15th Triennial World Congress, Barcelona, Spain

FAULT DIAGNOSIS FOR NONLINEAR HESSENBERG SYSTEMS

C. Verde [1]

Instituto de Ingeniería UNAM,
PB 70-410, Coyoacan, 04510 DF, México
verde@servidor.unam.mx

Abstract: This paper deals with the problem of fault detection and isolation for nonlinear Hessenberg systems. In particular, the first part of the FDI problem associated with the disturbance decoupling transformation is considered. The approach is based on a relation between the relative degree from the unknown input to the output variables for nonlinear Hessenberg systems. The major advantage for this class of systems is that a decoupled system can be obtained analytical without solving partial difference equations. Moreover this transformation can be used to define the structure of a reduced observer for the system where different observer design procedures can be used, since the subspace is uniform observable.

Keywords: Fault diagnosis, fault isolation problem, Hessenberg nonlinear systems, decoupling subspace, generic nonlinear decoupling transformation.

1. INTRODUCTION

In the last decade, the interest to determine reduced order models decoupled of unknown input for nonlinear systems has been increased. One of the reason, is that the robustness in the fault detection and isolation, FDI, problem for dynamic nonlinear systems requires a model without disturbance. The success of a robust FDI design is strong related with the accuracy and assumptions of the model used in the residual generator (Isermann, 1997).

Moreover, the disturbance decoupling solution requires the existence of m independent measurement to isolate $m-1$ unknown inputs in both linear and nonlinear case (Massoumnia *et al.*, 1989). In the case of state affine systems the sufficient conditions for the existence of a FDI solution using geometric approach are given in (Hammouri *et al.*, 1998). Seliger proposed to determine in a more general case the nonlinear transformation with a recursive approach (Seliger and Frank, 2000).

On the other hand, a general theory for the nonlinear FDI systems as well as the design of robust nonlinear observers is still missing. At present, there are attempts to overcome the difficulties of robust nonlinear analytical procedures by using diverse tools and particular class of nonlinearities as example (Hammouri *et al.*, 1998), (De-Persis and Isidori, 2000), (Seliger and Frank, December, 1991), (Alcorta, 1999), (Shields *et al.*, 2001). Some authors, as Wunnenberg (Wunnenberg, 8-222, 1990) tackled the robust design for FDI in an integrated way i.e. the existence conditions of the residual generation using observer with disturbance are determined. This formulation makes more difficult to satisfy in some cases the dynamic specification of the estimator error. Moreover, this fault diagnosis formulation do not help to study, which additional assumptions are required to get a residual generator.

[1] This work was funded by the IN115600 DGAPA-UNAM project and the CONACyT-35402A project. Support of collaboration between the UNAM and IMP was received under the FIES program

This problem has been overcame dividing the robust FDI procedure in three steps. The first associated to the disturbance decoupling approaches, the second related with the synthesis of observers or filter for nonlinear systems and finally build a residual generator according the estimation error. This separation of tasks has the advantage that allows to consider more general class of systems and to benefit from diverse approaches in each part of the problem.

On the other hand, there are some physical models associated to biological and transport processes with structural coupled properties in the states which can be exploited to simplify the task of the observer design (Olivier *et al.*, 1998). Similarly, the structural properties of this kind of systems can be used to simplify the procedure to determine the decoupling transformation. This fact motivated this contribution in which is shown that in the particular case of an upper and lower nonlinear Hessenberg system with a relationship between relative degree of the unknown input to the outputs, exists a decoupling transformation with a fixed structure which can be determined without the solution of a partial differential equation, PDE. Moreover, the structure of the decoupled subspace and measurement space can be directly obtained. Therefore, one can construct a fixed gain observer or if the system is control affine a high gain observer (Gauthier *et al.*, June, 1992) or a Kalman-like observer.

