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OPTIMALITY OF DECENTRALIZED SIMPLE DYNAMIC DISPLACEMENT FEEDBACK FOR LARGE SPACE STRUCTURES

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Abstract: This paper considers position and attitude control of large space structures composed of a number of subsystems which are interconnected by springs and dampers. It is assumed that sensors and actuators are collocated. A decentralized simple dynamic displacement feedback, proposed by Fujisaki, Ikeda, and Miki, is transformed to a controller whose input contains the control input and the measured output of the space structure. The objective of this paper is to present a condition under which the controller is optimal for a quadratic cost function.

Keywords: Large space structures, Interconnected systems, Decentralized control, Optimality, Dynamic displacement feedback.

1. INTRODUCTION

This paper considers position and attitude control of large space structures composed of a number of subsystems which are interconnected flexibly by springs and dampers. Flexible interconnection of subsystems is realistic when we deal with very large space structures such as solar power satellites (Mankins, 1997). For interconnected large space structures, it is reasonable to apply decentralized control compatible with subsystems, which allows us to expand a large space structure by connecting a new subsystem, or remove a failed subsystem from the structure without any change of the control law. In this paper, we consider optimality of the decentralized simple dynamic displacement feedback proposed by Fujisaki, Ikeda, and Miki (2001) under the assumption of sensors/actuators collocation.

The simple dynamic displacement feedback (SDDF B) (Fujisaki *et al.*, 2001) is an extension of the direct velocity and displacement feedback (DVDFB) (Ikeda, Koujitani, and Kida, 1993) which robustly stabilizes uncertain space structures with collocated sensors and actuators if the rigid modes are controllable and observable. To apply DVDFB, we do not need to know any parameter values of the structure. The SDDFB retains this desirable property for large space structures.

The objective of this paper is to present a condition under which optimality of the decentralized SDDFB is achieved for a quadratic cost function. We expect desirable response characteristics of the overall control system by such optimality. However, it is not possible for mechanical systems such as space structures to be optimally controlled by proper dynamic displacement feedback, because the relative degree of mechanical systems is 2, while the relative degree of the loop transfer function of an optimal control system is 1. Therefore, we propose a transformation of the SDDFB to a controller whose input contains not only the measured output, but also the control input of the space structure to be controlled as in the cases of observer-based and Kalman-filter-based controllers. Then, it is shown that the overall closed-loop system is optimized by tuning the controller parameters properly.

The organization of this paper is as follows. In Section 2, we describe subsystems of the large space structure and present a transformation of the local SDDFB. In Section 3, we connect the subsystems by springs and dampers and form the decentralized SDDFB. In Section 4, we derive a condition for optimality of the control system. An example is presented in Section 5, which suggests how to choose parameters in local controllers to achieve the overall optimality.

2. SDDFB CONTROL OF SUBSYSTEMS

In this section, we present subsystems with collocated sensors and actuators and the stabilizing SDDFB control law. The subsystems are described by

$$M_i\ddot{q}_i + D_i\dot{q}_i + K_i q_i = L_i u_i, \quad y_i = L_i^T q_i \qquad i = 1, 2, \ldots, \ell \qquad (1)$$

where $q_i \in R^{n_i}$, $u_i \in R^{r_i}$, and $y_i \in R^{r_i}$ are the displacement, control input, and measured output of the i-th subsystem, respectively. The mass matrix M_i is positive definite, and the damping and stiffness matrices D_i, K_i are positive semidefinite. The existence of rigid modes implies

$$\text{rank}\begin{bmatrix} D_i & K_i \end{bmatrix} = \text{rank}\, D_i = \text{rank}\, K_i < n_i. \quad (2)$$

The matrix L_i is defined by the locations and directions of actuators, and the matrix L_i^T expresses the locations and directions of sensors. This transposition relation is implied by the sensors/actuators collocation. We assume that

$$\text{rank}\begin{bmatrix} D_i & L_i \end{bmatrix} = n_i, \quad \text{rank}\begin{bmatrix} K_i & L_i \end{bmatrix} = n_i \quad (3)$$

hold, which means that the rigid modes of each subsystem are controllable and observable.

To stabilize the subsystem, we consider a local SDDFB controller (Fujisaki *et al.*, 2001)

$$\begin{aligned}\dot{\zeta}_i &= -\gamma_i V_i \zeta_i + \beta_i\gamma_i U_i y_i \\ u_i &= \beta_i\gamma_i U_i^T \zeta_i \\ &\quad - \left(\alpha_i^2 U_i^T V_i^{-2} U_i + \beta_i^2\gamma_i U_i^T V_i^{-1} U_i\right) y_i \quad (4)\end{aligned}$$

where $\zeta_i \in R^{r_i}$ is the state of the controller, V_i is a positive definite matrix, U_i is a nonsingular matrix, and $\alpha_i, \beta_i, \gamma_i$ are positive constants. The transfer function of (4) is written as

$$\begin{aligned}u_i(s) = &-\alpha_i^2 U_i^T V_i^{-2} U_i y_i(s) - \beta_i^2 U_i^T V_i^{-2} U_i \\ &\cdot \left\{ \left(\frac{1}{\gamma_i}\right) s U_i^{-1} V_i^{-1} U_i + I_{r_i} \right\}^{-1} s y_i(s) \quad (5)\end{aligned}$$

where the second term works as a differentiator in the low frequency range. The parameter γ_i determines the bandwidth of differentiation and α_i^2, β_i^2 respectively play as the gains of the proportional part and the pseudo-differential part of the controller.

