# An Application of Sliding Mode Controller to Nonminimum-Phase Nuclear Steam Generator Water Level Control

Kwangsoo Kim<sup>†</sup>, Hyungjin Kim<sup>†</sup>, Yunchul Kim<sup>†</sup>, Seungrohk Oh<sup>‡</sup>, and Dong-il "Dan" Cho<sup>†</sup>

<sup>†</sup>School of Electrical Engineering and Computer Science Automation and Systems Research Institute Seoul National University San56-1, Shillim-dong, Kwanak-gu, Seoul 151-744, KOREA Tel: +82-2-880-8371, E-mail: dicho@asri.snu.ac.kr

<sup>7</sup>School of Electronics & Computer Engineering Dankook University San 8, HanNam-dong, YongSan-Gu, Seoul, KOREA

Abstract: The difficulties in designing an effective water level controller for the nuclear steam generator arise from highly nonlinear and nonminimum-phase plant characteristics as well as nonnegative input constraint. In this paper, a modified sliding mode controller with gain scheduling is proposed for nuclear steam generator water level control. A conventional proportional-integral with feed-forward controller is also developed, and the performance of two controllers is compared in the low power range. The modified sliding mode controller shows improvements in the overshoot, undershoot, and settling time. *Copyright* © 2002 IFAC

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## 1. INTRODUCTION

Uninterrupted operations without frequent shutdowns of a nuclear power plant are desired for economic efficiency and safety. Among various causes of reactor trips, the feed-water system in the nuclear reactor has been reported to be a major contributor to the plant shutdowns (Kothare *et al.*, 2000). Too high water level causes damages to a turbine with over-moisturized outgoing steam. Also, too low water level exposes the U-tube in the steam generator outside the water and, in this case, the Utube could be damaged. In both cases, the plant is interrupted for safety, which results in economic losses.

Some characteristics of the steam generator make a proper controller design difficult. First, the system has a strong nonlinearity. The linearized plant model is a linear parameter varying system dependent on the demanded generating power, which means that it is difficult to design a global controller. Second, due to the so-called "swell and shrink" phenomena, the plant has a nonminimum-phase behavior. The swell and shrink effects are explained by the thermodynamic properties of the two-phase mixture rather than the mass balance relations between the feed-water and steam flow (Kothare *et al.*, 2000). The nonminimum-phase behavior limits the frequency bandwidth of the controller and thus makes the plant response slow (Skogestad *et al.*, 1996). Third, it is impossible to draw water from the steam generator, *i.e.* it is only possible to deliver water. We call this a nonnegative input constraint in this paper. This constraint imposes a limitation on the choice of available control methodology.

A lot of approaches to this control problem including classical PID controller have been proposed. Various fuzzy logic-based controller designs have been reported in the literatures. Akin (Akin et al., 1991) followed the classical design of fuzzy logic based control such that more than 50 linguistic rules were adopted from the human operator's experience. Na (Na et al., 1998) presented a genetic fuzzy controller in which the genetic algorithm was used for the design of membership functions and rule sets for a fuzzy control using the water level, feed-water flow rate, and steam flow rate. Cho and No (Cho et al., 1996) proposed a procedure construct stability-guaranteed fuzzy logic to controller rules on the fuzzy phase-plane with the Lyapunov 's stability criteria. Besides the fuzzy logic controllers, A LQG/LTR controller (Menon et al.,

1992), an adaptive observer-based controller (Na et al., 1992), a genetic algorithm-based hybrid feedforward and feedback controller (Zhao et al., 1997), and model predictive controller (MPC) (Kothare et al., 2000) were studied. These proposed controllers were designed based on the local model around some operating points. Therefore, they adopted a gain scheduling for covering the entire operating power range.

In this paper, we propose a modified sliding mode controller with a state observer. It differs from the standard sliding-mode controller in that its control input does not have any compensation terms using a kind of sign function, due to the nonnegative input constraint as explained above. However, the stability of the overall system is guaranteed by the specified positions of stable closed-loop poles. A full-order state observer is introduced under the assumption that the only measurable variable is output, i.e. water level.

The organization of this paper is as follows: main features of the nuclear power plant and the linear parameter varying model of a steam generator for a controller design are given in the next section. The controller design is presented in detail in section 3. The following section shows some simulation results in a few of operation scenarios. Concluding remarks is presented.

#### 2. PROBLEM FORMULATION

The nuclear power plant consists of a reactor, a steam generator, a turbine, a condenser and two control valves. The steam generator makes the steam with the thermal energy supplied by the reactor in which the fission reaction takes place. The turbine valve controls the amount of steam deriving the turbine. In addition to those systems, other components are needed to compensate the loss of water by the steam flow. Feed water pump and feed water control valve performs this role. The whole system is shown in figure 1.