The paper is organized as follows. In section 2 the reduced unknown input decoupling problem is formulated and solved in section 3. Section 4 shows the useful of the procedure to define the decoupled subsystems required to solve the leak detection an isolation problem in a pipeline with multi-leaks. Finally in section 5 the conclusions and significance of the transformation are given.

2. PROBLEM STATEMENT

Let U be the set of admissible controls. Consider the differential system defined on a domain $\Omega \in \Re^n$

$$\Sigma \begin{cases} \dot{x} = f(x,u) + F_1(x)p_1 + F_2(x)p_2 \\ x(0) = x_0 \quad u \in U \\ y = h(x) \in \Re^2 \end{cases} \tag{1}$$

where

(1) The system is both strictly linked lower and upper Hessenberg. This means for any indexes (i,j) such that

$$j > i+1, \quad \frac{\partial f_i(x,u)}{\partial x_{j+1}} = 0, \quad \frac{\partial f_i(x,u)}{\partial x_{i+1}} \neq 0$$

and if

$$j < i+1, \quad \frac{\partial f_i(x,u)}{\partial x_{j-1}} = 0, \quad \frac{\partial f_i(x,u)}{\partial x_{i-1}} \neq 0$$

(2) The system has only 2 outputs and for any $x \in \Omega$, $y_1 = h(x_1)$ with $\frac{dh(x_1)}{dx_1} \neq 0$ and $y_2 = h(x_n)$ with $\frac{dh(x_n)}{dx_n} \neq 0$. These output properties are called upper and lower measured respectively.
(3) The system satisfies some kind of bounded-input/bounded-output properties.
(4) Independent on the set of admissible $u(t)$, the existence of a fault p_i produces a deviation of the output such that the $\|y(t) - y_0(t)\| \neq 0$, where $y_0(t)$ is the output of the system without fault p_i. It means each fault produces a deviation of the nominal value of the output. This property is called the detectability of the fault (Chen and Patton, 1999).
(5) The distribution vector $F_i(x)$ of the faults has $n-1$ zero elements.
(6) f, F_1, F_2 and h are smooth functions.
(7) Assuming as unknown input p_j, the relative degrees from p_j to the output y_1 and y_2 satisfies the relation $\rho_1 + \rho_2 = n + 1$. This disturbance property plays an important role to determine the decoupling transformation.

From property (2) it is concluded that the system is uniformly input observable with any of the two outputs (Olivier *et al.*, 1998). It is to note that, in the case of more than 2 sensors, once the decoupled transformation proposed here is done, the extra measures can be used to reduce the dimension of the subspace. Moreover from properties (2) and (4) one concludes that the faults are detectable and can be isolated, since y_1 and y_2 are independent (White and Speyer, 1987).

Property (7) seems to be restricted, however it satisfies for processes which involve a transfer phenomena. Its lack means that the state space of the system cannot be described by n-2 derivatives of the outputs and only 2 states which depend on the unknown input p_j and combinations of the output derivatives.

In general, the FDI of Σ with relation to p_i admits a solution if exists a dynamic system called residual generator

$$\begin{aligned} \dot{z} &= F(u,z,y) \\ r &= H(y,z) \end{aligned} \tag{2}$$

satisfying

(1) if $p_i \neq 0$ then $r(t) \neq 0$
(2) if $p_i = 0$ then $\lim_{t\to\infty} r(t) = 0$ for any x_o and z_o and p_j, $j \neq i$.

According to the approach given in (Frank *et al.*, 1999) to design the residual generator (2), the first step is to separate the disturbed from the undisturbed portion of the model using a decoupling transformation $T(x,u)$ with p_j assumed as disturbance. The second task is the filter design for the smaller system obtained before. The existence condition to solve the FDI problem are given in the following theorem.

