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PARAMETERISED CONTROL FOR AN UNDERACTUATED BIPED ROBOT

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Abstract: This paper presents a control law for the tracking of an optimal reference trajectory by an underactuated biped robot. The degree of under-actuation is one during the single support phase. The control law is defined in the following way. Only the geometric evolution of the robot is controlled, not its temporal evolution. To achieve this objective, we consider a set of reference trajectories parameterised by a virtual time. The robot is under-actuated so its evolution is constrained and the evolution of the virtual time can be analysed. A analytical simple condition to assure convergence toward the optimal reference trajectory is deduced. For the biped and the studied optimal motion, this condition is naturally satisfied.

Keywords: Walking robot, underactuated mechanism, control, stability

1. INTRODUCTION

To define a simple and economic walking robot, it is interesting to reduce the number of actuators, and to use unactuated ankles. The robot studied in this paper is a planar biped with only four actuators, two on the haunch, two on the knees. During the single support phase, five independent configuration variables exist. Thus the robot is underactuated. This mechanical simplification makes the design of the control law difficult. One classical way to control a system consists in two steps. During the first step, an open loop joint reference trajectory is designed. In the second step a control law is defined to track this reference trajectory. In this context, a reference trajectory was obtained by an optimisation technique (Chevallereau and Aoustin, 2001) and now a control law is proposed.

Various studies for the control of an underactuated biped exist. A first group of methods (Grizzle *et al.*, 2001; Aoustin and Formal'sky, 1999) is based on the definition of the reference trajectory for m outputs (where m is the number of actuators), not as a function of time but as a function of a configuration variable independent of the m outputs. When such a control has converged, the configuration of the robot at the impact is the desired configuration but the velocity can differ from the cyclic one. The convergence of the motion toward a cyclic trajectory is then studied numerically using the Poincaré stability. Another approach involves parameterised reference trajectories. In this case, one derivative of the parameter is used as a supplementary input as it was shown in (Wieber, 2000; Canudas de Witt *et al.*, 2002; Gubina *et al.*, 1974). In (Wieber, 2000), the parameter is used to satisfy some constraints on the reaction between the feet and the ground. In (Canudas de Witt *et al.*, 2002), a parameter involved in the zero dynamics is used as a supplementary input.

In this paper, only the geometric evolution of the robot is controlled, not its temporal evolution like in (Grizzle *et al.*, 2001; Aoustin and Formal'sky, 1999). To achieve this objective a set of reference

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trajectories parameterised by a virtual time is considered. The second derivative of the virtual time is considered as a supplementary control input. Thus we deal with a model with the same number of inputs and independent configuration variables. But the robot is under-actuated so its evolution is constrained. Through the study of the dynamic model the evolution of the virtual time can be analysed. And an analytical simple condition to assure a convergence toward the optimal cyclic reference trajectory is deduced. For the optimal walking of a biped this condition is naturally satisfied.

In section 2, the modelling of the robot and an optimal reference trajectory are presented. In section 3 the control law is defined. The evolution of the virtual time is analysed in section 4 and a condition of convergence is deduced. Section 5 concludes this study.

2. THE ROBOT MODELLING

2.1 The studied robot

The biped studied walks in a vertical xz plane. It is composed of a trunk and two identical legs. Each leg is composed of two links articulated with a knee. The knees and the hips are one degree of freedom rotational joints. During the single

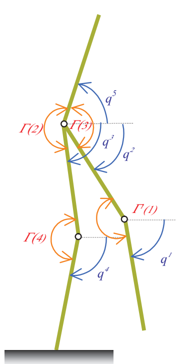

Fig. 1. The studied biped

support phase the vector $q = (q^1, q^2, q^3, q^4, q^5)^T$ describes the configuration of the robot. The vector Γ describes the torques applied at the hip and knee joints (Figure 1).

2.2 Dynamic modelling

The walk studied is composed of single support phases separated by instantaneous double support phases.

In single support on leg j (j=1 or 2), the dynamic model can be written as:

$$A(q)\ddot{q} + H(q, \dot{q}) = D\Gamma \qquad (1)$$

where $A(5 \text{ x } 5)$ is the inertia matrix, $H(5 \text{ x } 1)$ is the vector of Coriolis, centrifugal and gravity effects and D is a $(5 \text{ x } 4)$ matrix.