In the time domain, the term $\left\{ \left(\frac{1}{\gamma_i}\right) s U_i^{-1} V_i^{-1} U_i + I_{r_i} \right\}^{-1} s y_i(s)$ means the estimated value $\hat{y}_i$ of the velocity $\dot{y}_i$, and the SDDFB controller (4) can be wriiten as

$$u_i = -\alpha_i^2 W_i y_i - \beta_i^2 W_i \hat{y}_i \qquad (6)$$

where $W_i = U_i^T V_i^{-2} U_i$. Therefore, the SDDFB controller (4) is the approximation of the DVDFB controller

$$u_i = -\alpha_i^2 W_i y_i - \beta_i^2 W_i \dot{y}_i \qquad (7)$$

which can be applied in the case that the velocity of the subsystem is measurable as well as the displacement.

The closed-loop system obtained by applying the local controller (4) to the subsystem (1) is stable, and its stability is robust against the uncertainty in M_i, D_i, K_i of the structure (Fujisaki *et al.*, 2001). However, the closed-loop system cannot be optimal for any quadratic cost function, because the relative degree of its loop transfer function is 2. As well known, for a closed-loop system to be optimal, the relative degree of the loop transfer function has to be 1.

Now, we note that for optimal control, we generally use a Kalman filter or an observer to estimate the state, and apply state feedback control law. Such a Kalman-filter-based or observer-based controller's inputs are the measured output and control input of the system to be controlled. The controller can optimize systems of the relative degree 2 or even more. Seeing this fact, we use

$$\begin{aligned}&-\gamma_i V_i \zeta_i + \beta_i\gamma_i U_i y_i \\ &\quad = -\beta_i^{-1} V_i U_i^{-T} \{\beta_i\gamma_i U_i^T \zeta_i \\ &\qquad\quad - \left(\alpha_i^2 U_i^T V_i^{-2} U_i + \beta_i^2\gamma_i U_i^T V_i^{-1} U_i\right) y_i\} \\ &\qquad -\alpha_i^2\beta_i^{-1} V_i^{-1} U_i y_i \\ &\quad = -\alpha_i^2\beta_i^{-1} V_i^{-1} U_i y_i - \beta_i^{-1} V_i U_i^{-T} u_i\end{aligned}$$

to transform the controller (4) equivalently to

$$\begin{aligned}\dot{\zeta}_i &= -\alpha_i^2\beta_i^{-1} V_i^{-1} U_i y_i - \beta_i^{-1} V_i U_i^{-T} u_i \\ u_i &= \beta_i\gamma_i U_i^T \zeta_i \\ &\quad - \left(\alpha_i^2 U_i^T V_i^{-2} U_i + \beta_i^2\gamma_i U_i^T V_i^{-1} U_i\right) y_i \quad (8)\end{aligned}$$

whose inputs are the measured output y_i and control input u_i of the subsystem (1), where U_i^{-T} denotes $(U_i^T)^{-1}$. In this paper, we use (8) as the local controller.

3. SDDFB OF OVERALL SYSTEM

We connect the subsystems (1) by springs and dampers. The obtained overall system is described by

$$\begin{aligned}&M_i\ddot{q}_i + D_i\dot{q}_i + K_i q_i \\ &\quad = L_i u_i + \sum_{j=1}^{\ell} N_{ij}\{K_{Cij}\left(N_{ji}^T q_j - N_{ij}^T q_i\right)\end{aligned}$$

$$+ D_{Cij}\left(N_{ji}^T\dot{q}_j - N_{ij}^T\dot{q}_i\right)\}$$
$$i = 1, 2, \ldots, \ell \tag{9}$$

where N_{ij} is a matrix representing the locations and directions of the springs and dampers at the i-th subsystem, which connect the i-th subsystem with the j-th subsystem. The matrices K_{Cij} and D_{Cij} respectively denote spring and damper parameters, which are positive semi-definite.

In (9),

$$K_{Cij}\left(N_{ji}^T q_j - N_{ij}^T q_i\right) + D_{Cij}\left(N_{ji}^T\dot{q}_j - N_{ij}^T\dot{q}_i\right)$$

represents the force and/or torque affecting the i-th subsystem by the springs and dampers connected with the j-th subsystem, and $N_{ij}^T q_i$ is the displacement of the connecting points of springs and dampers in the i-th subsystem. We assume that when the i-th and j-th subsystems are at the origins of their displacements, $q_i = 0$ and $q_j = 0$, the springs between them are neither stretched nor compressed. The effects of springs and dampers are bilateral, which implies $K_{Cij} = K_{Cji}$ and $D_{Cij} = D_{Cji}$.

