy shaping as in (van der Schaft, 1986) the rigid body motion is expressed in the canonical form. The standard approach is to use a coordinate neighbourhood, e.g. Euler angles and their conjugate momenta. In this work a global approach is chosen. The rotational motion of a rigid body is parameterized by the unit quaternion $\mathbf{q} = \begin{bmatrix} q_1 & q_2 & q_3 & q_4 \end{bmatrix}^{\mathrm{T}}$ and the conjugate momenta $\mathbf{p} = \begin{bmatrix} p_1 & p_2 & p_3 & p_4 \end{bmatrix}^{\mathrm{T}}$. The idea adopted in this section was addressed earlier in celestial mechanics, (Cid and Saturio, 1988), (Morton, 1994). The authors studied a canonical transformation $y = f(x)$ of the state space $\mathbf{y} \in \mathbb{R}^{2n}$ to $\mathbf{x} \in \mathbb{R}^{2m}$ with $m > n$. The motion of the rigid body is a special case of this transformation for $m = 4$, $n = 3$. In other words the rigid body motion is no longer described locally in a 3 dimensional Euclidean space but rather globally in 4 dimensions. Following this idea the body angular velocity vector gets also an extra dimension, which is trivially 0 only on the unit sphere $S^3 = \{\mathbf{q} \in \mathbb{R}^4 : \mathbf{q}^{\mathrm{T}}\mathbf{q} = 1\}$.

The kinetic energy of a rigid body rotation is a function of the instant angular velocity ω

$$T = \frac{1}{2}\omega^{\mathrm{T}}\mathbf{J}\omega, \tag{1}$$

where $\mathbf{J}$ is the inertia tensor. The angular velocity vector may be regarded as an element of the quaternion vector space. Defining $\Omega := \begin{bmatrix} \omega^{\mathrm{T}} & 0 \end{bmatrix}^{\mathrm{T}}$ Eq. (1) becomes

$$T = \frac{1}{2}\Omega^{\mathrm{T}}\mathbf{J}^{*}\Omega, \tag{2}$$

where $\mathbf{J}^{*}$ is a block diagonal matrix

$$\mathbf{J}^{*} = \begin{bmatrix} \mathbf{J} & \mathbf{0} \\ \mathbf{0} & J_0 \end{bmatrix}. \tag{3}$$

The element J_0 takes in general an arbitrary nonsingular value. Using the standard quaternion parameterizations of kinematics

$$\dot{\mathbf{q}} = \frac{1}{2}\mathbf{Q}(\mathbf{q})\Omega, \tag{4}$$

where

$$\mathbf{Q}(\mathbf{q}) = \begin{bmatrix} q_4 & -q_3 & q_2 & q_1 \\ q_3 & q_4 & -q_1 & q_2 \\ -q_2 & q_1 & q_4 & q_3 \\ -q_1 & -q_2 & -q_3 & q_4 \end{bmatrix}$$

the kinetic energy is

$$T = 2\mathbf{q}^{\mathrm{T}}\mathbf{Q}(\dot{\mathbf{q}})\mathbf{J}^{*}\mathbf{Q}^{\mathrm{T}}(\dot{\mathbf{q}})\mathbf{q}. \tag{5}$$

In general the Hamiltonian, (Goldstein, 1980) is defined as

$$H(\mathbf{q},\mathbf{p}) = \langle \mathbf{p}, \dot{\mathbf{q}} \rangle - L(\mathbf{q},\dot{\mathbf{q}}), \tag{6}$$

where the Lagrangian $L = T(\mathbf{q},\dot{\mathbf{q}}) - U(\mathbf{q})$, and conjugate momentum $\mathbf{p}$ is

$$\mathbf{p} = \frac{\partial L}{\partial \dot{\mathbf{q}}} = \frac{\partial T}{\partial \dot{\mathbf{q}}} = 4\dot{\mathbf{q}}^{\mathrm{T}}\mathbf{Q}^{\mathrm{T}}(\mathbf{q})\mathbf{J}^{*}\mathbf{Q}^{\mathrm{T}}(\mathbf{q}), \tag{7}$$