Theorem 1: (Frank *et al.*, 1999) Exists a transformation in the space $z = T(x,u) \in \Re^p$ called disturbance decoupling transformation and an output transformation $y_n = h_x(x,y)$ that generates a subspace decoupled from p_1 and sensitive to p_2 for the system

$$\dot{x} = f(x,u) + F_1(x)\,p_1 + F_2(x)\,p_2 \qquad (3)$$

with output $y = h(x)$, if and only if

$$\frac{\partial T(x,u)}{\partial x} F_1(x) = 0 \qquad (4)$$

$$rank\left(\frac{\partial T(x,u)}{\partial x} F_2(x)\right) = rank\,(F_2(x)) \qquad (5)$$

for all (x,u) with $x = \Psi(z, y_a)$ and $y_a = h_a(z,y)$. Moreover, if the system is uniform observable exists an observer sensitive to p_2, and then exists a residual generator (2).

3. DECOUPLING TRANSFORMATION

There are diverse algorithms to find the transformation $T(x,u)$ and the output space $h_a(z,y)$ which generate an observable subsystem that is affected by the fault and not affected by the disturbance; in some cases condition (4) implies the analytical solution of PDE (De-Persis and Isidori, 2001), (Hammouri *et al.*, 1998).

Here, one exploits the strictly upper and lower Hessenberg structure of the system (1) to get the decoupling transformation without solving a PDE.

Considering property (5) and that the disturbance p_1 affects directly the state x_i, the three states equations associated to x_i can be written by

$$\begin{aligned} \dot{x}_{i-1} &= f_{i-1}(x_{i-2}, x_{i-1}, x_i, u) \\ \dot{x}_i &= f_i(x_{i-1}, x_i, x_{i+1}, u) + F_{1i}(x)p_1 \\ \dot{x}_{i+1} &= f_{i-1}(x_i, x_{i+1}, x_{i+2}, u) \end{aligned} \qquad (6)$$

From the structure of this set of states, one notes that the decoupling of the state x_i from the whole system (1) is sufficient to solve the decoupling for p_1, reducing its dimension to $nr = n-1$. i.e. x_i can be contemplated as a disturbance in the reduced system

$$\dot{x} = f_r(x,u,d) + F_{r2}(x)p_2 \qquad (7)$$

$$y = \begin{bmatrix} h_1(x_1) \\ h_2(x_{nr}) \end{bmatrix} \qquad (8)$$

where $f_r(x,u,d) = f(x,u)$ without the element i, $d = x_i$, and $F_{r2}(x) = F_2(x)$ without component i.

For the deduction of the decoupling transformation for x_i, without loss of generality, one can assume the autonomous case neglecting input u and p_2; then the formulation for the decoupling problem of (7) and (8) is equivalent to find the nonlinear transformation which generates a dynamic subset decoupled from d for

$$\dot{x} = f_r(x) + g_i(x)d \qquad (9)$$

$$y = \begin{bmatrix} h_1(x_1) \\ h_2(x_{nr}) \end{bmatrix} \qquad (10)$$

where the two state equations associated to the disturbance d in the reduced system are

$$\begin{aligned} \dot{x}_{i-1} &= f_{i-1}(x_{i-2}, x_{i-1}, d) \\ \dot{x}_i &= f_i(d, x_i, x_{i+1}) \end{aligned} \qquad (11)$$

The form of this dynamic relationship between states comes from the strictly upper and lower Hessenberg structure assumed for the system. This form together with (7) allow to construct a subspace decoupled from d.

The Lie derivative of h along f, written as $L_f h$, is defined by $L_f h = \frac{\partial h}{\partial x} f$ and thus, $L_g L_f h = \frac{\partial L_f h}{\partial x} g$.