Thus, the overall structure is described by

$$\tilde{M}\ddot{\tilde{q}} + \tilde{D}\dot{\tilde{q}} + \tilde{K}\tilde{q} = \tilde{L}\tilde{u}, \quad \tilde{y} = \tilde{L}^T\tilde{q} \tag{10}$$

where

$$\begin{aligned}
&\tilde{q} = [\, q_1^T \ q_2^T \ \cdots \ q_\ell^T \,]^T, \quad \tilde{u} = [\, u_1^T \ u_2^T \ \cdots \ u_\ell^T \,]^T \\
&\tilde{y} = [\, y_1^T \ y_2^T \ \cdots \ y_\ell^T \,]^T \\
&\tilde{M} = \mathrm{diag}\{M_i\}_{i=1,2,\ldots,\ell} \\
&\tilde{D} = \mathrm{diag}\{D_i\}_{i=1,2,\ldots,\ell} + \sum_{i=1}^{\ell-1}\sum_{j=i+1}^{\ell}\tilde{N}_{ij}D_{Cij}\tilde{N}_{ij}^T \\
&\tilde{K} = \mathrm{diag}\{K_i\}_{i=1,2,\ldots,\ell} + \sum_{i=1}^{\ell-1}\sum_{j=i+1}^{\ell}\tilde{N}_{ij}K_{Cij}\tilde{N}_{ij}^T \\
&\tilde{L} = \mathrm{diag}\{L_i\}_{i=1,2,\ldots,\ell}
\end{aligned}$$

Here, the matrix $\tilde{N}_{ij}$ is defined by N_{ij} and N_{ji} as

$$\tilde{N}_{ij} = \begin{bmatrix} \vdots \\ N_{ij} \\ \vdots \\ -N_{ji} \\ \vdots \end{bmatrix} \begin{matrix} \\ i \\ \\ j \\ \\ \end{matrix}$$

in which the elements except the submatrices N_{ij} and $-N_{ji}$ are all zero.

The local SDDFB controllers (8) are put together in the decentralized form as

$$\begin{aligned}
\dot{\tilde{\zeta}} &= -\bar{\alpha}^2\bar{\beta}^{-1}\tilde{V}^{-1}\tilde{U}\tilde{y} - \bar{\beta}^{-1}\tilde{V}\tilde{U}^{-T}\tilde{u} \\
\tilde{u} &= \bar{\beta}\bar{\gamma}\tilde{U}^T\tilde{\zeta} \\
&\quad - \left(\bar{\alpha}^2\tilde{U}^T\tilde{V}^{-2}\tilde{U} + \bar{\beta}^2\bar{\gamma}\tilde{U}^T\tilde{V}^{-1}\tilde{U}\right)\tilde{y}
\end{aligned} \tag{11}$$

where

$$\begin{aligned}
&\tilde{\zeta} = [\, \zeta_1^T \ \zeta_2^T \ \cdots \ \zeta_\ell^T \,]^T \\
&\tilde{V} = \mathrm{diag}\{V_i\}_{i=1,2,\ldots,\ell}, \quad \tilde{U} = \mathrm{diag}\{U_i\}_{i=1,2,\ldots,\ell} \\
&\bar{\alpha} = \mathrm{diag}\{\alpha_i I_{r_i}\}_{i=1,2,\ldots,\ell}, \ \bar{\beta} = \mathrm{diag}\{\beta_i I_{r_i}\}_{i=1,2,\ldots,\ell} \\
&\bar{\gamma} = \mathrm{diag}\{\gamma_i I_{r_i}\}_{i=1,2,\ldots,\ell}.
\end{aligned} \tag{12}$$

The overall closed-loop system is stable (Fujisaki *et al.*, 2001).

4. OPTIMALITY

The interconnected structure (10) and the decentralized controller (11) are described together in a state equation as

$$\dot{\tilde{\xi}} = \tilde{A}\tilde{\xi} + \tilde{B}\tilde{u} \tag{13}$$

$$\tilde{A} = \begin{bmatrix} 0 & I_{\tilde{n}} & 0 \\ -\tilde{M}^{-1}\tilde{K} & -\tilde{M}^{-1}\tilde{D} & 0 \\ -\bar{\alpha}^2\bar{\beta}^{-1}\tilde{V}^{-1}\tilde{U}\tilde{L}^T & 0 & 0 \end{bmatrix}$$

$$\tilde{B} = \begin{bmatrix} 0 \\ \tilde{M}^{-1}\tilde{L} \\ -\bar{\beta}^{-1}\tilde{V}\tilde{U}^{-T} \end{bmatrix}$$

$$\begin{aligned}
\tilde{u} = -\big[\, &\bar{\alpha}^2\tilde{U}^T\tilde{V}^{-2}\tilde{U}\tilde{L}^T + \bar{\beta}^2\bar{\gamma}\tilde{U}^T\tilde{V}^{-1}\tilde{U}\tilde{L}^T \\
&0 \quad -\bar{\beta}\bar{\gamma}\tilde{U}^T \,\big]\,\tilde{\xi}
\end{aligned} \tag{14}$$

where $\tilde{\xi} = \left[\, \tilde{q}^T \ \dot{\tilde{q}}^T \ \tilde{\zeta}^T \,\right]^T$ and $\tilde{n} = \sum_{i=1}^{\ell} n_i$. Then, we present the following theorem.

