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THE PARAMETERIZATION OF ALL STABILIZING REPETITIVE CONTROLLERS FOR A CERTAIN CLASS OF NON-MINIMUM PHASE SYSTEMS

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Abstract: We give an explicit parameterization of all causal stabilizing repetitive controllers for single-input/single-output continuous time non-minimum phase systems of a certain class. When the plant meets certain conditions, using the parallel compensation technique, we obtain the parameterization of all repetitive controllers. Finally, a numerical example shows the effectiveness of this parameterization.

Keywords: parameterization, interpolation problem, strong stability, parallel compensation, inverse system

1. INTRODUCTION

We examine the parameterization of all stabilizing repetitive controllers for a certain class of non-minimum phase systems. The repetitive control system is a type of servomechanism for repetitive reference signals. That is, the repetitive control system follows the periodic reference input without steady state error even if a periodic disturbance or uncertainty exists in the plant (Inoue, et al., 1980; Inoue, Iwai and Nakano, 1981; Hara, Omata, and Nakano, 1986; Yamamoto and Hara, 1987; Hara and Yamamoto, 1986; Hara, Yamamoto, Omata and Nakano, 1988; Omata, Hara and Nakano, 1987; Watanabe and Yamatari, 1986; Ikeda and Takano, 1988; Gotou, et al., 1987; Katoh and Funahashi, 1996). The repetitive control system was initially proposed for 'high accuracy control magnet power supply of proton synchrotron'(Inoue, et al., 1980). Subsequently, several papers on the theory and application of repetitive control systems have been published (Inoue, et al., 1980; Inoue, Iwai and Nakano, 1981; Hara, Omata, and Nakano, 1986; Yamamoto and Hara, 1987; Hara and Yamamoto, 1986; Hara, Yamamoto, Omata and Nakano, 1988; Omata, Hara and Nakano, 1987; Watanabe and Yamatari, 1986; Ikeda and Takano, 1988; Gotou, et al., 1987; Katoh and Funahashi, 1996). Because a repetitive control system follows any periodic reference input without steady state error is a neutral type of time-delay control system, it is difficult to design stabilizing controllers for the plant (Watanabe and Yamatari, 1986). To design a repetitive control system that follows any periodic reference input without steady state error, the plant needs to be biproper (Hara, Omata, and Nakano, 1986; Yamamoto and Hara, 1987; Hara and Yamamoto, 1986; Hara, Yamamoto, Omata and Nakano, 1988; Omata, Hara and Nakano, 1987; Watanabe and Yamatari, 1986). Ikeda and Takano (Ikeda and Takano, 1988) pointed out that it is physically difficult for the output to follow any periodic reference input without steady state error. In addition they showed that the repetitive control system is L_2 stable for periodic signals that do not include infinite frequency signals if the relative degree of the controller is one.

However, if the actual control system is strictly proper and has any relative degree, many design methods are given in (Hara, Omata, and Nakano, 1986; Hara and Yamamoto, 1986; Yamamoto and Hara, 1987; Hara, Yamamoto, Omata and Nakano, 1988; Omata, Hara and Nakano, 1987; Watanabe and Yamatari, 1986). These studies are divided into two types. One uses a low pass filter and the other uses an attenuator. Since the first type of repetitive control system has a simple structure, and is easy to design, this design method is used in many applications.

On the other hand, there remains the important control problem of finding all stabilizing controllers, called the parameterization problem (D.C. Youla, H.Jabr and J.J. Bongiorno, 1976; V. Kucera, 1979; C.A. Dedoer, R.W. Liu, J. Murray and R. Saeks, 1980; J.J. Glaria and G.C. Goodwin, 1994; M. Vidyasagar, 1985). Initially, the parameterization of a stabilizing repetitive compensator was studied by Hara and Yamamoto (Hara and Yamamoto, 1986). In (Hara and Yamamoto, 1986), since the stability sufficient condition of a repetitive control system is defined as an H_∞ norm problem, the parameterization for the repetitive control system is given by resolving the interpolation problem of Nevanlinna-Pick. Katoh and Funahashi give the parameterization of repetitive control system for minimum phase systems by solving the Bezout equation exactly (Katoh and Funahashi, 1996). In (Katoh and Funahashi, 1996), the parameterization is not based on sufficient stability condition. This result is important in the sense that the class of repetitive controllers is more extensive than the class of repetitive controller given in (Hara and Yamamoto, 1986). However, in (Katoh and Funahashi, 1996), the plant is assumed to be an asymptotically stable, or able to be stabilized by local feedback control. This implies that (Katoh and Funahashi, 1996) gave a parameterization of all stabilizing repetitive controllers for asymptotically stable and minimum phase plants. That is (Katoh and Funahashi, 1996) did not give the exact parameterization for minimum phase systems.

