15th Triennial World Congress, Barcelona, Spain

AN EXTENDED BCU METHOD TO POWER SYSTEM TRANSIENT STABILITY ASSESSMENT

Ignacio Luna López, Alexander Loukianov, J. Manuel Cañedo C.

CINVESTAV IPN, Unidad Guadalajara, Apartado Postal 31-438, Guadalajara, Jalisco. 45091, México, email: iluna@gdl.cinvestav.mx

Abstract: This paper shows that under some circumstances the so called BCU method may not be able to find the true CUEP for a specific fault-on trajectory of a power system. The reasons to that malfunction are analysed, and a methodology than can improve the capability of the BCU method to successfully find the CUEP is presented. Finally, several applications of the proposed method to power electrical systems, are shown. *Copyright © 2002 IFAC*

Keywords: power system, stability analysis.

1. INTRODUCTION

Transient stability analysis plays an important role in the planning and operation of power electric systems (PES). Recently, transient stability analysis had been performed by utilities exclusively by means of the numerical integration of non-linear differential equations describing the fault-on system and de post-fault system. An alternative approach to transient stability analysis employing Lyapunov function theory, called the direct methods, was proposed in the 60's (Chiang, *et. al*., 1994). Direct methods refer to those methods that determine the stability of a post-fault system based on energy functions without explicitly integrating differential equations.

There exist a variety of methods that are classified as direct methods. One of them, is the so-called *Controlling Unstable Equilibrium Point method* (CUEP method) (Chiang, *et. al.,* 1994; Chiang, *et. al.,* 1987; Chiang, 1995; Fouad and Vittal, 1992). The principal advantage of this method is that it is able to approximate the part of the boundary of stability that contains the point where the sustained fault-on trajectory crosses the boundary of stability.

The CUEP method is a generic method. There are distinct methods that actually are the CUEP method. The main difference between those methods is the way that they follow to find the CUEP. As example of them, there is the called Mode of Disturbance Method (MOD) proposed by Fouad and Vittal, (1992). The basis of this method is to find the oscillation mode that makes the system unstable, and the energy that is associated with that mode.

Another method called as BCU method (Boundary of stability region Controlling Unstable equilibrium point method), is based on the differential geometry theory (Chiang, *et. al*., 1988, 1994, 1995; Zaborsky, *et. al*., 1988). The BCU method has been very attractive because it has a robust theoretical background.

In this paper it is shown that in spite of the above background the BCU method may be unable to correctly find the CUEP under some circumstances, and two motives that can cause this inability, are demonstrated. This paper presents conditions that are used to distinguish when the BCU method fails or hit in finding the CUEP, and propose an algorithm that may improve the capabilities of the BCU method. This algorithm will be called as the EBCU method (Extended BCU method) because it use the BCU method algorithm in combination with a back-up method. The last method will be used when the BCU method fails in finding the CUEP.

2. THE CUEP METHOD

This section presents briefly the generic CUEP method. The objective of the CUEP method is to determinate the stability of a post-fault system.

Definition 1. *A power system is said to be stable if after a disturbance it reaches an acceptable steady-state condition.*

In other words, a power system is stable if the fault-on trajectory at clearing time lies inside the stability region of a desired Stable Equilibrium Point (SEP) of its post-fault system.

The point where the fault-on trajectory leaves the stability region of the SEP is called the exit point. This point is contained into the SEP's boundary of stability. It has been shown (see Chiang, *et. al*., 1988; Zaborsky, *et. al*., 1988) that the SEP's boundary of stability of a PES is equal to the union of the stable manifolds of the unstable equilibrium points (UEP) that lies into the SEP's boundary of stability.

Definition 2. *The UEP whose stable manifold contains to the exit point is called the Controlling Unstable Equilibrium Point (CUEP).*

It has been shown that the CUEP is an UEP whose unstable manifold has dimension one, that is, the Jacobian matrix evaluated into the UEP has only one eigenvalue with real part greater than zero. This kind of UEP is called as type-1 equilibrium point.

In order to make an approximation of the relevant part of the SEP's boundary of stability, it is necessary to know the CUEP for a specific fault-on trajectory (Chiang, 1995). Each fault-on trajectory together with every post-fault system has a specific CUEP. The issue of the CUEP method (and the issue of this paper) is to find the true CUEP for the each condition of the PES.