Theorem 2: Consider the nonlinear uniform observable reduced system given by (9) and (10), where the disturbance $d = x_i$ and with relative degree from the state x_i to the outputs, $\rho_1 = i-1$, $\rho_2 = nr - \rho_1$ respectively. Under these conditions if $\rho_1 \geq \rho_2$, the nonlinear transformation

$$T(x) = \begin{bmatrix} L_{fr}^0 h_1 \\ \vdots \\ L_{fr}^{\rho_1-2} h_1 \\ L_{fr}^{\rho_1-1} h_1 - \kappa L_{fr}^{\rho_2-1} h_2 \\ L_{fr}^0 h_2 \\ \vdots \\ L_{fr}^{\rho_2-2} h_2 \end{bmatrix} \qquad (12)$$

and if $\rho_2 < \rho_1$ the mapping

$$T(x) = \begin{bmatrix} L_{fr}^0 h_2 \\ \vdots \\ L_{fr}^{\rho_2-2} h_2 \\ L_{fr}^{\rho_2-1} h_2 - \frac{1}{\kappa} L_{fr}^{\rho_1-1} h_1 \\ L_{fr}^0 h_1 \\ \vdots \\ L_{fr}^{\rho_1-2} h_1 \end{bmatrix} \qquad (13)$$

satisfies the condition $\frac{\partial T(x)}{\partial x} g_i(x) = 0$ with

$$\kappa(x) = L_{g_i} L_{fr}^{\rho 1-1} h_1(x) / (L_{g_i} L_{fr}^{\rho 2-1} h_2(x))$$

$$\beta_1(x) = L_{fr}^{\rho_1 - 1} h_1, \quad b_{\rho_1}(x^*) = L_{fr}^{\rho_1} h_1$$

$$\beta_2(x) = L_{fr}^{\rho_2 - 1} h_2, \quad b_{\rho_2}(x^*) = L_{fr}^{\rho_1} h_2$$

and the subspace decoupled from d is given by

$$\dot{x}^* = \begin{cases} \begin{bmatrix} x_2^* \\ \vdots \\ L_{fr}^{\rho_1} h_1 - \kappa L_{fr}^{\rho_2} h_2 \\ \vdots \\ x_{nr}^* \end{bmatrix} & \text{if } \rho_1 \geq \rho_2 \\ \begin{bmatrix} x_2^* \\ \vdots \\ L_{fr}^{\rho_2} h_2 - \frac{1}{\kappa} L_{fr}^{\rho_1} h_1 \\ \vdots \\ x_{nr}^* \end{bmatrix} & \text{if } \rho_2 > \rho_1 \end{cases} \tag{14}$$

with an auxiliary output injection

$$y_a = \begin{cases} y_2 & \text{if } \rho_1 \geq \rho_2 \\ y_1 & \text{if } \rho_2 > \rho_1 \end{cases} \tag{15}$$

Moreover, if $\rho_1 = \rho_2$ and $b_{\rho_1}(x^*)$ is linear in $x_{\rho_1}^*$ the minimal realization of the subspace is of dimension ρ_1.

Proof: The proof is constructive for any state. Let the state i be the disturbance, under consideration. Because of the uniform observability with $y_1 = h_1(x) = x_1$ and the relative degree of d with respect to y_1, the mapping

$$z = \Phi(x) = \begin{bmatrix} \Phi_U(x) \\ L_{fr}^{\rho_1 - 1} h_1 \\ \Phi_L(x) \end{bmatrix} = \begin{bmatrix} x_1 \\ \vdots \\ L_{fr}^{\rho_1 - 1} h_1 \\ --- \\ \phi_i(x) \\ \vdots \\ \phi_{nr}(x) \end{bmatrix} \tag{16}$$

with $\phi_i(x), \ldots, \phi_{nr}(x)$ chosen such that the Jacobian matrix of $\Phi(x)$ is nonsingular, transforms the system to

$$\begin{bmatrix} \dot{z}_1 \\ \vdots \\ \dot{z}_{\rho_1} \\ -- \\ \dot{z}_{\rho_1+1} \\ \vdots \\ \dot{z}_{nr} \end{bmatrix} = \begin{bmatrix} z_2 \\ \vdots \\ b_{\rho_1}(z) + a_{\rho_1}(z) d \\ --- \\ b_i(z) + a_i(z) d \\ \vdots \\ b_{nr}(z) + a_{nr}(z) d \end{bmatrix} \tag{17}$$

with $a_{\rho_1} = L_{g_i} L_{fr}^{\rho 1-1} h_1 = a_{\rho_1}(z_1, z_2, \ldots, z_{\rho 1})$.