In this paper, we consider the parameterization for all stabilizing repetitive controllers for a certain class of non-minimum phase systems that has never considered. Using the fusion of the parallel compensation technique and the solution of the Bezout equation, the parameterization of all stabilizing controllers can be obtained.

Notation

- $R(s)$ the set of real rational function with s.
- RH_∞ the set of stable proper real rational functions.
- H_∞ a set of stable causal functions.
- U the unimodular procession in RH_∞. That is, $U(s) \in U$ means that $U(s) \in H_\infty$ and $1/U(s) \in H_\infty$.

2. PROBLEM FORMULATION

Let us consider unity feedback system given by

$$\begin{cases} y = G(s)u \\ u = C(s)(r-y) \end{cases}, \tag{1}$$

where $G(s)$ is a plant, $G(s)$ is controllable and observable, and $s/G(s)$ is assumed to be strongly stabilizable. That is, a stable controller to stabilize $1/G(s)$ exists. y is the output. u is the control input. r is the periodic reference input with period T written as

$$r(t+T) = r(t). \ \ (\forall t \geq 0) \tag{2}$$

If the plant $G(s)$ has a periodic disturbance, with period T and uncertainty, and the output y follows the periodic reference input, r, with period T without steady state error, then the controller $C(s)$ must be described by (Yamamoto and Hara, 1987)

$$C(s) = C_r(s)\hat{C}(s), \tag{3}$$

where $C_r(s)$ is an internal model for the periodic reference input, with period T, written as

$$C_r(s) = \frac{a(s)}{1-q(s)e^{-sT}} \tag{4}$$

and $a(s) \in RH_\infty$ is minimum phase and $q(s)$ is strictly proper, asymptotically stable low pass filter satisfying $q(0) = 1$. It is assumed a necessary condition exists in the internally stabilizing controllers, that is, $G(s)C_r(s)$ is assumed to have no unstable pole-zero cancellation.

The problem considered in this paper is to give all causal controllers $\hat{C}(s)$ such that the system (1) is internally stable under the assumption that the repetitive controller $C_r(s)$ has settled beforehand.

3. THE PARAMETERIZATION OF ALL STABILIZING REPETITIVE CONTROLLERS

Using the controller $C(s)$ given by (3), the parameterization of all causal controllers so that the system in (1) is internally stable is given by following theorem.

Theorem 1. Using $K(s)$ such that $G(s)a(s)/(1-q(s)e^{-sT})+K(s)$ is of minimum phase and asymptotically stable and both $K(s)$ and $1/K(s)$ are causal, the parameterization of all controllers $\hat{C}(s)$ in (3) that stabilize $G(s)$ is given by

$$\hat{C}(s) = \frac{\bar{C}(s)}{1+\bar{C}(s)K(s)}, \tag{5}$$

$$\left(\lim_{\omega\to\infty} \bar{C}(j\omega)K(j\omega) \neq -1\right)$$

where $\bar{C}(s)$ is written by

$$\bar{C}(s) = \frac{1}{Q(s)} - \frac{1}{\frac{a(s)}{1-q(s)e^{-sT}}G(s)+K(s)}. \tag{6}$$

Here, $Q(s)$ is any asymptotically stable non-zero function such that both $Q(s)$ and $1/Q(s)$ are causal.

Proof of this theorem requires the following theorem.