Theorem 1. For the system (13), the state feedback (14) is a stabilizing control law, and minimizes the quadratic cost function

$$\tilde{J} = \int_0^\infty (\tilde{\xi}^T\tilde{Q}\tilde{\xi} + \tilde{u}^T\tilde{R}\tilde{u})dt \tag{15}$$

$$\tilde{Q} = \tilde{T}^T\begin{bmatrix} \tilde{Q}_{11} & \tilde{Q}_{12} & \tilde{Q}_{13} \\ \tilde{Q}_{12}^T & \tilde{Q}_{22} & \tilde{Q}_{23} \\ \tilde{Q}_{13}^T & \tilde{Q}_{23}^T & \tilde{Q}_{33} \end{bmatrix}\tilde{T} \tag{16}$$

$$\begin{aligned}
\tilde{Q}_{11} &= \tilde{\alpha}^2\tilde{K} + \tilde{K}\tilde{\alpha}^2 + \tilde{\alpha}^4\tilde{L}\tilde{U}^T\tilde{V}^{-2}\tilde{U}\tilde{L}^T \\
\tilde{Q}_{12} &= \frac{1}{2}(\tilde{\alpha}^2\tilde{D} - \tilde{D}\tilde{\alpha}^2) + \frac{1}{2}(\tilde{K}\tilde{\beta}^2 - \tilde{\beta}^2\tilde{K}) \\
&\quad -(\tilde{K} + \tilde{\alpha}^2\tilde{L}\tilde{U}^T\tilde{V}^{-2}\tilde{U}\tilde{L}^T) \\
&\quad\quad \cdot\tilde{M}^{-1}\tilde{L}\tilde{U}^T\tilde{V}^{-3}\tilde{U}\tilde{L}^T\tilde{\beta}^4\tilde{\gamma}^{-1} \\
\tilde{Q}_{13} &= (\tilde{K} + \tilde{\alpha}^2\tilde{L}\tilde{U}^T\tilde{V}^{-2}\tilde{U}\tilde{L}^T) \\
&\quad \cdot\tilde{M}^{-1}\tilde{L}\tilde{U}^T\tilde{V}^{-3}\tilde{U}\tilde{\beta}^4\bar{\gamma}^{-1} \\
\tilde{Q}_{22} &= \tilde{\beta}^2\tilde{D} + \tilde{D}\tilde{\beta}^2 + \tilde{\beta}^4\tilde{L}\tilde{U}^T\tilde{V}^{-2}\tilde{U}\tilde{L}^T \\
&\quad -2\tilde{\alpha}^2\tilde{M} - \tilde{\beta}^4\tilde{\gamma}^{-1}\tilde{L}\tilde{U}^T\tilde{V}^{-3}\tilde{U}\tilde{L}^T\tilde{M}^{-1} \\
&\quad\quad \cdot(\tilde{D} + \tilde{\beta}^2\tilde{L}\tilde{U}^T\tilde{V}^{-2}\tilde{U}\tilde{L}^T) \\
&\quad -(\tilde{D} + \tilde{\beta}^2\tilde{L}\tilde{U}^T\tilde{V}^{-2}\tilde{U}\tilde{L}^T) \\
&\quad\quad \cdot\tilde{M}^{-1}\tilde{L}\tilde{U}^T\tilde{V}^{-3}\tilde{U}\tilde{L}^T\tilde{\beta}^4\tilde{\gamma}^{-1} \\
\tilde{Q}_{23} &= (\tilde{D} + \tilde{\beta}^2\tilde{L}\tilde{U}^T\tilde{V}^{-2}\tilde{U}\tilde{L}^T)
\end{aligned}$$

$$\cdot\tilde{M}^{-1}\tilde{L}\tilde{U}^T\tilde{V}^{-3}\tilde{U}\bar{\beta}^4\bar{\gamma}^{-1}$$

$$\tilde{Q}_{33} = \bar{\beta}^4\tilde{U}^T\tilde{V}^{-2}\tilde{U}$$

$$\tilde{T} = \begin{bmatrix} I_{\tilde{n}} & 0 & 0 \\ 0 & I_{\tilde{n}} & 0 \\ -\bar{\gamma}\tilde{U}^{-1}\tilde{V}\tilde{U}\tilde{L}^T & \tilde{L}^T & \bar{\beta}^{-1}\bar{\gamma}\tilde{U}^{-1}\tilde{V}^2 \end{bmatrix}$$

$$\tilde{\alpha} = \mathrm{diag}\{\alpha_i I_{n_i}\}_{i=1,2,\ldots,\ell}$$

$$\tilde{\beta} = \mathrm{diag}\{\beta_i I_{n_i}\}_{i=1,2,\ldots,\ell}$$

$$\tilde{\gamma} = \mathrm{diag}\{\gamma_i I_{n_i}\}_{i=1,2,\ldots,\ell}$$

$$\tilde{R} = (\tilde{U}^T\tilde{V}^{-2}\tilde{U})^{-1} \quad (17)$$

if the parameters $\alpha_i, \beta_i, \gamma_i$ are chosen so that the matrix $\tilde{Q}$ is positive definite.

(Proof) Since the state feedback (14) is equivalent to the controller (11), it is a stabilizing control law for the system (13). Then, we introduce a matrix

$$\tilde{P} = \tilde{T}^T \begin{bmatrix} \tilde{P}_{11} & \tilde{P}_{12} & \tilde{P}_{13} \\ \tilde{P}_{12}^T & \tilde{P}_{22} & \tilde{P}_{23} \\ \tilde{P}_{13}^T & \tilde{P}_{23}^T & \tilde{P}_{33} \end{bmatrix} \tilde{T} \quad (18)$$

$$\tilde{P}_{11} = \frac{1}{2}(\tilde{D}\tilde{\alpha}^2 + \tilde{\alpha}^2\tilde{D}) + \frac{1}{2}(\tilde{K}\tilde{\beta}^2 + \tilde{\beta}^2\tilde{K}) + \tilde{\alpha}^2\tilde{\beta}^2\tilde{L}\tilde{U}^T\tilde{V}^{-2}\tilde{U}\tilde{L}^T$$