When the free leg j touches the ground at the end of single support, an impact exists. This impact is assumed instantaneous and inelastic. During the impact, the supporting leg i (with $i \neq j$) takes off. The velocities just before and just after the impact, denoted $\dot{q}^-$ and $\dot{q}^+$ respectively, are related by a linear model (Furusho *et al.*, 1995):

$$\dot{q}^+ = I(q)\dot{q}^- \qquad (2)$$

During the single support phase, the number of torques is four but there are five independent configuration variables. Thus, a relation on the robot evolution independent of the torques can be written. To obtain this relation, the dynamic model is projected into the space generated by a vector orthogonal to D denoted $D^\perp$. By definition $D^\perp D = 0$, thus: .

$$D^\perp A(q)\ddot{q} + D^\perp H(q, \dot{q}) = 0 \qquad (3)$$

An other way to find such a relation is to remark that the derivative of the angular momentum of the robot written around the supporting leg tip depends only on the gravity effects. In the case of a planar motion, the angular momentum is perpendicular to the motion plane. Its value is denoted σ, we have.

$$\dot{\sigma} = mg(x_g - x_s) \qquad (4)$$

where m is the total mass of the robot, g is the gravity acceleration, x_s is the abscissa of the contact point, x_g is the abscissa of the robot mass center.

remark : for a special choice of $D^\perp$, σ can be calculated by: $\sigma = D^\perp A(q)\dot{q}$

2.3 An optimal trajectory

The definition of some optimal reference trajectory is detailed in (Chevallereau and Aoustin, 2001) for example. An optimal trajectory is assumed to be known by numerical values for a given sampling time. The duration of an half step is denoted T. For each time between 0 and T the joint configuration $q_r(t)$, the joint velocity $\frac{dq_r(t)}{dt}$, and the joint acceleration $\frac{d^2q_r(t)}{dt^2}$ are recorded.

The optimal trajectory is cyclic, thus the configuration of the robot is a continuous periodic function and the legs swap roles from one half step to the following one, thus:

$$q_r(t + kT) = E^k q_r(t) \qquad (5)$$

for $0 < t < T$, where E is a permutation matrix which allows to take into account the exchange of legs. The condition (2) is satisfied for each impact.

The characteristics of the proposed control law will be illustrated on a reference trajectory corresponding to an energetic criterion (Chevallereau and Aoustin, 2001) for a motion velocity equal to $1.25m/s$. For this optimal trajectory, the angular momentum σ can be calculated as function of $q_r(t)$ and $\frac{dq_r(t)}{dt}$ and is denoted σ_r.

$$\sigma_r(t) = D^{\perp} A(q_r(t)) \frac{dq_r(t)}{dt}$$

$\sigma_r(t)$ is periodic with a period equal to T and is presented in figure 2 for an half step. For each sin-

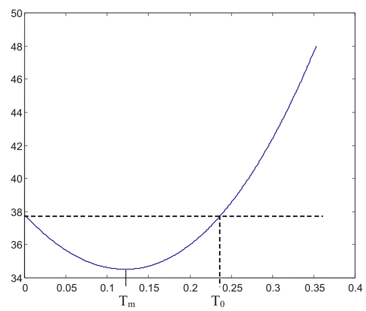

Fig. 2. Angular momentum $\sigma_r(t)$ (in kgm^2/s)

gle support phase, the initial configuration of the robot is such that $x_g < x_s$. During the first part of the motion the angular momentum decreases (see equation (4)) and its initial angular momentum must be high enough to reach a configuration such that $x_g > x_s$. In the other case it falls down back (Kajita and Tani, 1995). After, as soon as $x_g > x_s$ the angular momentum increases. Thus for any cyclic walk of a biped, the evolution of the angular momentum is close to the evolution presented in figure 2 and never crosses zero.

Two times deserve attention (see figure 2). They are denoted T_m and T_0. They are defined by: $\frac{d\sigma_r(T_m)}{d\tau} = 0$ and $\sigma_r(T_0) = \sigma_r(0)$ with $T_0 > 0$. A ttime such that $\tau(t) = T_m$, the configuration of the robot is such that its mass centre abscissa coincides with the contact point abscissa. For any studied walking trajectory of the biped, T_m exists. T_0 may exist or not for cyclic walking trajectories. If T_0 exists, $T_0 > T_m$. For the studied trajectory, $T_m \simeq 0.12s$ and $T_0 \simeq 0.24s$.