One of the methods developed to find the CUEP is the BCU Method. This method is based on the properties of a gradient system that is related with the original PES model.

3. POWER ELECTRICAL SYSTEM MODEL

This section presents the PES mathematical model and the gradient system of PES; this gradient system is an approximation of the PES mathematical model.

3.1. Power electrical system model.

Synchronous machines will be modelled with the classical model. Considering loads as constant impedances that can be added to the transmission network, and using a synchronous machine of the PES (for simplicity, the nth machine) as reference, it is possible to model a n-machine PES by the following $2(n-1)$ differential equations system:

$$\dot{\mathbf{x}} = \mathbf{F}(\mathbf{x}) \tag{1}$$

where $\mathbf{x} = \left[\delta_{1n} \cdots \delta_{n-1,n}\ \omega_{1n} \cdots \omega_{n-1,n}\right]^T$,

$$\mathbf{F}(\mathbf{x}) = \left[\omega_0\omega_{1n} \cdots \omega_0\omega_{n-1,n}\ \ f_1(\delta) - D\omega_{1n} \cdots f_{n-1}(\delta) - D\omega_{n-1,n}\right]^T \tag{2}$$

$$f_i(\boldsymbol{\delta}) = \frac{Tm_i - Te_i}{M_i} - \frac{Tm_n - Te_n}{M_n}, \quad i = 1,\ldots,n-1 \tag{3}$$

δ_{in} is the relative angle between the i-th machine and the reference machine, ω_{in} is the relative speed between the ith machine and the reference machine, ω_0 is the synchronous angular speed, that is $\omega_0 = 2\pi f$, M_i is the inertia constant, D is the damping coefficient, Tm_i is the mechanical torque applied, and Te_i is the electrical torque defined as

$$Te_i = E_i^2 G_{ii} + \sum_{\substack{j=1 \\ j\neq i}}^{n} E_i E_j Y_{ij} \cos\left(\theta_{ij} + \delta_j - \delta_i\right) \tag{4}$$

Since the aim of this work is around the CUEP method it is necessary to have an energy function of PES mathematical model. The system (1) has a real energy function only if transfer conductances, are avoided. In other case, it is necessary to approximate the energy function. The most common approximation (Fouad and Vittal, 1992) of energy function of (1) is based on the assumption that the fault-on trajectories are straight lines. This expression was derived under COI reference frame; using this result an expression under the nth machine reference frame has been obtained. The resulting expression is fully equivalent to the given in (Fouad and Vittal, 1992) although it is different to the given in (Chiang, *et. al.,* 1994). The expression used in this work is

$$V(\delta,\omega) = V_c(\omega) + V_p(\delta) \tag{5}$$

where

$$V_c(\omega) = \frac{\omega_0}{2M_T} \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} M_i M_j \left(\omega_{in} - \omega_{jn}\right)^2 \tag{6}$$

$$\begin{aligned} V_p(\delta) = &-\frac{1}{2M_T} \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \left[\left(\delta_{in} - \delta_{jn}\right) - \left(\delta_{in}^s - \delta_{jn}^s\right)\right]\left[T_i M_j - T_j H_i\right] \\ &- \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} E_i E_j B_{ij} \left[\cos\left(\delta_{in} - \delta_{jn}\right) - \cos\left(\delta_{in}^s - \delta_{jn}^s\right)\right] + V_{path}(\delta) \end{aligned} \tag{7}$$

$$V_{path}(\delta) = \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} E_i E_j G_{ij} \frac{A_{ij} - A_{ij}^s}{\left(\delta_{in} - \delta_{jn}\right) - \left(\delta_{in}^s - \delta_{jn}^s\right)} \left[\operatorname{sen}\left(\delta_{in} - \delta_{jn}\right) - \operatorname{sen}\left(\delta_{in}^s - \delta_{jn}^s\right)\right] \tag{8}$$

$$A_{ij} = \delta_{in} + \delta_{jn} - 2\sum_{k=1}^{n-1} \frac{M_k}{M_T} \delta_{kn} \tag{9}$$

$$T_i = Tm_i - E_i^2 G_{ii} \tag{10}$$

Quantities super-indexed with s refer to post-fault stable equilibrium point. All network quantities (reduced admittance matrix, G_{ij} and B_{ij}) correspond to post-fault system.