Theorem 2. $G(s)a(s)/(1-q(s)e^{-sT})+K(s)$ is assumed to be of minimum phase and both

$$\frac{a(s)}{1-q(s)e^{-sT}}G(s)+K(s)$$

and

$$\frac{1}{\dfrac{a(s)}{1-q(s)e^{-sT}}G(s)+K(s)}$$

are causal. The parameterization of all controller $\hat{C}(s)$ that stabilize the unity feedback control system in

$$\begin{aligned} y &= \left\{\frac{a(s)}{1-q(s)e^{-sT}}G(s)+K(s)\right\}u \\ u &= -\hat{C}(s)y \end{aligned} \tag{7}$$

is given by

$$\hat{C}(s)=\frac{1}{Q(s)}-\frac{1}{\dfrac{a(s)}{1-q(s)e^{-sT}}G(s)+K(s)}. \tag{8}$$

Here $Q(s)$ is any non-zero asymptotically stable rational function such that both $Q(s)$ and $1/Q(s)$ are causal.

Theorem 3. If $s/G(s)$ are strongly stabilizable, there exists $K(s)$ such that $G(s)a(s)/(1-q(s)e^{-sT})+K(s)$ is of minimum phase and asymptotically stable and both $K(s)$ and $1/K(s)$ are causal.

Proof of Theorem 3 requires the following theorem.

Theorem 4. Let $\sigma_1,\cdots,\sigma_l$ be distinct nonnegative extended real numbers, and let $s_{l+1},\cdots,s_n$ be distinct complex numbers having a positive imaginary part. Let

$$S=\{\sigma_1,\ldots,\sigma_l,s_{l+1},\ldots,s_n\},$$

let

$$M=\{m_1,\ldots,m_n\}$$

be a corresponding set of positive integers, and let

$$R=\{r_{ij},j=0,\ldots,m_j-1;i=1,\ldots,n\}$$

be a set of complex numbers, where r_{ij} is real whenever $j=0,\cdots,m_i-1$, $i=1,\cdots,l$, and $r_{i0}\neq 0$ for all i. Under these conditions, there exists a unimodular matrix $U(s)$ in S that satisfies

$$\frac{\mathrm{d}^j}{\mathrm{d}s^j}U(s_i)=r_{ij} \tag{9}$$

if, and only if, $r_{10},\cdots,r_{l0}$ are all of the same sign (M. Vidyasagar, 1985).

Using above mentioned theorems, we shall show the proof of Theorem 3.
(Proof) We will show there exists $K(s)$ such that $G(s)a(s)/(1-q(s)e^{-sT})+K(s)$ is of minimum phase, $K(s)$ is asymptotically stable and both $K(s)$ and $1/K(s)$ are causal.

$G(s)a(s)/(1-q(s)e^{-sT})+K(s)$ is rewritten by

$$\begin{aligned} &\frac{a(s)}{1-q(s)e^{-sT}}G(s)+K(s) \\ &=\frac{N(s)a(s)+K(s)D(s)(1-q(s)e^{-sT})}{D(s)(1-q(s)e^{-sT})}, \end{aligned} \tag{10}$$

where, $D(s)\in RH_\infty$ and $N(s)\in RH_\infty$ are coprime factor on RH_∞ of $G(s)$ written by

$$G(s)=D^{-1}(s)N(s). \tag{11}$$

Therefore, the condition for the existence of an asymptotically stable $K(s)$ so that $G(s)a(s)/(1-q(s)e^{-sT})+K(s)$ is of minimum phase is equal to the condition for the existence of an asymptotically stable $K(s)$ and $U(s)$ satisfying

$$U(s)=N(s)a(s)+K(s)D(s)(1-q(s)e^{-sT})\in\mathcal{U}. \tag{12}$$

This condition is equal to the interpolation problem written by

$$\frac{\mathrm{d}^j}{\mathrm{d}s^j}U(s_i)=\frac{\mathrm{d}^j}{\mathrm{d}s^j}N(s_i), \quad (j=0,\ldots,m_i-1,i=1,\ldots,l) \tag{13}$$

where $\sigma_1,\cdots,\sigma_l$ are unstable zeros of $(1-q(s)e^{-sT})D(s)$.