3. THE PROPOSED CONTROL LAW

The optimal reference trajectory is composed essentially of single support phases. During these phases, the robot studied is under-actuated. The objective of the control law presented in this section is not to track exactly the reference trajectory but only the path associated. Only a geometrical tracking is desired. A set of trajectories corresponding to the same configuration path as the optimal trajectory is considered. The control is defined to insure the convergence to the set of reference trajectories in a finite time.

3.1 A set of reference trajectories

We consider a set of trajectories defined by :

$$\begin{cases} q_d(t) = q_r(\tau(t)) \\ \dot{q}_d(t) = \frac{dq_r(\tau(t))}{d\tau}\dot{\tau} \\ \ddot{q}_d(t) = \frac{dq_r(\tau(t))}{d\tau}\ddot{\tau} + \frac{d^2 q_r(\tau(t))}{d\tau^2}\dot{\tau}^2 \end{cases} \quad (6)$$

where τ is a function of the time called "virtual time". The knowledge of $\tau(t)$ defines a unique trajectory .Any trajectory defined by (6) corresponds to the same path in the joint space as the optimal trajectory but the evolution of the robot with respect to time may differ. The optimal trajectory belongs to the considered set of reference trajectories with: $\tau = t$, $\dot{\tau} = 1$, $\ddot{\tau} = 0$

The desired configuration of the robot at time t is the optimal configuration at time $\tau(t)$. Thus for all trajectories defined by (6) the free leg tip does not touch the ground for any time t such that $\tau(t) < T$. In consequence the impact with the ground occurs for the first step at time t_1 such that $\tau(t_1) = T$ and for the step k at time t_k such that $\tau(t_k) = kT$. The configuration at impact is the same as the configuration of the optimal trajectory. But the velocity can be different from the optimal one.

All the reference trajectories (6) respect the impact equation (2) and $\dot{\tau}$ is continuous at the impact time. This property is obtained because (i) the optimal trajectory is cyclic and takes into account the impact and (ii) the velocity after the impact is linear with respect to the velocity before impact (2).

3.2 The control law

The second derivative $\ddot{\tau}$ includes in the definition of the reference trajectories will be treated as a supplementary control input. Thus the control law will be designed for a system with the same number of outputs and inputs. The controlled variables are simply the five joint variables q. The control inputs are the four torques Γ and $\ddot{\tau}$.

The control law is a computed torque control law classically used in robotics. But to have a finite time stabilisation around one of the desired trajectories, the feedback function proposed in (Bhat and Bernstein, 1998; Grizzle *et al.*, 2001) is used. The tracking errors are defined with respect to the trajectories belonging to (6):

$$e_q(t) = q_r(\tau(t)) - q(t)$$
$$\dot{e}_q(t) = \frac{dq_r(\tau(t))}{d\tau}\dot{\tau} - \dot{q}(t) \quad (7)$$

The desired behaviour in closed loop is :

$$\ddot{q} = \ddot{q}_d + \frac{1}{\epsilon^2}\psi \quad (8)$$

where ψ is a vector of 5 components $\psi(i)$ with:

$$\psi(i) = -sign(\epsilon\dot{e}_q(i))|\epsilon\dot{e}_q(i)|^\nu - sign(\phi(i))|\phi(i)|^\nu$$

and $0 < \nu < 1$, $\epsilon > 0$, $\phi(i) = e_q(i) + \frac{1}{2-\nu}sign(\epsilon\dot{e}_q(i))|\epsilon\dot{e}_q(i)|^{2-\nu}$, the expression $x(i)$ denotes the i^th component of a vector x with i=1,...5. ν and ϵ are parameters to adjust the settling time of the controller.