3.2. Gradient system of PES.

The gradient system can be derived from equations (1) to (10) of the form (Chiang, 1995)

$$\dot{\delta} = \mathbf{F}_g(\delta) \tag{11}$$

where $\mathbf{F}_g = \left[\omega_0 f_1(\delta) \cdots \omega_0 f_{n-1}(\delta)\right]^T$. This gradient system has an energy function that has the form

$$V_{pot}(\delta) = V_p(\delta) + V_{path}(\delta) \tag{12}$$

where $V_p(\delta)$ and $V_{path}(\delta)$ are defined in equations (7) and (8) respectively. The energy function of the gradient system is the potential energy function of PES model. It is easy to see that if δ_E is an equilibrium point of (11) then the point (δ_E , 0) is an equilibrium point of (1).

4. THE BCU METHOD

4.1 The BCU method algorithm.

The BCU method consists on finding the gradient system's CUEP δ_{co}. The PES model's CUEP will be (δ_{co} , 0). The ideal algorithm to find δ_{co} is

- First, to find the gradient system's exit point. It means to find the point where the fault-on trajectory crosses the gradient system's boundary of stability.
- Then, this point is used as initial condition to solve the gradient system. Since by definition the exit point lies into the CUEP's stable manifold, the solution trajectory will tend to the CUEP.

The exit point is not easy to find (in computation terms). Therefore, an approximation to this point is used instead of the true exit point. The point where the gradient system's energy function (12) reaches its first maximum (along the fault-on trajectory) approximates the true exit point. This point will be represented by δ^*. Because the information about fault-on trajectory is a set of points, and it is not continuous, a second point to approximate the true exit point is used. This point, represented by δ^+, is the last point after the gradient system's energy function reaches its first maximum. The gradient system's solution trajectories ($\phi^*(t)$ and $\phi^+(t)$ respectively) that are calculated using the points δ^* and δ^+ as initial conditions, will not tend to the CUEP, because they do not lie into the CUEP's stable manifold. Although, if the points δ^* and δ^+ are sufficiently close to the true exit point then the trajectories $\phi^*(t)$ and $\phi^+(t)$ will go towards the CUEP for a while, and then will go away from the CUEP. This behaviour is the basis to the BCU method.

For simplicity, the Euclidean norm of (11) will be named as the *gradient function*. Since the CUEP is an equilibrium point of (11) the gradient function at CUEP is equal to zero. Furthermore, while the trajectories $\phi^*(t)$ and $\phi^+(t)$ tends to the CUEP, the gradient function tends to zero. That happens until the trajectories go away the CUEP, and at this moment the gradient function reach a local minimum.

Let be δ_0^* and δ_0^+ the points where gradient function along $\phi^*(t)$ and $\phi^+(t)$ reach a local minimum respectively. Then the point where gradient function is the lowest, is the point closest to the CUEP, and it can be used as a guest point to solve the algebraic equation system

$$0 = \mathbf{F}_g(\delta) \tag{13}$$

Let δ^i be the solution of (13). Then δ^i is the gradient system's CUEP, and the CUEP of PES is (δ^i,0).

4.2 Troubles with the BCU method.

The success of BCU method in finding the CUEP is mostly determined by its ability to find a suitable guest point as a solution of (13). If this point is unsuitable then the found equilibrium point (when the used method can find one) could not be the true gradient system's CUEP.

In order to solve the equations (13) the Newton-Raphson method is commonly used. It has been shown that the vicinities of equilibrium points obtained by this method are of fractal nature (Liu and Thorp, 1997). This fractal nature implies that although the guest point (δ_0^* or δ_0^+) lies into a vicinity of the CUEP, it is possible that the Newton-Raphson method will find an equilibrium point different from the CUEP. So, in order to find the true CUEP, the guest point must lie into the fractal that corresponds to the CUEP. Figure 1 shows the Newton-Raphson method's fractal nature when it is applied to the gradient system of a 3-machine PES.