Similarly for the output $y_2 = h_2(x) = x_{nr}$, the mapping

$$w = \Psi(x) = \begin{bmatrix} \Psi_U(x) \\ L_{fr}^{\rho_2 - 1} h_2 \\ \Psi_L(x) \end{bmatrix} = \begin{bmatrix} x_{n-1} \\ \vdots \\ L_{fr}^{\rho_2 - 1} h_2 \\ --- \\ \psi_{n-i+1}(x) \\ \vdots \\ \psi_{nr}(x) \end{bmatrix} \tag{18}$$

with the functions $\psi_{n-i+1}(x), \ldots, \psi_{nr}(x)$ chosen such that its Jacobian matrix is nonsingular, transforms the system to

$$\begin{bmatrix} \dot{w}_1 \\ \vdots \\ \dot{w}_{\rho_2} \\ -- \\ \dot{w}_{\rho_2+1} \\ \vdots \\ \dot{w}_{nr} \end{bmatrix} = \begin{bmatrix} w_2 \\ \vdots \\ b_{\rho_2}(w) + a_{\rho_2}(w) d \\ ---- \\ b_{n-\rho_2}(w) + a_{n-\rho_2}(w) d \\ \vdots \\ b_{nr}(w) + a_{nr}(w) d \end{bmatrix} \tag{19}$$

with $a_{\rho_2} = L_{g_i} L_{fr}^{\rho_2 - 1} h_2 = a_{\rho_2}(w_1, w_2, \ldots, w_{\rho_2})$.

Taking into account that the state z_{ρ_1} in (17) and w_{ρ_2} in the representation (19) depend on d, and $a_{\rho_1}(z)$ and $a_{\rho_2}(w)$ are different from zero (from the relative degree property), one can combine the states z_{ρ_1} and w_{ρ_2} to generate a new state decoupled from d, holding the rest of the states of the two normal forms which are independent on d. In particular, one suggests $x^* = T(w, z) =$

$$T(x) = \begin{cases} \begin{bmatrix} \Phi_U \\ \beta_1 - \frac{a_{\rho 1}}{a_{\rho_2}} \beta_2 \\ \Psi_U \end{bmatrix} & \text{if} \quad \rho_1 \geq \rho_2 \\ \begin{bmatrix} \Psi_U(x) \\ \beta_2 - \frac{a_{\rho_2}}{a_{\rho 1}} \beta_1 \\ \Phi_U(x) \end{bmatrix} & \text{if} \quad \rho_1 < \rho_2 \end{cases} \tag{20}$$

in which $x^* \epsilon \Re^{nr-1}$ and its Jacobian matrix has the structure

$$T_J = \begin{bmatrix} 1 & \cdots & 0 & 0 & | & 0 & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & | & \vdots & \vdots & \vdots & \vdots \\ \times & \cdots & \times & 0 & | & 0 & 0 & \cdots & 0 \\ \times & \cdots & \times & \frac{\partial \beta_1}{\partial x_{i-1}} & | & -\kappa \frac{\partial \beta_2}{\partial x_i} & \times & \cdots & \times \\ 0 & \cdots & 0 & 0 & | & 0 & 0 & \cdots & 1 \\ \vdots & \ddots & \vdots & \vdots & | & 0 & 0 & \cdots & \vdots \\ 0 & \cdots & 0 & 0 & | & 0 & \times & \cdots & \times \end{bmatrix}$$