$$\tilde{P}_{12} = \tilde{\alpha}^2\tilde{M}, \qquad \tilde{P}_{13} = 0$$

$$\tilde{P}_{22} = \tilde{\beta}^2\tilde{M} - \tilde{\beta}^4\tilde{\gamma}^{-1}\tilde{L}\tilde{U}^T\tilde{V}^{-3}\tilde{U}\tilde{L}^T + \tilde{\beta}^6\tilde{\gamma}^{-2}\tilde{L}\tilde{U}^T\tilde{V}^{-3}\tilde{U}\tilde{L}^T\tilde{M}^{-1}\tilde{L}\tilde{U}^T\tilde{V}^{-3}\tilde{U}\tilde{L}^T$$

$$\tilde{P}_{23} = -\tilde{\beta}^6\tilde{\gamma}^{-2}\tilde{L}\tilde{U}^T\tilde{V}^{-3}\tilde{U}\tilde{L}^T\tilde{M}^{-1}\tilde{L}\tilde{U}^T\tilde{V}^{-3}\tilde{U}$$

$$\tilde{P}_{33} = \bar{\beta}^4\bar{\gamma}^{-2}\tilde{U}^T\tilde{V}^{-1} \cdot(\bar{\gamma}\tilde{V}^{-1} + \bar{\beta}^2\tilde{V}^{-2}\tilde{U}\tilde{L}^T\tilde{M}^{-1}\tilde{L}\tilde{U}^T\tilde{V}^{-2})\tilde{V}^{-1}\tilde{U}.$$

Using this $\tilde{P}$, the stabilizing feedback gain of (14) can be described as

$$\tilde{R}^{-1}\tilde{B}^T\tilde{P} = \left[\ \bar{\alpha}^2\tilde{U}^T\tilde{V}^{-2}\tilde{U}\tilde{L}^T + \bar{\beta}^2\bar{\gamma}\tilde{U}^T\tilde{V}^{-1}\tilde{U}\tilde{L}^T \quad 0 \quad -\bar{\beta}\bar{\gamma}\tilde{U}^T\ \right]. \quad (19)$$

Furthermore, $\tilde{P}$ satisfies the following Riccati equation with $\tilde{Q}$ of (16) and $\tilde{R}$ of (17) for the system (13).

$$\tilde{P}\tilde{A} + \tilde{A}^T\tilde{P} - \tilde{P}\tilde{B}\tilde{R}^{-1}\tilde{B}^T\tilde{P} + \tilde{Q} = 0 \quad (20)$$

We rewrite (20) as

$$\tilde{P}(\tilde{A} - \tilde{B}\tilde{R}^{-1}\tilde{B}^T\tilde{P}) + (\tilde{A} - \tilde{B}\tilde{R}^{-1}\tilde{B}^T\tilde{P})^T\tilde{P} + \tilde{P}\tilde{B}\tilde{R}^{-1}\tilde{B}^T\tilde{P} + \tilde{Q} = 0, \quad (21)$$

where $\tilde{A} - \tilde{B}\tilde{R}^{-1}\tilde{B}^T\tilde{P}$ is stable and $\tilde{Q}$ is positive definite. Thus, $\tilde{P}$ is a unique positive definite solution of the Riccati equation (20). Therefore, the feedback gain (19) of the control law (14) is optimal for the cost function (15). The proof is completed.

The existence of the parameters α_i, β_i, and γ_i which satisfy the condition of Theorem 1 is shown as follows. Let us choose the parameters as

$$\alpha_i = \alpha, \quad \beta_i = \beta, \quad \gamma_i = \gamma, \quad i = 1, 2, \ldots, \ell. \quad (22)$$

Then, $\tilde{Q}$ of (16) becomes

$$\tilde{Q} = \tilde{T}^T \begin{bmatrix} \tilde{Q}_{11} & \tilde{Q}_{12} & \tilde{Q}_{13} \\ \tilde{Q}_{12}^T & \tilde{Q}_{22} & \tilde{Q}_{23} \\ \tilde{Q}_{13}^T & \tilde{Q}_{23}^T & \tilde{Q}_{33} \end{bmatrix} \tilde{T} \quad (23)$$

$$\tilde{Q}_{11} = \alpha^2\left(2\tilde{K} + \alpha^2\tilde{L}\tilde{U}^T\tilde{V}^{-2}\tilde{U}\tilde{L}^T\right)$$

$$\tilde{Q}_{12} = -\beta^4\gamma^{-1}\left(\tilde{K} + \alpha^2\tilde{L}\tilde{U}^T\tilde{V}^{-2}\tilde{U}\tilde{L}^T\right) \cdot\tilde{M}^{-1}\tilde{L}\tilde{U}^T\tilde{V}^{-3}\tilde{U}\tilde{L}^T$$

$$\tilde{Q}_{13} = \beta^4\gamma^{-1}\left(\tilde{K} + \alpha^2\tilde{L}\tilde{U}^T\tilde{V}^{-2}\tilde{U}\tilde{L}^T\right) \cdot\tilde{M}^{-1}\tilde{L}\tilde{U}^T\tilde{V}^{-3}\tilde{U}$$