Taking into account the expression (6) of the reference trajectory, the equation (8) can be rewritten as:

$$\ddot{q} = \frac{dq_r(\tau(t))}{d\tau}\ddot{\tau} + v(\tau, \dot{\tau}, q, \dot{q}) \quad (9)$$

with $v(\tau, \dot{\tau}, q, \dot{q}) = \frac{d^2q_r(\tau(t))}{d\tau^2}\dot{\tau}^2 + \frac{1}{\epsilon^2}\psi$. The dynamic model of the robot is described by equation (1), thus the control law must be such that:

$$A(q)(\frac{dq_r(\tau(t))}{d\tau}\ddot{\tau} + v) + H(q, \dot{q}) = D\Gamma \quad (10)$$

or :

$$A(q)v + H(q, \dot{q}) = -A(q)\frac{dq_r(\tau(t))}{d\tau}\ddot{\tau} + D\Gamma \quad (11)$$

As the control input are the torques and $\ddot{\tau}$, we obtain :

$$\begin{bmatrix} \ddot{\tau} \\ \Gamma \end{bmatrix} = \Lambda^{-1}\{A(q)v + H(q, \dot{q})\} \quad (12)$$

with

$$\Lambda = \begin{bmatrix} -A(q)\frac{dq_r(\tau(t))}{d\tau} & D \end{bmatrix}$$

Like the matrices Λ and $A(q)$ are invertible, without modelling error, the control law insures that eq. (8) is satisfied or that $q(t)$ goes towards $q_d(\tau(t))$ in a finite time. Without disturbance, a perfect tracking of $q_d(\tau(t))$ is obtained. This control law defines $\ddot{\tau}$. Knowing initial values for τ and $\dot{\tau}$, the evolution of $\tau(t)$ can be calculated but not chosen. The initial values are $\tau(0) = 0$, and $\dot{\tau}(0) = \frac{\dot{q}(0)^T\frac{dq_r(0)}{d\tau}}{|\frac{dq_r(0)}{d\tau}|^2}$ to minimize the error on the joint velocity $|\dot{q}(0) - \dot{q}_r(0)|^2$.

Remark: The singularities for this control law. The control is not defined if the matrix Λ is singular. The matrix D is a constant (5x4) matrix with rank 4. The matrix Λ is invertible if and only if $(A(q)\frac{dq_r(\tau(t))}{d\tau})$ does not belong the space generated by D or:

$$D^\perp(A(q)\frac{dq_r(\tau(t))}{d\tau}) \neq 0 \quad (13)$$

For a trajectory belonging to the set described by equation (6):

$$D^\perp(A(q_r(\tau(t)))\frac{dq_r(\tau(t))}{d\tau}) = \sigma_r(\tau(t))$$

Thus, for the optimal trajectory, the angular momentum is far from zero, (see figure 2), so no singularity occurs. With small tracking errors, no singularity appears.

4. CONVERGENCE TOWARDS THE OPTIMAL TRAJECTORY

The control law insures that the motion of the robot converges in a finite time towards a reference trajectory described by (6). As soon as the control has converged, we have: $q(t) = q_r(\tau(t))$, $\dot{q}(t) = \dot{q}_r(\tau(t), \dot{\tau})$, $\ddot{q}(t) = \dot{q}_r(\tau(t), \dot{\tau}, \ddot{\tau})$, and these properties will be kept for all the following steps. In the following section, the behaviour of the robot is studied **after** the convergence of the control law, **when the robot follows a trajectory satisfying (6)**. The robot velocity is $\dot{q}(t) = \frac{dq_r(\tau(t))}{d\tau}\dot{\tau}$ and the optimal velocity is $\frac{dq_r(\tau(t))}{d\tau}$. The difference between the two velocities is proportional to $e = \dot{\tau} - 1$, this term is referred to as "velocity difference". The robot converges towards the optimal trajectory if and only if $\dot{\tau}$ converges to 1 or e converges to 0.

4.1 Evolution of the virtual time

During the single support phase, the robot studied is under-actuated, thus it can not follow any trajectory described by (6). The motion of the robot can be studied with the evolution of the angular momentum (4)The angular momentum σ is linear with respect to the velocity components. The real velocity of the robot is proportional to the optimal velocity. Thus the angular momentum can be expressed by:

$$\sigma(t) = \sigma_r(\tau(t))\dot{\tau} \quad (14)$$

Using this equation, the derivative of the angular momentum can be written as:

$$\dot{\sigma}(t) = \frac{d\sigma_r(\tau(t))}{d\tau}\dot{\tau}^2 + \sigma_r(\tau(t))\ddot{\tau} \quad (15)$$