Next, we show that interpolation problem in (13) is solvable. From the assumption that $q(s)$ is strictly proper, the number of zeroes of $1-q(s)e^{-sT}$ in the closed right half plane is finite. Therefore, Theorem4 is applicable to the interpolation problem in (13). From the assumption that $s/G(s)$ is strongly stabilizable, $N(s_i)$ is always the same sign as s_i on the real axis. From the assumption that $a(s)$ is minimum phase, $N(s_i)a(s_i)$ is always the same sign. From Theorem4, a $U(s)$ that satisfies (13) exists.

Next, we show that if $U(s)$ satisfies (13), then both $K(s)=\dfrac{U(s)-N(s)a(s)}{D(s)(1-q(s)e^{-sT})}$ and $1/K(s)$

are causal. From the assumption that $N(s)$ is strictly proper, $U(s)-N(s)a(s)$ and $1/(U(s)-N(s)a(s))$ are causal. From the assumption that $D(s)$ is biproper, both $D(s)(1-q(s)e^{-sT})$ and $1/D(s)(1-q(s)e^{-sT})$ are causal. Accordingly, $K(s) = \dfrac{U(s)-N(s)a(s)}{D(s)(1-q(s)e^{-sT})}$ and $1/K(s)$ are causal.

We have, therefore, proved Theorem3 ∎

Using the above theorem, we shall show the proof of Theorem1.
(Proof) From Theorem3, there exists $K(s) \in H_\infty$ such that both $K(s)$ and $1/K(s)$ are causal. Therefore, we show that the following expressions hold.

(1) If the causal controller $\hat{C}(s)$ in (3) stabilize the plant $G(s)$, then

$$\bar{C}(s) = \frac{\hat{C}(s)}{1-\hat{C}(s)K(s)}$$

stabilizes $G(s)C_r(s)+K(s) = G(s)a(s)/(1-q(s)e^{-sT})+K(s)$.

(2) If the causal controller $\bar{C}(s)$ stabilizes

$$\begin{aligned} &G(s)C_r(s)+K(s) \\ &= G(s)a(s)/(1-q(s)e^{-sT})+K(s), \end{aligned}$$

then

$$\hat{C}(s) = \frac{\bar{C}(s)}{1+\bar{C}(s)K(s)}$$

stabilizes $G(s)C_r(s)$.

The former expression is proved as follows. From the assumption that $\hat{C}(s)$ is causal and

$$\lim_{w\to\infty} \bar{C}(jw)K(jw) \neq -1,$$

$\bar{C}(s)$ is causal. We have

$$\begin{aligned} &\frac{1}{1+\bar{C}(s)\left(G(s)C_r(s)+K(s)\right)} \\ &= \frac{1}{1+\dfrac{\hat{C}(s)}{1-\hat{C}(s)K(s)}\left(G(s)C_r(s)+K(s)\right)} \\ &= \frac{1-\hat{C}(s)K(s)}{1+\hat{C}(s)G(s)C_r(s)}. \end{aligned} \tag{14}$$

From the assumption that $\hat{C}(s)C_r(s)$ stabilizes $G(s)$, $\bar{C}(s) = \dfrac{\hat{C}(s)}{1-\hat{C}(s)K(s)}$ stabilizes $G(s)C_r(s)+K(s) = G(s)a(s)/(1-q(s)e^{-sT})+K(s)$. Therefore, the former expression is proved.

Next, the latter expression is proved. From

$$\hat{C}(s) = \frac{\bar{C}(s)}{1+\bar{C}(s)K(s)},$$

$$\left(1+\bar{C}(s)K(s)\right)\left(1-\hat{C}(s)K(s)\right) = 1 \tag{15}$$

holds. From the assumption of

$$\left(1+\bar{C}(s)K(s)\right)\left(1-\hat{C}(s)K(s)\right) = 1$$

and

$$\lim_{w\to\infty} \bar{C}(jw)K(jw) \neq -1,$$

$$\lim_{w\to\infty} \hat{C}(jw)K(jw) \neq -1 \tag{16}$$

is satisfied. From the assumption $\bar{C}(s)$ is causal, $\hat{C}(s)$ is causal. We have

$$\begin{aligned} &\frac{1}{1+\hat{C}(s)G(s)C_r(s)} \\ &= \frac{1}{1+\dfrac{\bar{C}(s)}{1+\bar{C}(s)K(s)}G(s)C_r(s)} \\ &= \frac{1+\bar{C}(s)K(s)}{1+\bar{C}(s)\left(G(s)C_r(s)+K(s)\right)}. \end{aligned} \tag{17}$$

From the assumption that $\bar{C}(s)$ stabilizes $G(s)C_r(s)+K(s)$, $\hat{C}(s) = \dfrac{\bar{C}(s)}{1+\bar{C}(s)K(s)}$ stabilizes $G(s)C_r(s)$. The latter part is proved.