$$\tilde{Q}_{22} = \beta^2\left(2\tilde{D} + \beta^2\tilde{L}\tilde{U}^T\tilde{V}^{-2}\tilde{U}\tilde{L}^T\right) - 2\alpha^2\tilde{M} - \beta^4\gamma^{-1}\tilde{L}\tilde{U}^T\tilde{V}^{-3}\tilde{U}\tilde{L}^T\tilde{M}^{-1} \cdot\left(\tilde{D} + \beta^2\tilde{L}\tilde{U}^T\tilde{V}^{-2}\tilde{U}\tilde{L}^T\right) - \beta^4\gamma^{-1}\left(\tilde{D} + \beta^2\tilde{L}\tilde{U}^T\tilde{V}^{-2}\tilde{U}\tilde{L}^T\right) \cdot\tilde{M}^{-1}\tilde{L}\tilde{U}^T\tilde{V}^{-3}\tilde{U}\tilde{L}^T$$

$$\tilde{Q}_{23} = \beta^4\gamma^{-1}\left(\tilde{D} + \beta^2\tilde{L}\tilde{U}^T\tilde{V}^{-2}\tilde{U}\tilde{L}^T\right) \cdot\tilde{M}^{-1}\tilde{L}\tilde{U}^T\tilde{V}^{-3}\tilde{U}$$

$$\tilde{Q}_{33} = \beta^4\tilde{U}^T\tilde{V}^{-2}\tilde{U}.$$

By increasing γ, the off-diagonal blocks $\tilde{Q}_{12}$, $\tilde{Q}_{13}$, and $\tilde{Q}_{23}$ in the matrix between $\tilde{T}^T$ and $\tilde{T}$ of $\tilde{Q}$ converge at zero matrices. On the other hand, the diagonal blocks $\tilde{Q}_{11}$ and $\tilde{Q}_{33}$ are always positive definite, and $\tilde{Q}_{22}$ becomes positive definite by choosing a sufficiently large β for α. Therefore, there exist α, β, and γ of (22), which satisfy the condition of Theorem 1.

The choices of $\alpha_i, \beta_i, \gamma_i$ in (22) are the simplest ones. The condition of Theorem 1 provides the freedom in choosing the parameters.

If the velocity of each subsystem in the structure is measurable as well as the displacement by sensors collocated with actuators, we can apply stabilizing decentralized DVDFB

$$\tilde{u} = -\bar{\alpha}^2\tilde{W}\tilde{y} - \bar{\beta}^2\tilde{W}\dot{\tilde{y}} \quad (24)$$

$$\tilde{W} = \mathrm{diag}\{W_i\}_{i=1,2,\ldots,\ell}$$

composed of the local DVDFB (7), where $\bar{\alpha}$ and $\bar{\beta}$ have been defined in (12). It has been shown (Ikeda, *et al.*, 1993) that if we set

$$\alpha_i = \alpha, \quad \beta_i = \beta, \quad i = 1, 2, \ldots, \ell \quad (25)$$

and choose α, β suitably, the decentralized DVDFB (24) becomes an optimal control law. We state the following lemma.

Lemma 1. For the space structure (1), the decentralized stabilizing control law (24) with (25) minimizes the quadratic cost function

$$\hat{J} = \int_0^\infty \left(\begin{bmatrix} \tilde{q}^T & \dot{\tilde{q}}^T \end{bmatrix} \hat{Q} \begin{bmatrix} \tilde{q} \\ \dot{\tilde{q}} \end{bmatrix} + \tilde{u}^T \hat{R} \tilde{u} \right) dt \qquad (26)$$

$$\hat{Q} = \begin{bmatrix} \alpha^2(2\tilde{K} + \alpha^2 \tilde{L}\tilde{W}\tilde{L}^T) & 0 \\ 0 & \beta^2(2\tilde{D} + \beta^2 \tilde{L}\tilde{W}\tilde{L}^T) - 2\alpha^2 \tilde{M} \end{bmatrix}$$

$$\hat{R} = \tilde{W}^{-1} > 0$$

where α and β are chosen so that $\hat{Q}$ is positive definite.

In this DVDFB case also, we need to increase β sufficiently for α to make the control law optimal.

Comparing $\hat{Q}$ and $\hat{R}$ in Lemma 1 with $\tilde{Q}$ and $\tilde{R}$ in Theorem 1 under the constraint (22), we see a relation. If we choose the matrices in (4) and (7) so that

$$W_i = U_i^T V_i^{-2} U_i, \qquad (27)$$

the matrix $\hat{Q}$ is equal to the upper left 2×2 block of the matrix between $\tilde{T}^T$ and $\tilde{T}$ of $\tilde{Q}$, and $\hat{R} = \tilde{R}$. Furthermore, the upper left 2×2 block of the matrix between $\tilde{T}^T$ and $\tilde{T}$ of $\tilde{P}$ in (18) is equal to

$$\hat{P} = \begin{bmatrix} \alpha^2 \tilde{D} + \beta^2 \tilde{K} + \alpha^2 \beta^2 \tilde{L}\tilde{W}\tilde{L}^T & \alpha^2 \tilde{M} \\ \alpha^2 \tilde{M} & \beta^2 \tilde{M} \end{bmatrix} \quad (28)$$

which has been used as the positive definite solution to the Riccati equation to prove Lemma 1 (Ikeda *et al.*, 1993).