if $\rho_1 \geq \rho_2$, and

$$T_J = \begin{bmatrix} 0 & \cdots & 0 & 0 & | & 0 & 0 & \cdots & 1 \\ \vdots & \vdots & \vdots & \vdots & | & \vdots & \vdots & \cdots & \vdots \\ 0 & \cdots & 0 & 0 & | & 0 & \times & \cdots & \times \\ \times & \cdots & \times & -\frac{1}{\kappa}\frac{\partial \beta_1}{\partial x_{i-1}} & | & \frac{\partial \beta_2}{\partial x_i} & \times & \cdots & \times \\ 1 & \cdots & 0 & 0 & | & 0 & 0 & \cdots & 0 \\ \vdots & \cdots & \vdots & \vdots & | & \vdots & \vdots & \vdots & \vdots \\ \times & \cdots & \times & 0 & | & 0 & 0 & \cdots & 0 \end{bmatrix}$$

if $\rho_1 < \rho_2$. From the structure of $g_i(x)$ one can see that

$$\text{if } \rho_1 \geq \rho_2, \quad a_{\rho 1} = \frac{\partial \beta_1}{\partial x_{i-1}} g_{i_{i-1}} = -\kappa \frac{\partial \beta_2}{\partial x_i} g_{i_i}$$

and

$$\text{if } \rho_1 \geq \rho_2, \quad a_{\rho 2} = \frac{1}{\kappa}\frac{\partial \beta_1}{\partial x_{i-1}} g_{i_{i-1}} = -\frac{\partial \beta_2}{\partial x_i} g_{i_i}$$

and then, it is concluded that the product of the Jacobian matrix T_J by the column vector $g_i(x)$ satisfies (4), and this proves that $T(x)$ is a decoupling transformation for d.

Finally, from the structures (17) and (19), one obtains using the auxiliary output injection (15) the subspace:

$$\begin{bmatrix} \dot{z}_1 \\ \vdots \\ \dot{z}_{\rho_1 - 1} \\ \dot{z}_{\rho_1} - \kappa \dot{w}_{\rho_2} \\ \dot{w}_1 \\ \vdots \\ \dot{w}_{\rho_2} \end{bmatrix} = \begin{bmatrix} x_2^* \\ \vdots \\ x_{\rho_1}^* + \kappa x_{nr-1}^* \\ b_{\rho_1}(x^*) - \kappa b_{\rho_2}(x^*) \\ x_{\rho_1+2}^* \\ \vdots \\ x_{nr}^* \end{bmatrix} \quad (21)$$

If $\rho_1 \geq \rho_2$ with $y_a = w_1$. On the contrary, if $\rho_2 > \rho_1$

$$\begin{bmatrix} \dot{w}_1 \\ \vdots \\ \dot{w}_{\rho_2 - 1} \\ \dot{w}_{\rho_2} - \frac{1}{\kappa}\dot{z}_{\rho_1} \\ \dot{z}_1 \\ \vdots \\ \dot{z}_{\rho_1} \end{bmatrix} = \begin{bmatrix} x_2^* \\ \vdots \\ x_{\rho_2}^* + \frac{1}{\kappa} x_{nr-1}^* \\ b_{\rho_2}(x^*) - \frac{1}{\kappa} b_{\rho_1}(x^*) \\ x_{\rho_2+2}^* \\ \vdots \\ x_{nr}^* \end{bmatrix} \quad (22)$$

with $y_a = z_1$. This state representation corresponds with the subsystem given in (14). Since the condition used to write eqs. (7) and (8) is only sufficient, in some cases, the transformation could be conservative.

To be sure that the maps (12) and (13) solve the decoupling problem and the sensitivity to the faults the rank condition (5) must be verified.

The demonstration of the dimension reduction for the case $\rho_1 = \rho_2$ is based on the fact that $b_{\rho_2}(w)\big|_{w=\Gamma(x^*)} = b_{\rho_1}(z)\big|_{z=\Gamma(x^*)}$.