The Hamiltonian for the rigid body motion is then

$$\begin{aligned} H(\mathbf{q},\mathbf{p}) &= \mathbf{p}^{\mathrm{T}}\dot{\mathbf{q}} - L(\mathbf{q},\mathbf{p}) \\ &= \frac{1}{8}\mathbf{p}^{\mathrm{T}}\mathbf{Q}(\mathbf{q})\mathbf{J}^{*-1}\mathbf{Q}^{\mathrm{T}}(\mathbf{q})\mathbf{p} + U(\mathbf{q}). \end{aligned} \tag{8}$$

Having Hamiltonian the canonical equations are calculated

$$\begin{aligned} \dot{\mathbf{q}} &= \frac{1}{4}\mathbf{Q}(\mathbf{q})\mathbf{J}^{*-1}\mathbf{Q}^{\mathrm{T}}(\mathbf{q})\mathbf{p} \\ \dot{\mathbf{p}} &= -\frac{1}{4}\mathbf{Q}(\mathbf{p})\mathbf{J}^{*-1}\mathbf{Q}^{\mathrm{T}}(\mathbf{p})\mathbf{q} - \frac{\partial U(\mathbf{q})}{\partial \mathbf{q}} + \mathbf{M}_p, \end{aligned} \tag{9}$$

where $\mathbf{M}_p$ is the generalized moment.

The spacecraft control torque is denoted by $\mathbf{M}_c$. To find the correspondence between the generalized moment and the control torque, the invariance of the work done by this two fields is used. It follows that the time derivatives of the work done by the torque $\mathbf{M}_p$ and $\mathbf{M}_c$ are equal

$$\dot{\mathbf{q}}^{\mathrm{T}}(t)\mathbf{M}_p(t) = \dot{W}(t) = \Omega^{\mathrm{T}}(t)\mathbf{M}_c(t). \tag{10}$$

Applying Eq. (4), the right hand side of Eq. (10) becomes

$$\dot{\mathbf{q}}^{\mathrm{T}}(t)\mathbf{M}_p(t) = \dot{W}(t) = 2\dot{\mathbf{q}}^{\mathrm{T}}(t)\mathbf{Q}\left(\mathbf{q}(t)\right)\mathbf{M}(t), \tag{11}$$

where $\mathbf{M} = \begin{bmatrix} \mathbf{M}_c^T & 0 \end{bmatrix}^T$. It follows from Eq. (11) that

$$\mathbf{M}_p(t) = 2\mathbf{Q}(\mathbf{q}(t))\mathbf{M}(t) \tag{12}$$

or equivalently

$$\mathbf{M}(t) = \frac{1}{2}\mathbf{Q}^{\mathrm{T}}(\mathbf{q}(t))\mathbf{M}_p(t). \tag{13}$$

3. ATTITUDE CONTROL

The energy shaping, (van der Schaft, 1986) suggests a feedback control of the form

$$\mathbf{M}_p = -\frac{\partial V(\mathbf{q})}{\partial \mathbf{q}} + \mathbf{M}_d, \tag{14}$$

where $V(\mathbf{q})$ is a continuously differentiable scalar valued function. The term $\mathbf{M}_d$ is a dissipative force, and the time derivative of its work $\dot{W} = \mathbf{M}_d^{\mathrm{T}}\dot{\mathbf{q}}$ is negative definite. The control law (14) makes the system asymptotically stable to the equilibrium point $(\mathbf{q}_e, \mathbf{0})$ if $\mathbf{q}_e$ is the minimum of the sum of the potential energies $U(\mathbf{q}) + V(\mathbf{q})$ as in Fig. 1.

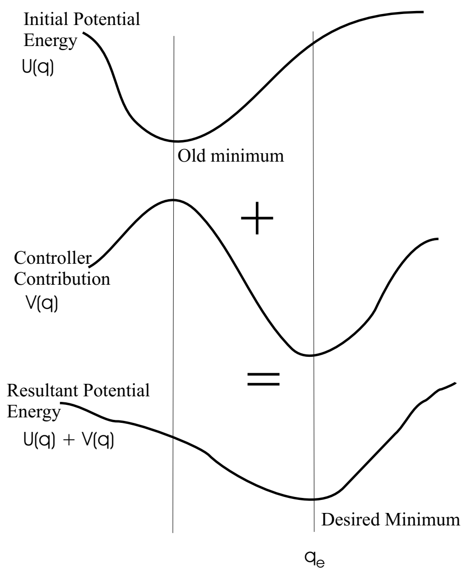

Fig. 1. A control torque equal the negative gradient of potential energy gives a contribution to the total potential energy in the system.