From Theorem2, the parameterization of all controllers $\bar{C}(s)$ to stabilize $G(s)C_r(s)+K(s)$ is written as

$$\bar{C}(s) = \frac{1}{Q(s)} - \frac{1}{\dfrac{a(s)}{1-q(s)e^{-sT}}G(s)+K(s)}. \tag{18}$$

This completes the proof of this theorem. ∎

4. A DESIGN METHOD OF FREE PARAMETER $Q(S)$

In this section, we present a design method of free parameter $Q(s)$ to specify the sensitivity characteristic.

The sensitivity function $S(s)$ is written as

$$\begin{aligned} &S(s) \\ &= \frac{Q(s)\dfrac{a(s)}{1-q(s)e^{-sT}}G(s)}{\left(\dfrac{a(s)}{1-q(s)e^{-sT}}G(s)+K(s)\right)^2} \\ &+ \frac{K(s)\left(\dfrac{a(s)}{1-q(s)e^{-sT}}G(s)+K(s)\right)}{\left(\dfrac{a(s)}{1-q(s)e^{-sT}}G(s)+K(s)\right)^2} \end{aligned} \tag{19}$$

In order to specify the sensitivity function $S(s)$ using $Q(s)$, $Q(s)$ is settled by

$$Q(s)=-\frac{K(s)\left(\dfrac{a(s)}{1-q(s)e^{-sT}}G(s)+K(s)\right)}{\dfrac{a(s)}{1-q(s)e^{-sT}}\hat{G}(s)}\bar{q}(s). \tag{20}$$

Here,

$$G(s)=G_i(s)\hat{G}(s), \tag{21}$$

$G_i(s)$ is inner function satisfying $|G_i(jw)|=1(\forall\omega\in R)$ and $\hat{G}(s)$ is of minimum phase and $\bar{q}(s)\in RH_\infty$ is a strictly proper low pass filter. $Q(s)$ defined by (20) is obviously an asymptotically stable causal function. Using $Q(s)$ in (20), the sensitivity function $S(s)$ is rewritten as

$$S(s)=\frac{K(s)\left(1-\bar{q}(s)G_i(s)\right)}{\dfrac{a(s)}{1-q(s)e^{-sT}}G(s)+K(s)}. \tag{22}$$

The desirable sensitivity characteristic is obtained using $\bar{q}(s)\in RH_\infty$.

5. NUMERICAL EXAMPLE

In this section, we present a numerical example to show effectiveness of this result.

Let $G(s)$ be

$$G(s)=\frac{-s+1}{2s^2+3s+1}. \tag{23}$$

Let us consider to obtain the parameterization of all repetitive controllers for $G(s)$ in (23), where $T=1$[sec] and the repetitive compensator $C_r(s)$ is given by

$$C_r(s)=\frac{1}{1-q(s)e^{-sT}} \tag{24}$$

$$q(s)=\frac{1}{0.001s+1}. \tag{25}$$

One condition for $K(s)$ to hold the condition in Theorem3 is given by

$$K(s)=\frac{10s+5}{s+2}. \tag{26}$$

From Theorem 1, the parameterization of all controller $\hat{C}(s)$ is given by

$$\hat{C}(s)=\frac{\bar{C}(s)}{1+\bar{C}(s)K(s)} \tag{27}$$

$$\left(\lim_{\omega\to\infty}\bar{C}(j\omega)K(j\omega)\neq -1\right)$$

$$\bar{C}(s)=\frac{1}{Q(s)}-\frac{1}{G(s)\dfrac{a(s)}{1-q(s)e^{-sT}}+K(s)} \tag{28}$$