4. EXAMPLE

Consider a model of a pipeline with 3 leaks given in (Verde, 2001), in which one seeks a subsystem which is decoupled of the middle leak, i.e. $i = 2$. In this case the smooth function $f_r \in \Re^6$ in (9) is given by

$$\dot{x} = \begin{bmatrix} -\mu x_1^2 - \alpha_1 x_2 \\ \alpha_2 (x_1 - x_3) \\ -\mu x_3^2 + \alpha_1 x_2 \\ -\mu x_4^2 - \alpha_1 x_5 \\ \alpha_2 (x_4 - x_6) \\ -\mu x_6^2 + \alpha_1 x_5 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -1 \\ 1 \\ 0 \\ 0 \end{bmatrix} d \quad (23)$$

with $y = [x_1 \quad x_6]'$ and it satisfies all the assumptions ((1) to (7)). Calculating the relative degree of the disturbance d to the 2 outputs one gets $\rho_1 = \rho_2 = 3$ and $\kappa = -1$. Applying Theorem (2) to (23), one obtains the following structure of the system decoupled of d

$$\begin{aligned} \dot{x}_{r1} &= x_{r2} \\ \dot{x}_{r2} &= x_{r3} - y_a \\ \dot{x}_{r3} &= L_{fr}^3 h_1 + L_{fr}^3 h_2 = b(x_r, y_a) \quad (24) \\ \dot{x}_{r4} &= x_{r5} \\ \dot{x}_{r5} &= y_a \\ y &= x_{r1} \quad (25) \end{aligned}$$

in which only the derivatives of h_1 and h_2 along f must be evaluated. In this case the auxiliary output injection is given $y_a = \ddot{x}_6$ and the nonlinear function $b(x_r, y_a)$

$$\begin{aligned} &= -2\mu(x_{r2}^2 + x_{r5}^2 + x_{r1}(x_{r3} - y_a) + x_{r4} y_a) \\ &\quad -2a\left(x_{r2} + x_{r5}\right) - \mu a(x_{r1}^2 + x_{r4}^2) \\ &\quad \frac{4\mu^2}{a}\left(x_{r1}x_{r2}(x_{r3} - y_a) + x_{r4}x_{r5}y_a\right) \\ &\quad \frac{\mu}{a}\left((x_{r3} - y_a + a x_{r1})^2 + (y_a + a x_{r4})^2\right) \\ &\quad -4\mu^2(x_{r1}^2 x_{r2} + x_{r4}^2 x_{r5}) \\ &\quad -\frac{4\mu^3}{a}\left((x_{r1}x_{r2})^2 + (x_{r4}x_{r5})^2\right) \end{aligned}$$

with $a = \alpha_1 \alpha_2$ and

$$x_1 = x_{r1}$$

$$x_2 = \frac{-1}{\alpha_1}(x_{r2} + \mu x_{r1}^2)$$

$$x_3 = \frac{1}{a}(x_{r3} - y_a + 2\mu x_{r1} x_{r2} + a x_{r1})$$

$$x_4 = \frac{1}{a}(y_a + 2\mu x_{r4} x_{r5} + a x_{r4})$$

$$x_5 = \frac{1}{\alpha_1}(x_{r5} + \mu x_{r4}^2)$$

$$x_6 = x_{r4}$$

This example shows the simplicity of the proposal, since the subspace structure is given by the relative degrees of the outputs (i.e property (7)) and only $b(x_r, y_a)$ must be calculated. Note that for this example does not exist a linear decoupling transformation and therefore, the fault detector nonlinear filter for polynomial systems proposed by (Ashton *et al.*, 1998) cannot be applied. To design the nonlinear estimator of (24) diverse procedures can be used, as example (Gauthier and Kupta, 1994) or if the system is control affine a high gain or a Kalman-like obverver. This shows the advantage of the tasks separation in the residual generator problem for fault detection.