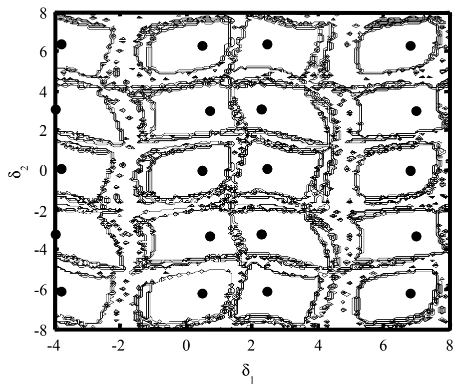

Fig. 1. Newton-Raphson method's fractal nature in a 3-machine PES.

Figure 1 shows all of the equilibrium points that lie into the subspace defined by $-4 \le \delta_1 \le 8$ and $-8 \le \delta_2 \le 8$. Each of them is surrounded by a set of points that form their solution region. The interface between these solution regions is composed by two kinds of points. The first one is the set of points that do not belong to any solution region. The points which have a solution, but this solution can be very different for two points although both points are closed each other compose the second kind.

If the points δ^* and δ^+ are very close to the true exit point there is a great possibility that the points δ_0^* and δ_0^+ lie into the fractal that corresponds to the CUEP. But it has been shown that in some situations δ^* and δ^+ could be far away the true exit point (Struggs and Mili, 2001). When it is the case, it is possible that the gradient function does not have a minimum along the trajectories $\phi^*(t)$ and $\phi^+(t)$. If this minimum exists it could be far away the CUEP or it could lie outside the fractal that corresponds to the CUEP. Moreover, it is possible that even though the point δ^* or δ^+ are close to the true exit point, the gradient function along the post-fault gradient system's solutions $\phi^*(t)$ or $\phi^+(t)$ does not have a local minimum. Figure 2 shows three different post-fault gradient system trajectories in a 3-machine system, and Figure 3 shows the gradient function for the three post-fault trajectories.

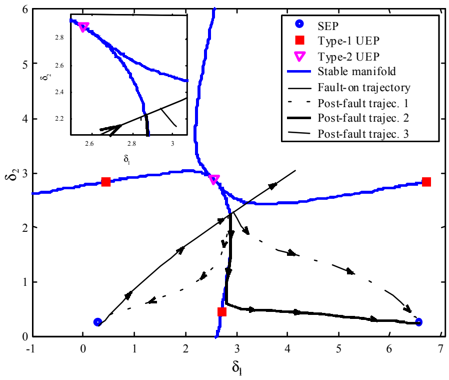

Fig. 2. Post-fault gradient system's trajectories in a 3-machine PES.

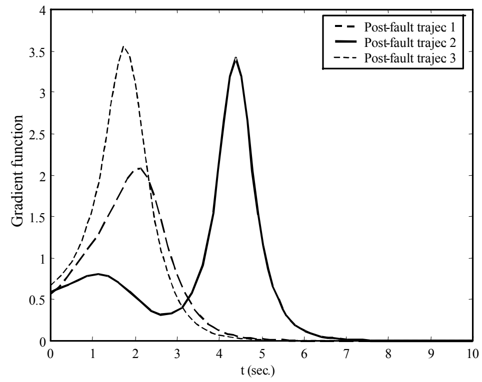

Fig. 3. Gradient function along 3 post-fault gradient system trajectories.

Each trajectory shown on Fig. 2 has as initial condition a point that is close to a type-1 UEP's stable manifold, that is, close to a SEP's boundary of stability. Each trajectory moves nearly the stable manifold for a while but only the second one passes sufficiently close to the type-1 UEP as the gradient function along the trajectory has a local minimum, see Fig. 3. Trajectories 1 and 2 do not have a local minimum of gradient function even their initial conditions are close to the stable manifold.

Another situation that can cause a malfunctioning of the BCU method can be found when the initial condition to the post-fault gradient system is close to a type-2 UEP. In this case it is possible to detect a local minimum of gradient function because the presence of the equilibrium point (which by definition is not the CUEP), but this point will be close to the type-2 UEP and the solution method will find that point.