3.1 *Control Synthesis*

The controller proposed in this paper is designed for a spacecraft equipped with a star camera, which bore axis shall never point to the Sun. This attitude is denoted as forbidden. For simplicity of the exposition we assume that the reference quaternion is the identity quaternion $\mathbf{q}_e = \mathbf{e}$, otherwise the quaternion $\mathbf{q}$ shall be substituted by $\mathbf{Q}(\mathbf{q}_e)\mathbf{q}$ in the subsequent formulas.

Forbidden attitudes in the slew maneuver problem are not only a certain point $\mathbf{q}_f$ and its antipode $-\mathbf{q}_f$, but rather the whole geodesics G_f on the 3-sphere. Having a forbidden attitude $\mathbf{q}_f$ the whole family can be generated by a product with a rotation about the bore axis. The control law proposed shall make use of two orthogonal vectors $\mathbf{W}_1, \mathbf{W}_2 \in \mathbb{R}^4$ normal to the geodesics G_f. They are constructed in the following procedure:

Procedure 1.

(1) Determine the unit vector $\mathbf{b}$ in direction of the bore axis and the unit vector $\mathbf{s}$ pointing towards the Sun.

(2) Compute a quaternion $\mathbf{q}_f$ corresponding to such a rotation $\mathbf{R}_f : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ that $\mathbf{b} = \mathbf{R}_f(\mathbf{s})$. For this purpose we employ a definition of a unit quaternion, (Goldstein, 1980)

$$\mathbf{q}_f = \left[n_1 \sin\frac{\psi}{2} \;\; n_1 \sin\frac{\psi}{2} \;\; n_3 \sin\frac{\psi}{2} \;\; \cos\frac{\psi}{2} \right]^{\mathrm{T}}, \tag{15}$$

where the triad $\mathbf{n} = [n_1 \; n_2 \; n_3]^T$ is the unit vector of the rotation axis and ψ is the angle of rotation. The vector $\mathbf{n}$ is orthogonal to $\mathbf{s}$ and $\mathbf{b}$, $\mathbf{n} = \mathbf{b} \times \mathbf{s}/|\mathbf{b} \times \mathbf{s}|$. The angle $\psi \in [0, \pi]$ is computed using the scalar product of $\mathbf{s}$ and $\mathbf{b}$, $\psi = \mathrm{acos}(\mathbf{s}^{\mathrm{T}}\mathbf{b})$.

(3) Compute the geodesics G_f as the product of $\mathbf{q}_f$ and quaternions corresponding to the rotations about the bore axis $\mathbf{b}$

$$G_f = \left\{ \mathbf{Q}(\mathbf{q}_f) \begin{bmatrix} b_1 \sin\phi \\ b_2 \sin\phi \\ b_3 \sin\phi \\ \cos\phi \end{bmatrix} : \phi \in [-\pi, \pi) \right\}. \tag{16}$$

(4) Find two orthogonal vectors $\mathbf{W}_3$ and $\mathbf{W}_4$ both belonging to the geodesics G_f: $\mathbf{W}_3 = [b_1 \; b_2 \; b_3 \; 0]^{\mathrm{T}}$ and $\mathbf{W}_4 = \mathbf{e}$. Now the wanted vectors $\mathbf{W}_1$, $\mathbf{W}_2$ are chosen such that together with $\mathbf{W}_3$ and $\mathbf{W}_4$ they form orthonormal basis for $\mathbb{R}^4$. The unit vectors $\mathbf{W}_1$, $\mathbf{W}_2$ have the following form

$$\mathbf{W}_1 = \mathbf{Q}(\mathbf{q}_f)[c_1 \; c_2 \; c_3 \; 0]^{\mathrm{T}}, \tag{17}$$

$$\mathbf{W}_2 = \mathbf{Q}(\mathbf{q}_f)[d_1 \; d_2 \; d_3 \; 0]^{\mathrm{T}}, \tag{18}$$

where the vectors $\mathbf{b}$, $\mathbf{c}$, $\mathbf{d}$ form orthonormal basis in $\mathbb{R}^3$.