5. EXAMPLE

Let us consider an interconnected space structure, which is composed of two subsystems connected in the x direction by springs and dampers as illustrated in Fig. 1. For simplicity, we assume that each subsystem consists of two rigid bodies of rectangular shapes, which are connected in the y direction by springs and dampers. The rigid bodies may not be of the same size.

The mass and moment of inertia of the j-th rigid body in the i-th subsystem is denoted by m_{ij} and J_{ij}. The motion of the rigid body is described by the displacements x_{ij}, y_{ij} of the center of mass and the rotational angle θ_{ij} around the center of mass. It is assumed that the input forces and torque are applied at the center of mass in each rigid body.

In the i-th subsystem ($i = 1, 2$), rigid body has two connecting points labeled as ijk meaning that the connecting point is the k-th one ($k = 1, 2$) at the j-th rigid body ($j = 1, 2$). The length of the line segment between the center of mass and the connecting point ijk is denoted by ℓ_{ijk}. The angle between the line segment and the edge of the rigid body at the connecting point is denoted by ψ_{ijk}.

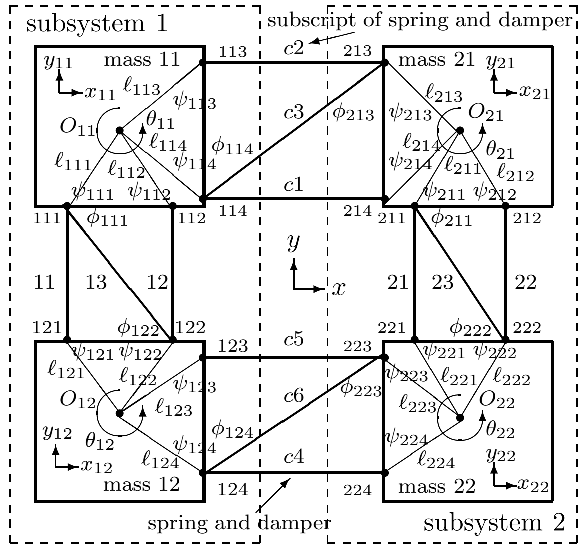

Fig. 1. An interconnected space structure

The springs and dampers are represented by lines between rigid bodies in Fig. 1. The angle between the edge of the rigid body and the direction of the spring and damper attached not rectangularly at the point ijk is denoted by ϕ_{ijk}. Then, the displacement vector of the i-th subsystem (1) is

$$q_i = \begin{bmatrix} x_{i1} & y_{i1} & \theta_{i1} & x_{i2} & y_{i2} & \theta_{i2} \end{bmatrix}^T$$

and the coefficient matrices of (1) are

$$M_i = \mathrm{diag}\{m_{i1},\ m_{i1},\ J_{i1},\ m_{i2},\ m_{i2},\ J_{i2}\}$$

$$D_i = \begin{bmatrix} \hat{N}_{i1} \\ -\hat{N}_{i2} \end{bmatrix} D_{si} \begin{bmatrix} \hat{N}_{i1}^T & -\hat{N}_{i2}^T \end{bmatrix}$$

$$K_i = \begin{bmatrix} \hat{N}_{i1} \\ -\hat{N}_{i2} \end{bmatrix} K_{si} \begin{bmatrix} \hat{N}_{i1}^T & -\hat{N}_{i2}^T \end{bmatrix}$$

$$\hat{N}_{i1} = \begin{bmatrix} 0 & 0 & -\cos\phi_{i11} \\ 1 & 1 & \sin\phi_{i11} \\ -\ell_{i11}\cos\psi_{i11}, & \ell_{i12}\cos\psi_{i12}, & -\ell_{i11}\sin(\psi_{i11} + \phi_{i11}) \end{bmatrix}$$

$$\hat{N}_{i2} = \begin{bmatrix} 0 & 0 & -\cos\phi_{i22} \\ 1 & 1 & \sin\phi_{i22} \\ -\ell_{i22}\cos\psi_{i22}, & \ell_{i21}\cos\psi_{i21}, & \ell_{i22}\sin(\psi_{i22} + \phi_{i22}) \end{bmatrix}$$

$$D_{si} = \mathrm{diag}\{d_{i1},\ d_{i2},\ d_{i3}\}$$

$$K_{si} = \mathrm{diag}\{k_{i1},\ k_{i2},\ k_{i3}\}$$

$$L_i = I_6$$

Here, the matrices $\hat{N}_{i1}$ and $\hat{N}_{i2}$ are defined by the location and direction of springs and dampers in the i-th subsystem. The scalar values k_{ik}, d_{ik} respectively represent the spring and damper parameters. The subscripts ik indicate the positions of the spring and damper as indicated in Fig. 1.