5. CONCLUSIONS

It has been introduced, without solving a PDE, a decoupling transformation for strictly upper and lower Hessenberg nonlinear systems when exists a relationship between the relative degrees from the perturbation to the outputs. Moreover, the structure of a subsystem which is not affected by the disturbance by the coordinate change in the state and output spaces is given. The results developed in this paper can be used directly in the design of FDI filter for a class of nonlinear systems, allowing the application of diverse procedure to design the residual generator. Therefore, for a system with a Hessenberg structure, the advantage to divide the robust residual generator problem in steps is evident. The transformation is applied to the case of isolation of leaks in a pipeline. The solution allows to determine analytical expressions for any number of leaks and parameters of the pipeline without any constraint in the procedure to design the residual generator. The key of the proposal is that each output allows to define a set of states in which the derivatives of the outputs are the states and in each subspace the disturbance is injected only in on state (z_{ρ_1} and w_{ρ_2}).

6. REFERENCES

Alcorta, E. (1999). Modellgestuetzte residuengenerierung fuer die diagnose von additiven and multiplikativen fehlen in dynamischen systemen. Vol. Reihe 8, Nr 781. Fortschritt-Berichte VDI. Germany.

Ashton, S.A., D. N. Shields and S. Daley (1998). Fault detection in pipelines using nonlinear observers. *UKACC International Conference on Control'98* **455 IEE**, 135–140.

Chen, J. and R. J. Patton (1999). *Robust Model-Based Fault Diagnosis for Dynamic Systems.* Kluwer Academic Publishers. London.

De-Persis, C. and A. Isidori (2000). A geometric approach to nonlinear fault detection and isolation. *SAFEPROCESS'2000* pp. 209–214.

De-Persis, C. and A. Isidori (2001). A geometric approach to nonlinear fault detection and isolation. *IEEE Trans Aut. Control* **46-6**, 853–866.

Frank, P. M., G. Schreier and E. Alcorta Garc a (1999). Nonlinear observers for fault detection and isolation. *Lecture Notes in Control and Information Science 244, eds H. Nijmeijer and T.I. Fossen* pp. 399–466.

Gauthier, J. and A. K. Kupta (1994). Observability and observers for nonlinear systems. *SIAM J. Contr. Optimization* **32**, 975–994.

Gauthier, J., H. Hammouri and S. Othman (June, 1992). A simple observer for nonlinear systems applications to bioreactors. *IEEE-Trans. on Automatic Control* **37**, 875–880.

Hammouri, H., M. Kinnaert and E.H. El Yaagoubi (1998). Fault detection and isolation for state affine systems. *European Journal of Control* **4**, 2–16.

Isermann, R. (1997). Supervision, fault -detection and fault-diagnosis methods: an introduction.. *Contr. Eng. Practice* **5(5)**, 639–652.

Massoumnia, M., G.C. Veghese and A. S. Willsky (1989). Failure detection and identification. *IEEE Trans. Autom. Control.*

Olivier, B., S. Gauthier and A. Sciandra (1998). Nonlinear observers for a class of biological systems: Application to validation of phytoplanktonic growth model. *IEEE TAC* **43-(8)**, 1056–1065.

Seliger, R. and P. Frank (2000). Robust observer-based fault diagnosis in non-linear uncertin systems. *Issues of Fault Diagnosis for Dynamic Systems, R, Patton, et al* pp. 145–187.

Seliger, R. and P. Frank (December, 1991). Fault-diagnosis by disturbance decoupled nonlinear observers. *30th Conference on Decision and Control, Brighton, England.*

Shields, D. N., S. A. Ashton and S. Daley (2001). Design of nonlinear observers for detecting faults in hydraulic sub-sea pipelines. *Control Engineering Practice* **9**, 297–311.

Verde, C. (2001). Multi-leak detection and isolation in fluid pipelines. *Control Eng. Practice,* **9**, 673–682.

White, J. and J. Speyer (1987). Detection filter design: spectral theory and algorithms. *IEEE, TAC* **32(7)**, 593–603.

Wunnenberg, J. (8-222, 1990). *Observer based fault detection in dynamic systems.* VDI-Fortschrittsberichte. Germany.