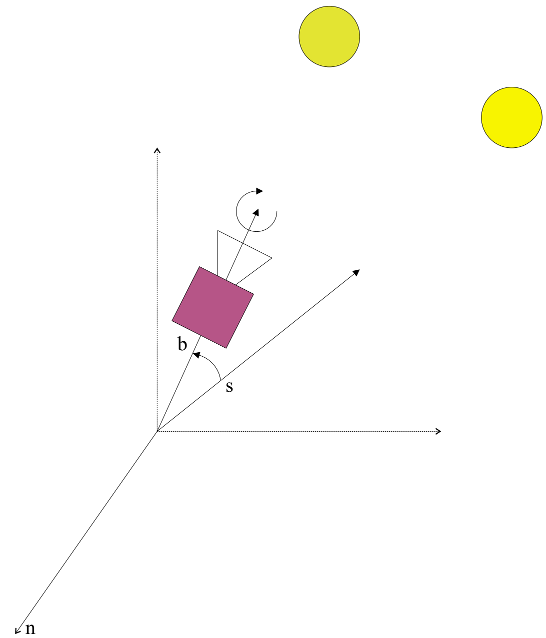

Fig. 2. q_f defines attitude quaternion rotating the sun vector $\mathbf{s}$ to the bore axis $\mathbf{b}$.

The procedure above provides a definition of two orthonormal vectors $\mathbf{W}_1$ and $\mathbf{W}_2$, which have a remarkable feature that

$$\mathbf{q} \in G_f \Leftrightarrow (\mathbf{q}^{\mathrm{T}}\mathbf{W}_1)^2 + (\mathbf{q}^{\mathrm{T}}\mathbf{W}_2)^2 = 0. \tag{19}$$

A potential function $V(\mathbf{q})$ suggested for the feedback is the function $1 - q_4$, having the minimum for $\mathbf{q} = \mathbf{e}$, combined with the feature in Eq. (19)

$$V(\mathbf{q}) = \frac{k_p(1 - q_4)}{(\mathbf{q}^{\mathrm{T}}\mathbf{W}_1)^2 + (\mathbf{q}^{\mathrm{T}}\mathbf{W}_2)^2}, \tag{20}$$

where k_p is a positive real serving as a design parameter. The expected performance is such that the control torque conforming to Eqs. (13) and (14) will be repellent to the geodesics G_f and the system is globally asymptotically stable to the identity $\mathbf{e}$. Using Eq. (13) the explicit form for the proportional part of the control torque is derived

$$\begin{bmatrix} M_{g1} \\ M_{g2} \\ M_{g3} \\ M_{g4} \end{bmatrix} = -\frac{1}{2}\mathbf{Q}^{\mathrm{T}}(\mathbf{q})\frac{\partial V(\mathbf{q})}{\partial \mathbf{q}} \tag{21}$$

$$= \frac{\mathbf{Q}^{\mathrm{T}}(\mathbf{q})\left((\tilde{q}_1^2+\tilde{q}_2^2)\mathbf{e}+2(1-q_4)(\tilde{q}_1\mathbf{W}_1+\tilde{q}_2\mathbf{W}_2)\right)}{-2\left(\tilde{q}_1^2+\tilde{q}_2^2\right)^2}$$

where $\tilde{q}_j = \mathbf{q}^{\mathrm{T}}\mathbf{W}_j$.

The control torque is then

$$\mathbf{M}_c = \begin{bmatrix} M_{g1} \\ M_{g2} \\ M_{g3} \end{bmatrix} + \mathbf{K}_d\omega, \tag{22}$$

where $\mathbf{K}_d$ is a negative definite matrix. Notice that the time derivative of the work done by the field $\mathbf{K}_d\omega$

$$\dot{W} = \omega^{\mathrm{T}}\mathbf{K}_d\omega \tag{23}$$

is negative definite.

We wrap up this subsection by reformulating Eqs. (20) and (21) for an arbitrary reference $\mathbf{q}_e$. The nominator of the potential energy is modified

$$V(\mathbf{q}) = k_p\frac{1-\mathbf{e}^{\mathrm{T}}\mathbf{Q}^{\mathrm{T}}(\mathbf{q}_e)\mathbf{q}}{(\mathbf{q}^{\mathrm{T}}\mathbf{W}_1)^2+(\mathbf{q}^{\mathrm{T}}\mathbf{W}_2)^2}. \tag{24}$$

Then the controller is given by Eq. (22), where the vector $\left[M_{g1}\ M_{g2}\ M_{g3}\ M_{g4}\right]^T$ is

$$\frac{k_p\mathbf{Q}^{\mathrm{T}}(\mathbf{q})\left((\tilde{q}_1^2+\tilde{q}_2^2)\mathbf{q}_e+2(1-\mathbf{q}_e^{\mathrm{T}}\mathbf{q})(\tilde{q}_1\mathbf{W}_1+\tilde{q}_2\mathbf{W}_2)\right)}{-2\left(\tilde{q}_1^2+\tilde{q}_2^2\right)^2}$$