But the derivative of the angular momentum depends only on the configuration of the robot (see equation (4)). Since the configuration of the robot is the optimal one $q(t) = q_r(\tau(t))$, we have:

$$\dot{\sigma}(t) = -mg(x_g(q_r(\tau(t))) - x_s) \qquad (16)$$

We can also write the same equation (4), for the optimal trajectory. Thus, we deduce that: $\dot{\sigma}(t) = \frac{d\sigma_r(\tau(t))}{d\tau}$. Using this equation into equation (15), we have:

$$\frac{d\sigma_r(\tau(t))}{d\tau}(\dot{\tau}^2 - 1) + \sigma_r(\tau(t))\ddot{\tau} = 0 \qquad (17)$$

The "velocity difference" has been defined by: $e = \dot{\tau} - 1$, so its derivative is $\dot{e} = \ddot{\tau}$. From equation (17), the behavior of the velocity difference is defined by:

$$\dot{e} = -\frac{\frac{d\sigma_r(\tau(t))}{d\tau}}{\sigma_r(\tau(t))}(\dot{\tau} + 1)e \qquad (18)$$

Using the relation $(\dot{\tau} + 1) = \dot{\tau}(\frac{2+e}{1+e})$, the equation (18) is rewritten as:

$$\frac{\dot{e}}{e}(\frac{1+e}{2+e}) = -\frac{\dot{\sigma}_r(\tau(t))}{\sigma_r(\tau(t))} \qquad (19)$$

σ_r is a periodical function discontinuous at the impact time. So the equation (19) can be integrated step by step only .The integration during step k gives for $t_k < t < t_{k+1}$:

$$\frac{1}{2}[Log((e+1)^2 - 1)]_{t_k}^{t} = -[Log(\sigma_r(\tau))]_{t_k}^{t} \qquad (20)$$

To simplify the notation the velocity difference at impact k, $e(t_k)$ is denoted e_k. Using the initial condition we have, for $t_k < t < t_{k+1}$:

$$e(t) = \sqrt{1 + e_k(e_k + 2)\left(\frac{\sigma_r(0)}{\sigma_r(\tau(t))}\right)^2} - 1 \qquad (21)$$

this function includes a square root and is defined if $e_k > e_{min}$ with:

$$e_{min} = -1 + \sqrt{1 - \left(\frac{\sigma_r(T_m)}{\sigma_r(0)}\right)^2} \qquad (22)$$

Since the evolution of σ_r is cyclic, the behavior of $e(t)$ is defined by the evolution of σ_r during one step. The different steps can be taken in to account using the following iterative equation:

$$e_k = \sqrt{1 + e_{k-1}(e_{k-1} + 2)\left(\frac{\sigma_r(0)}{\sigma_r(T)}\right)^2} - 1 \qquad (23)$$

A typical evolution of $e(t)$ is presented in figure 3.

- For $kT < \tau < kT + T_m$, $|e|$ increases because σ_r decreases.

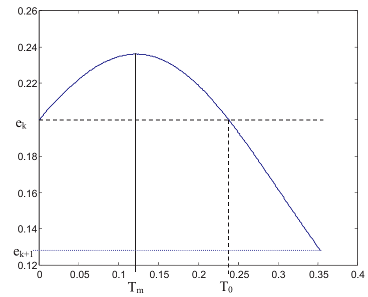

Fig. 3. Evolution of the velocity difference $e = 1 - \dot{\tau}$ as function of τ, for one step

- For $\tau = kT + T_m$, $|e|$ has a local maximum denoted e_k^m.

$$e(t_m^k) = \sqrt{1 + e_k(e_k + 2)\left(\frac{\sigma_r(0)}{\sigma_r(T_m)}\right)^2} - 1 \qquad (24)$$

- For $T_m + kT < \tau < (k+1)T$, $|e|$ decreases because σ_r increases.
- For $\tau = kT + T_0$, $\sigma(kT + T_0) = \sigma(0)$, $e = e_k$. This characteristics is clearly explained by equations (21).

Remark : Minimal initial velocity for one step. The definition of the parameterised reference trajectories assumes implicitly that the parameter is monotonic. In the case studied, the parameter is a virtual time so it must increase. In fact if the parameter decreases, this means that the robot goes back. The evolution of τ is monotonic if: $\dot{\tau} > 0$ or $e > -1$. Since $e(t)$ is expressed by equation (21), $e(t)$ is always greater than -1 when it is defined. Thus the condition of monotony is $e_k > e_{min}$. For the studied optimal trajectory, the minimal velocity difference is $e_{min} = -0.58$.