Free parameter $Q(s)$ is settled by

$$Q(s)=-\frac{K(s)\left(\dfrac{a(s)}{1-q(s)e^{-sT}}G(s)+K(s)\right)}{\dfrac{a(s)}{1-q(s)e^{-sT}}\hat{G}(s)}\bar{q}(s), \tag{29}$$

where $\bar{q}(s)$ is written by

$$\bar{q}(s)=\frac{1}{(0.01s+1)^2}. \tag{30}$$

The response to the reference input $r(t)=\sin(2\pi t)$ is shown in Fig. 1 . Here, a solid line shows the output y and a dotted line shows the reference input r. It is shown that the output y follows the reference input r without steady state error.

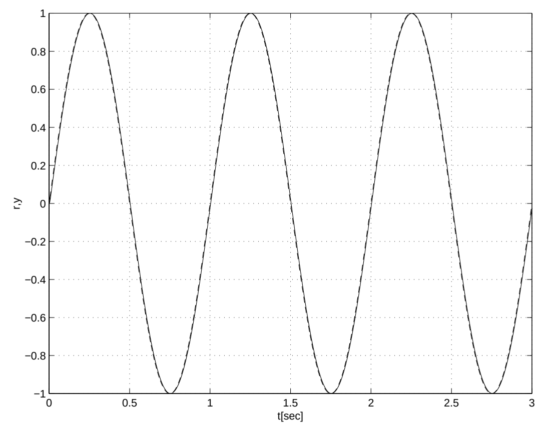

Fig. 1. Response for the reference input $r=\sin(2\pi t)$

Next, when disturbance $d=\sin(2\pi t)$ exists, the response for the disturbance is shown in Fig. 2 . Here, a solid line shows the output y and a dotted

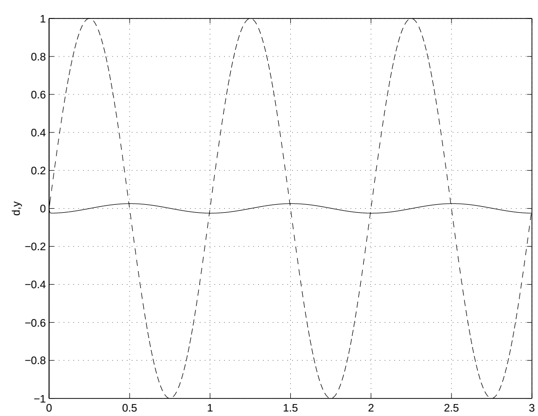

Fig. 2. Response for the disturbance $d=\sin(2\pi t)$

line shows the disturbance d. Fig. 2 shows that the disturbance is attenuated effectively.

From a practical point of view, the period of the disturbance is sometimes not equal to T. Next, when disturbance $d = \sin(1.02 \times 2\pi t)$, the response for the disturbance is shown in Fig. 3 . Here, a

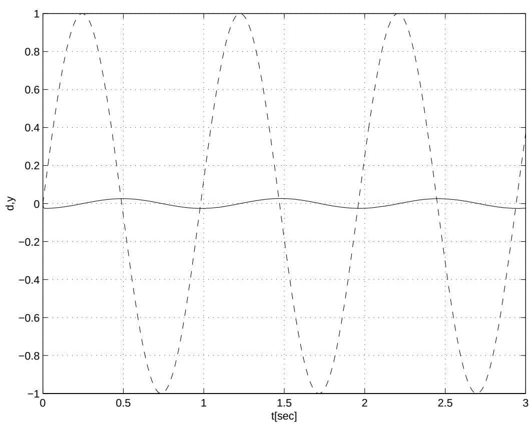

Fig. 3. Response for the disturbance $d = \sin(1.02 \times 2\pi t)$

solid line shows the output y and a dotted line shows the disturbance d. Fig. 3 shows that even if the period of disturbance is not equal to T, the disturbance is attenuated effectively.

6. CONCLUSIONS

In this paper, we proposed the parameterization of all stabilizing repetitive controllers for a certain class of all causal repetitive controllers of non-minimum phase systems such that the system is internally stable. A numerical example shows the effectiveness of the proposed method.

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