3.2 *Control Torque Command*

To implement the control law in Eqs. (21) and (22) for a spacecraft actuated by reaction wheels, an additional computation has to be carried out. The term $\omega \times \mathbf{h}_w$, where $\mathbf{h}_w$ is the angular momentum vector contributing from all four reaction wheels, has to be feed-forwarded by the controller. This is done in order to incorporate the angular momentum of the wheels in dynamics of the spacecraft. As the result, the torque generated by the wheels is computed according to

$$\mathbf{M}_w = \mathbf{M}_c + \omega \times \mathbf{h}_w. \tag{25}$$

3.3 *Control Algorithm*

(1) Compute the damping term

$$\mathbf{M}_{Damp} = \mathbf{K}_d\ \omega. \tag{26}$$

(2) Compute the conservative term

$$\mathbf{M}_{Conserv} = -\mathbf{K}_p\left[M_{g1}\ M_{g2}\ M_{g3}\right]^{\mathrm{T}}, \tag{27}$$

where the vector $\left[M_{g1}\ M_{g2}\ M_{g3}\ M_{g4}\right]^T$ is

$$\frac{\mathbf{Q}^{\mathrm{T}}(\mathbf{q})\left((\tilde{q}_1^2+\tilde{q}_2^2)\mathbf{q}_e+2(1-\mathbf{q}_e^{\mathrm{T}}\mathbf{q})(\tilde{q}_1\mathbf{W}_1+\tilde{q}_2\mathbf{W}_2)\right)}{-2\left(\tilde{q}_1^2+\tilde{q}_2^2\right)^2} \tag{28}$$

(3) Compute the angular momentum compensation

$$\mathbf{M}_{Compen} = \omega \times \mathbf{h}_w. \tag{29}$$

(4) Compute the control torque

$$\mathbf{M}_{Control} = \mathbf{M}_{Damp} + \mathbf{M}_{Conserv} + \mathbf{M}_{Compen}. \tag{30}$$

4. CONTROL TORQUE ALLOCATION

The problem of angular momentum distribution will be formulated and subsequently solved in this section. The control torque allocation provides ability to allocate the control torque computed by the attitude controller among the reaction wheels in tetrahedron configuration.

The problem considered is to find minimum of the function J

$$\min_{\mathbf{h}_w} J = \min_{\mathbf{h}_w} ||\mathbf{h}_w - \bar{\mathbf{h}}_w|| \tag{31}$$

subject to the constraint equation

$$\mathbf{D}\mathbf{h}_w = \mathbf{h}, \tag{32}$$

where $||\cdot||$ denotes the standard Euclidean norm, $\mathbf{h}_w$ is the vector which i-th component h_w^i is the angular momentum vector of i-th momentum wheel, $\bar{\mathbf{h}}_w$ is the nominal value of $\mathbf{h}_w$. We shall denote the problem (31), (32) as the Optimal Momentum Distribution Problem (OMDP).

Knowing $\mathbf{h}_w(k)$ at the time instant t_k and a constant value of the control torque $\mathbf{M}_c$ in the time interval $[t_k, t_{k+1}[$ the increment of the angular momentum is calculated $\Delta\mathbf{h}_w = \mathbf{M}_c T_s$, where $T_s = t_{k+1} - t_k$ is the sampling time in the discrete time implementation.

The difference between the present value of the angular momentum $\mathbf{h}_w(t_k)$ and the nominal value is denoted by $\Delta\mathbf{H}_w = \mathbf{h}_w(t_k) - \bar{\mathbf{h}}_w$. To formulate the OMDP as one of the standard static optimization problems, two vectors $\Delta\mathbf{L}_w$ and $\Delta\mathbf{L}$ are defined

$$\begin{aligned} \Delta\mathbf{L}_w &= \Delta\mathbf{h}_w + \Delta\mathbf{H}_w \\ \Delta\mathbf{L} &= \Delta\mathbf{h} + \mathbf{D}\Delta\mathbf{H}_w. \end{aligned} \tag{33}$$