4.2 Convergence towards the optimal trajectory

With the proposed control, the motion of the robot converges to the set of reference trajectories (6) in a finite time which can be chosen to be less than the duration of one step.

Theorem

If $e(t_1) > e_{min}$ and if $\left(\frac{\sigma_r(0)}{\sigma_r(T)}\right) < 1$, the velocity difference $e(t)$ goes to zero, i.e.,

$$\forall \epsilon > 0, \exists t_l, \text{ such that for } t > t_l, |e(t)| < \epsilon$$

In the defined attraction domain, the robot motion tends towards the optimal reference trajectory .

Proof

Step 1: The absolute value of the initial velocity difference decreases

If e_{k-1} is positive, like $\frac{\sigma(0)}{\sigma(T)} \leq 1$, we have:

$$1 + e_{k-1}(e_{k-1}+2)\left(\frac{\sigma(0)}{\sigma(T)}\right)^2 \leq 1 + \left(\frac{\sigma(0)}{\sigma(T)}\right)^2 e_{k-1}^2 + 2\frac{\sigma(0)}{\sigma(T)}e_{k-1} \quad (25)$$

thus, since the evolution of the velocity difference from one step to the following one is given by the iterative equation (23), we have:

$$e_k \leq \frac{\sigma_r(0)}{\sigma_r(T)} e_{k-1} \quad (26)$$

If e_{k-1} is negative, similar calculations can be made. Since the velocity difference does not change sign, we have:

$$|e_k| \leq \frac{\sigma_r(0)}{\sigma_r(T)} |e_{k-1}| \quad (27)$$

Step 2: The absolute value of the maximal velocity difference decreases

By hypothesis $e(0) > e_{min} > -1$, thus the maximal value e_m^k is an increasing function of e_k (eq. 24). Besides if $e_k = 0$ then $e_m^k = 0$. Thus since $|e_k| \leq |e_{k-1}|$, $|e_m^k| \leq |e_m^{k-1}|$.

In consequence

- since $e(0) > e_{min} > -1$, the evolution of τ is increasing during all the motion.
- since during step k, $t_k < t < t_{k+1}$; $|e(t)| < |e(t_m^k)|$, for $t > t_k$, $|e(t)| < |e(t_m^k)|$,

Step 3: The time t_l is calculated

A number of steps l is defined such that if $t > t_l$, $|e(t)| < \epsilon$. Using the step 2 of this proof, l is chosen such that $|e(t_m^l)| < \epsilon$. An intermediate positive value ε is defined. This value corresponds to the maximal value of $|e(t_l)|$, such that $|e(t_m^l)| < \epsilon$. ε is calculated by:

$$\varepsilon = \sqrt{1 + \epsilon(\epsilon+2)\left(\frac{\sigma_r(T_m)}{\sigma_r(0)}\right)^2} - 1 \quad (28)$$

Using equation (27), l is calculated such that $|e(t_l)| < \varepsilon$:

$$\left(\frac{\sigma_r(0)}{\sigma_r(T)}\right)^l |e(0)| \leq \varepsilon \quad (29)$$

thus

$$l \geq \frac{Log\left(\frac{\varepsilon}{|e(0)|}\right)}{Log\left(\frac{\sigma_r(0)}{\sigma_r(T)}\right)} \quad (30)$$

Remark : The equation (30) gives some information about the convergence rate, it is clear that the smaller $\frac{\sigma_r(0)}{\sigma_r(T)}$, the faster the convergence towards the optimal trajectory.

5. CONCLUSION

An original control law for the tracking of a desired joint reference trajectory has been proposed. The robot studied is a planar biped, underactuated during the single support phases. A cyclic desired reference trajectory (may be optimal) satisfying the dynamic equations is assumed to be known. A stable tracking of this trajectory is obtained if this trajectory is such that $\sigma_r(T) > \sigma_r(0)$ where σ_r is the angular momentum around the contact point with the ground for the optimal motion, and T is the duration of the single support. This result is very interesting because this condition can be easily tested or used to modify a given reference trajectory .It has been observed that for this robot, the optimal motion for an energetic criterion naturally satisfies this condition. The proposed control law have been tested in simulation with large initial velocity error. Good results are obtained. Due to space limitation, the simulation results are not presented in this paper.

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