The feed water and the steam flow rate are the two factors that decide the steam generator water level. The water level is the superposition of these two factors.

However, the mathematical modeling of the plant is not easy mainly because of the swell and shrink phenomena. In this paper, the model of steam generator that was introduced by Lee (Lee., 1991) is used. The transfer function from steam flow to water level,  $H_s(s)$ , and that from feed water and water

level,  $H_{f}(s)$ , are described as

$$H_s(s) = -\frac{K_1}{s} + Vs_2(p)\frac{0.05}{s+0.05}$$
(1)

$$H_{f}(s) = \frac{K_{1}}{s} - Vs_{1}(p) \frac{\mathbf{w}_{n}(p)^{2}}{s^{2} + 2a(p)s + \mathbf{w}_{n}(p)^{2}}$$
(2)

, where  $K_1$  is a constant and  $V_{S_1}$ ,  $V_{S_2}$ , a and  $W_n$  are functions of the reactor power. That is, the model itself changes as the reactor power varies.



Figure 1. The whole system and steam generator (adopted from Kothare et al., 2000)

The first term of (1) indicates the integration of the steam flow and the negative sign means that as the steam flows out, the water level will come lower and lower. The second term of (1) represents the swell and shrink phenomena.  $H_{s}(s)$  can be explained in the similar way. The control objective is to keep the water level between the specified range in the face of varying reactor power. Figure 2 shows the plant block diagram where reactor power takes an effect on the water level in two ways.

### 3. CONTROLLER DESIGN

The transfer function from the feed water to the water level can be represented as

$$y(s) = \frac{K_1(s-z_1)(s-z_2)}{s(s-p_1)(s-p_2)}u(s) = T(s)u(s)$$
(3)





The poles and the zeros vary as the reactor power changes. (3) can be realized in a canonical form as follows

$$\begin{bmatrix} \dot{x}_1\\ \dot{x}_2\\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0\\ 0 & 0 & 1\\ 0 & -p_1p_2 & (p_1+p_2) \end{bmatrix} \begin{bmatrix} x_1\\ x_2\\ x_3 \end{bmatrix} + \begin{bmatrix} 0\\ 0\\ K_1 \end{bmatrix} u$$
(4)

$$y = \begin{bmatrix} z_1 z_2 & -(z_1 + z_2) & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + v$$
(5)

The state variables are

$$x_1 = \frac{y - v}{N(s)}, \ x_2 = s \frac{y - v}{N(s)}, \ x_3 = s^2 \frac{y - v}{N(s)}$$
(6)

, where  $N(s) = s^2 - (z_1 + z_2)s + z_1z_2$  and *v* represent the disturbance caused by the steam flow which varies randomly. A sliding surface is chosen as follows:

$$S = c_1 x_1 + c_2 x_2 + c_3 x_3 + c_4 \int y dt + c_5 \iint y dt \tag{7}$$

Assuming  $c_3 = 1$  and differentiating (7), the following equation is obtained.

$$\hat{S} = (c_1 - p_1 p_2) x_2 + (c_2 + p_1 + p_2) x_3 + K_1 u_{eq}$$
(8)

 $+c_4y+c_5\int ydt$ 

Setting (8) to 0, the controller input is obtained as follows:

$$K_1 u_{eq} = g_2 x_2 + g_3 x_3 + g_4 y + g_5 \int y dt \tag{9}$$

Using the input (9), it is possible to compute the transfer function which defines the relation between the disturbance caused by steam flow, v and water level. The transfer function is obtained as follows

$$y(s) = \frac{N_{cl}(s)}{D_{cl}(s)}v(s)$$
(10)

, where

$$N_{cl}(s) = s^{4} - (p_{1} + p_{2} + g_{3})s^{3} + (p_{1}p_{2} - g_{2})s^{2}$$
  
and  
$$D_{cl}(s) = s^{4} - (p_{1} + p_{2} + g_{3})s^{3} + (p_{1}p_{2} - g_{2})s^{2}$$

$$D_{c1}(s) = s^{4} - (p_{1} + p_{2} + g_{4} + g_{3})s^{3} + \{p_{1}p_{2} + g_{4}(z_{1} + z_{2}) - g_{5} - g_{2}\}s^{2} + \{-g_{4}z_{1}z_{2} + g_{5}(z_{1} + z_{2})\}s - g_{5}z_{1}z_{2}$$
(11).