Now, the OMDP is expressed: Find $\Delta\mathbf{L}_w$ such that

$$\min_{\mathbf{L}_w} ||\mathbf{L}_w|| \tag{34}$$

subject to

$$\mathbf{D}\Delta\mathbf{L}_w = \Delta\mathbf{L}. \tag{35}$$

The optimization problem (34) and (35) has the solution (Griffel, 1989)

$$\Delta\mathbf{L}_w = \mathbf{D}^{\mathrm{R}}\Delta\mathbf{L}, \tag{36}$$

where $\mathbf{D}^{\mathrm{R}} = \mathbf{D}^{\mathrm{T}}(\mathbf{D}\mathbf{D}^{\mathrm{T}})^{-1}$ is the right pseudo-inverse of $\mathbf{D}$.

After substitution of Eq. (33) into Eq. (36), the solution to the OMDP is

$$\Delta\mathbf{h}_w = \mathbf{D}^{\mathrm{R}}\Delta\mathbf{h} - (\mathbf{E} - \mathbf{D}^{\mathrm{R}}\mathbf{D})\Delta\mathbf{H}_w. \qquad (37)$$

Eq. (37) has an elegant geometric interpretation. The image of $\mathbf{D}^{\mathrm{R}}$ coincides with the image of $\mathbf{D}^{\mathrm{T}}$ and the image of $(\mathbf{E} - \mathbf{D}^{\mathrm{R}}\mathbf{D})$ is the kernel of $\mathbf{D}$. Hence the two terms on the right hand side of Eq. (37) are orthogonal and $\Delta\mathbf{h}_w$ satisfying $\min_{\mathbf{h}_w} ||\mathbf{h}_w||$ subject to $\mathbf{D}\Delta\mathbf{h}_w = \Delta\mathbf{h}$ is $\Delta\mathbf{h}_w = \mathbf{D}^{\mathrm{R}}\Delta\mathbf{h}$. The second term in Eq. (37) is used to remove the excess of the angular momentum in the wheels from their nominal value.

Finally, Eq. (37) shall be rewritten using information about the computed control torque $\mathbf{M}_c$ and the torque generated by the wheels $\mathbf{M}_w$. This can be done using an observation that the control torque is constant between samples

$$\mathbf{M}_w = \mathbf{D}^{\mathrm{R}}\mathbf{M}_c - (\mathbf{E} - \mathbf{D}^{\mathrm{R}}\mathbf{D})\frac{\Delta\mathbf{H}_w}{T_s}. \qquad (38)$$

5. SIMULATION VALIDATION

The control algorithm is validated by the simulation test performed in Matlab®/ Simulink® environment. The spacecraft principal moments of inertia are $I_{xx} = 5$, $I_{yy} = 6$, $I_{zz} = 7$ kgm^2; the proportional control gain used in the test is $\mathbf{K}_p = 2.16\ 10^{-3}\mathbf{E}$, and the derivative control gain is $\mathbf{K}_d = -0.14\mathbf{E}$. When choosing the control parameters, matrices $\mathbf{K}_d$ and $\mathbf{K}_p$ the following considerations are taken into account:

- maximum torque produced by the reaction wheel assembly,
- maximum allowable angular velocity of the reaction wheel,
- large $\mathbf{K}_p$ gain contributes to quick initialization of the spacecraft slew maneuver (fast slew maneuver),
- large $\mathbf{K}_d$ gain contributes to good disturbance attenuation.

Two examples of a simulation tests are shown in Fig. 3 and Fig. 4. Fig. 3 depicts the test for the initial attitude $[0.5\ 0.5\ 0.5\ -0.5]^{\mathrm{T}}$, and the reference at the identity quaternion. It is seen that the inclination angle between the bore axis of the star camera and the Sun vector increases to 125 deg. Afterwards, it is reduced to 45 deg, which is the inclination angle at the reference. The simulation test for the initial attitude quaternion $[0\ 0\ 1\ 0]^{\mathrm{T}}$, and the reference $[0.16\ 0.32\ 0.48\ 0.80]^{\mathrm{T}}$ is illustrated in Fig. 4. Again the inclination between the star camera's bore axis and the sun vector increases to 90 deg then converges to the value at the reference.

The control torque allocation is designed to keep the angular momentum of the reaction wheels near their nominal values. Figure 5 illustrates this functionality.