The subsystems are connected at four points labeled as ijk denoting the k-th point ($k = 3, 4$) at the j-th rigid body ($j = 1, 2$) in the i-th subsystem ($i = 1, 2$). Then the interconnection term in (9) is expressed by

Table 1. Parameters of the structure

i	j	m_{ij}	J_{ij}	k_{ik}, k_{cl}	d_{ik}, d_{cl}
1	1	20.0	100.0	0.1	0.01
	2	40.0	400.0		
2	1	30.0	225.0		
	2	60.0	900.0		

$k = 1, 2, 3,\ l = 1, \ldots, 6$

Table 2. Parameters of the controller

Case 1

i	α_i	β_i	γ_i
1	3.0	9.3	23.0
2	3.0	11.3	43.0

Case 2

i	α_i	β_i	γ_i
1	3.0	6.0	23.0
2	3.0	7.0	43.0

$$\tilde{N}_{12} = \begin{bmatrix} N_{12} \\ -N_{21} \end{bmatrix}$$

$$N_{12} = \text{diag}\left\{ \begin{bmatrix} 1 & 1 & \sin\phi_{1j4} \\ 0 & 0 & \cos\phi_{1j4} \\ \ell_{1j4}\cos\psi_{1j4} & -\ell_{1j3}\cos\psi_{1j3} & \ell_{1j4}\sin(\psi_{1j4} + \phi_{1j4}) \end{bmatrix} \right\}_{j=1,2}$$

$$N_{21} = \text{diag}\left\{ \begin{bmatrix} 1 & 1 & \sin\phi_{2j3} \\ 0 & 0 & \cos\phi_{2j3} \\ \ell_{2j3}\cos\psi_{2j3} & -\ell_{2j4}\cos\psi_{2j4} & -\ell_{2j3}\sin(\psi_{2j3} + \phi_{2j3}) \end{bmatrix} \right\}_{j=1,2}$$

$$D_{C12} = \text{diag}\{d_{c1},\ d_{c2},\ d_{c3},\ d_{c4},\ d_{c5},\ d_{c6}\}$$

$$K_{C12} = \text{diag}\{k_{c1},\ k_{c2},\ k_{c3},\ k_{c4},\ k_{c5},\ k_{c6}\} \quad (29)$$

The scalar values k_{cl} and d_{cl} $(l = 1, \ldots, 6)$ respectively represent the parameters of the spring and damper connecting the subsystems. The subscript l indicates the l-th connection shown in Fig. 1. The notations $\ell_{ijk}, \psi_{ijk}, \phi_{ijk}$ for $k = 3, 4$ are defined as those for $k = 1, 2$.

For simulation, we use the values of mass, moment of inertia, stiffness, and damping given in Table 1. The length ℓ_{ijk} and angles $\psi_{ijk}, \phi_{ijk}\,(i, j = 1, 2,\ k = 1, 2, 3, 4)$ are 1.0, $\pi/4$ [rad], $\pi/3$ [rad], respectively.

The controller parameters $\alpha_i, \beta_i, \gamma_i$ $(i = 1, 2)$ are chosen as in Table 2. The matrices U_i, V_i $(i = 1, 2)$ in the controller are 6×6 identity matrices. In Table 2, the parameters of Case 1 make $\tilde{Q}$ of (16) positive definite, and those of Case 2 do not. In Case 1, the parameters are chosen as follows. First, α_i are arbitrarly fixed. Then, β_i are increased so that the term $\tilde{\beta}^2\tilde{D} + \tilde{D}\tilde{\beta}^2 + \tilde{\beta}^4\tilde{L}\tilde{U}^T\tilde{V}^{-2}\tilde{U}\tilde{L}^T - 2\tilde{\alpha}^2\tilde{M}$ of $\tilde{Q}_{22}$ in (16) becomes positive definite. Finally, γ_i are increased so that the matrix between $\tilde{T}^T$ and $\tilde{T}$ of $\tilde{Q}$ in (16) becomes positive definite.

Setting the initial displacements of subsystems as

$$q_1(0) = (0\ \ 0\ \ 0\ \ 0.05\ \ 0.07\ \ 0.1)^T$$
$$q_2(0) = (0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0)^T,$$

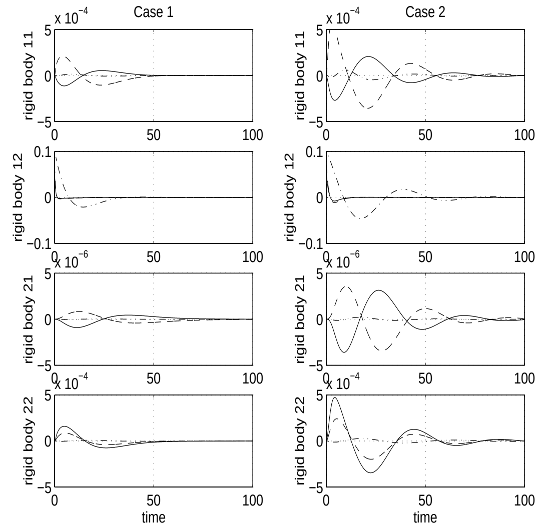

Fig. 2. Initial-state responses of displacements

the responses are computed as in Fig. 2. The left column shows behaviors in Case 1. The right column is those of Case 2. The top, second, third, and bottom figures respectively show the responses of rigid bodies 11, 12, 21, and 22. Solid lines, broken lines, and chained lines respectively indicate the displacements in the direction x_{ij}, y_{ij}, and the rotational angle θ_{ij}. It is seen that the settling time and amplitude of vibration is much less in Case 1 than in Case 2.

6. CONCLUSION

This paper has considered position and attitude control of large space structures composed of a number of subsystems which are interconnected by springs and dampers. A decentralized SDDFB control law compatible with subsystems has been applied under the assumption that sensors and actuators are collocated. A condition has been derived for the overall control system to be optimal for a quadratic cost function. This result can be used in choosing the controller parameters on the design stage, and also in tuning them on the operation stage where a new subsystem may be connected to the space structure or a failed subsystem may be disconnected from the structure.

7. REFERENCES

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