Let the desired poles of the steam flow disturbance transfer function  $p_{c1}$ ,  $p_{c2}$ ,  $p_{c3}$ , and  $p_{c4}$ . Then, the controller gains  $g_2$ ,  $g_3$ ,  $g_4$ , and  $g_5$  are determined by equating (11) and the equation,  $\overline{D}_{c1}(s) = (s - p_{c1})(s - p_{c2})(s - p_{c3})(s - p_{c4})$ . In order to cover the entire power range and achieve a good performance, the desired poles are chosen differently in the different reactor power ranges and then a gain scheduling is adopted.

As the state variable  $x_1$ ,  $x_2$  and  $x_3$  cannot be obtained directly from the plant, a state observer is needed. The observer equation is chosen as

$$\hat{x} = A\hat{x} + Bu + L(y - \hat{y})$$

$$\hat{y} = C\hat{x} + v$$
(12)

, where L is the observer gain matrix which reduces the errors between plant output, y and the observer output, y-hat to zero. In order to determine L easily, the plant state equation is transformed to observability canonical form as follows

$$T = (WN)^{-1} \quad , \quad W = \begin{bmatrix} p_1 p_2 & -(p_1 + p_2) & 1 \\ -(p_1 + p_2) & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$
$$N = \begin{bmatrix} C \\ CA \\ CA^2 \end{bmatrix}$$

The matrices in an observability canonical form would be

$$T^{-1}AT = A_{obs} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & -p_1p_2 \\ 0 & 1 & (p_1 + p_2) \end{bmatrix} , \quad T^{-1}L = L_{obs} ,$$

 $CT = C_{obs} = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$ 

Then, the error dynamics is characterized by the following equation.

$$\dot{e} = (A_{obs} - L_{obs}C_{obs})e \tag{13}$$

For the error to converge to zero, the eigenvalue of the matrix,  $A_{obs} - L_{obs}C_{obs}$  should be placed in the left half plane. By deciding the eigenvalue first,  $L_{obs}$  can be deduced inversely. And then L is calculated from  $L_{obs}$  using T matrix. It is desirable to fix the eigenvalue differently in the various power ranges like the controller.

The whole system is realized in Matlab/Simulink as shown in figure 3. This includes the plant, a controller and an observer. The controller and the observer are implemented using S-Function to represent the variation of parameters as the power changes. The controller block diagram is shown in figure 4. It is also noted that the gain scheduling scheme is realized using the S-function as shown in figure 4.



Figure 4. Controller block diagram in Matlab/Simulink

#### 4. SIMULATION RESULTS

To test the performance, the developed controller is compared with the PI+feed-forward controller. The controller input reflects the amount of steam flow too. The PI controller is designed on the local operating points and then adopt a gain scheduling scheme. The system with the PI controller is also implemented in Matlab/Simulink environment and the block diagram is shown in figure 5. in the figure 5, S-Function is also used to gain schedule. The saturator reflects the feasible feed water amount considering the nonnegative input constraint.

Since it is known to be more difficult to control the water level in the low power ranges, The performances of the developed sliding mode controllers and the PI controllers are compared in the power range of between 5% and 25%. The first scenario is to increase or decrease the power range from 15% to 25% by step. The second scenario is to change the power range between of 5% and 6%. Figure 6 and figure 7 show the results in the case of the first scenario and the second scenario, espectively. In these figures, the dotted line shows the result of the PI + feed forward controller and solid line indicates that of sliding mode like controller.

As shown in figure 6 and figure 7, the proposed modified sliding mode controller shows the less overshoot and undershoot, and also faster settling time, compared with the PI+feed-forward controller. The results are summarized in table 1.

## 5. CONCLUSIONS

In this paper, we proposed a modified sliding mode controller for steam generator water level control. A gain schedule method was adopted for covering the entire power range and the state variables were estimated with a state observer. The controller gains were determined by choosing the proper poles of the transfer function, relating the disturbance by the steam flow and the water level. A conventional PI+feed-forward controller was designed for comparing the performance of the developed controller. The simulations were conducted in the four scenarios and the results reveal that the proposed sliding mode controller showed an improvement in the overshoot, the undershoot, and the settling time.



Figure 5. PI+feed-water controller block diagram in Matlab/Simulink





(a) The power is increased from 5% to 6%





Figure 7. Water level response in the power range of between 5% and 6%

Table 1. Numerical comparison of simulation results			
	Criterion	PI+feed	Modified
		forward	sliding
			mode
From	Overshoot	0.48	0.45
15% to	Undershoot	-0.57	-0.57
25%	Settling time	244	202
From	Overshoot	0.31	0.17
25% to	Undershoot	-0.49	-0.44
15%	Settling time	445	330
From	Overshoot	0.048	0.055
5% to	Undershoot	-0.042	-0.041
6%	Settling time	750	328
From	Overshoot	0.037	0.026
6% to	Undershoot	-0.043	-0.046
5%	Settling time	845	255

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