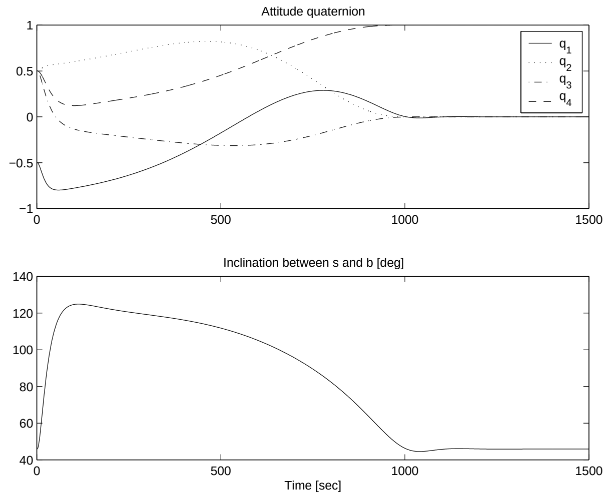

Fig. 3. Slew maneuver for the initial attitude $[0.5\ 0.5\ 0.5\ -0.5]^{\mathrm{T}}$, and the reference **e**.

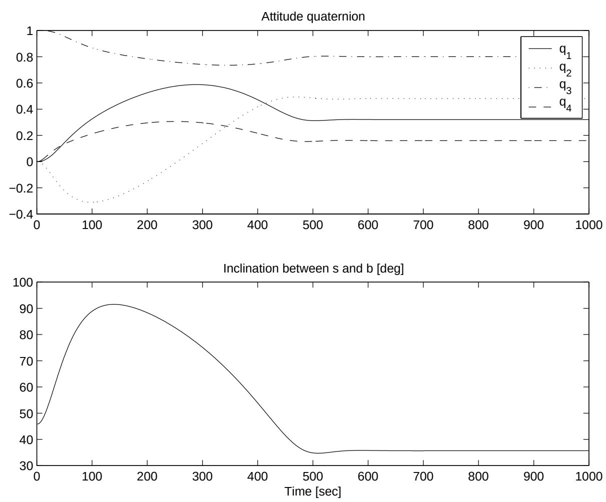

Fig. 4. Slew maneuver for initial attitude quaternion $[0\ 0\ 1\ 0]^{\mathrm{T}}$, and the reference $[0.32\ 0.48\ 0.80\ 0.16]^{\mathrm{T}}$.

The slew maneuver controller is activated during 2000 sec. The initial angular momentum is $[-140\ -140\ -140\ -140]^{\mathrm{T}}$. The algorithm distributes the control torque such that the angular momentum converges towards the nominal value $[140\ 140\ 140\ 140]^{\mathrm{T}}$.

6. CONCLUSIONS

The slew maneuver controller was proposed for a spacecraft equipped with a star camera and four reaction wheels in the tetrahedron configuration. The controller development was based on the energy shaping method. The desired potential function was carefully designed using a physical insight into the nature of the problem. The controller was designed to satisfy requirement that during the maneuver the camera should never be exposed to the Sun light. A second task of the controller was to distribute the control torque among the reaction wheels in such a way that the result angular momentum of each wheel was nearest to its

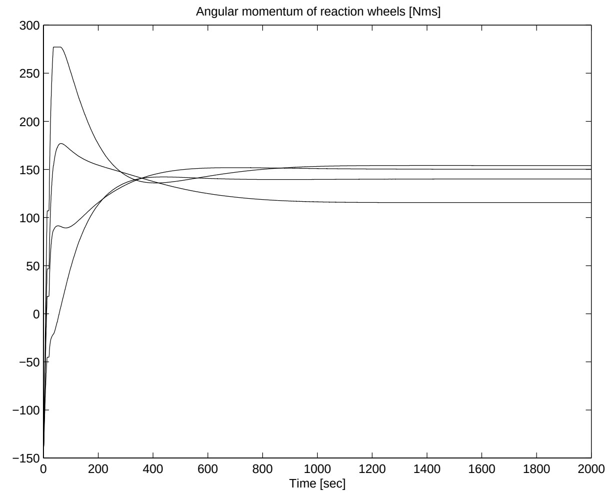

Fig. 5. The slew maneuver controller is activated during 2000 sec. The algorithm distributes the control torque such that the angular momentum of the wheels converges towards the nominal value $[140\ 140\ 140\ 140]^T$.

nominal value. A detailed simulation study showed convincing results for entire envelope of spacecraft operation.

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