5. A DYNAMICAL METHOD TO FIND TYPE-1 UEP

Liu and Thorp have developed a method useful to find the Closest UEP (Liu and Thorp, 1997). In order to avoid the BCU method troubles that can happen and using the property that the Closest UEP and the CUEP are both type-1 UEP, that method was adapted to find the CUEP instead of the Closed UEP

The only one condition to do it is that, the used guest point must be adequate to find the CUEP instead of the Closest UEP.

The basis of the method developed by Liu and Thorp, (1997), is the spectral decomposition of Jacobian matrix of gradient system (11), that is

$$\mathbf{J}(\delta) = \frac{\partial \mathbf{F}_g(\delta)}{\partial \delta} = \sum_{j=1}^{n-1} \lambda_j(\delta)\mathbf{P}_j(\delta) \qquad (14)$$

where $\lambda_j(\delta)$ is the j-eigenvalue of $\mathbf{J}(\delta)$, and $\mathbf{P}_j(\delta)$ is the vectorial product of the right and left j-eigenvectors of $\mathbf{J}(\delta)$.

The Jacobian matrix of gradient system has n-1 real eigenvalues. Let $\lambda_{n-1}(\delta)$ be the largest eigenvalue. Then an **A**-matrix can be constructed of the form

$$\mathbf{A}(\delta) = [\mathbf{P}_1(\delta) + \mathbf{P}_2(\delta) + \ldots + \mathbf{P}_{n-2}(\delta) - \mathbf{P}_{n-1}(\delta)]$$

to obtain the following new dynamical system

$$\dot{\delta} = \mathbf{A}(\delta)\mathbf{F}_g(\delta) \qquad (15)$$

It can be shown that the new system (15) has the same equilibrium points that the gradient system (11). The Jacobian matrix of the new system (15) can be calculated as

$$\mathbf{J}_n(\delta^e) = \left[\mathbf{A}(\delta)\frac{\partial \mathbf{F}_g(\delta)}{\partial \delta}\right]_{\delta^e} = \mathbf{A}(\delta^e)\mathbf{J}(\delta^e) \qquad (16)$$

where δ^e is an equilibrium point. Using the spectral decomposition of $\mathbf{J}(\delta)$ and the fact that the matrices $\mathbf{P}_i$, $i = 1,\ldots,n-1$ are orthogonal, equation (16) can be written as

$$\mathbf{J}_n(\delta^e) = \left(\sum_{j=1}^{n-2} \lambda_j(\delta^e)\mathbf{P}_j(\delta^e)\right) - \lambda_{n-1}(\delta^e)\mathbf{P}_{n-1}(\delta^e) \qquad (17)$$

From (17) and (14) it is easy to see that if δ^e is a type-1 equilibrium point of gradient system. Then δ^e will be a SEP of the new system (15). This property of the new system is used to find the type-1 equilibrium points of the gradient system.

Dynamical Method. *In order to find the type-1 UEP it is necessary to solve the differential equations (15). Let δ_0 be the initial condition. Let the solution of (15) tends to a SEP of this system. This SEP is equivalent to a type-1 UEP of the gradient system.*

Figure 4 shows the attraction regions of the SEP when the proposed method (*Dynamical Method*) is applied to a 3-machine PES. Since this method is developed to find type-1 UEP, the number of attraction regions contained into the intervals

-4≤δ_1≤8 and -8≤δ_2≤8, are less than the solution regions of the Newton-Raphson method (see Fig. 1).

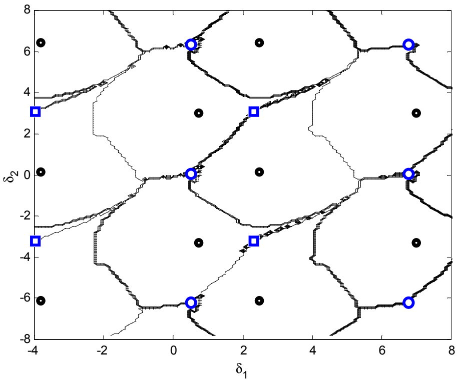

Fig. 4. Attraction regions of Dynamical Method in a 3-machine PES.

The type-1 UEP are shown using a filled circle; the SEP are shown with a non-filled circle, and type-2 UEP are shown with a non-filled square. A continuos attraction region surrounds every type-1 UEP. The boundaries between these attraction regions are perfectly distinguished. Both SEP and type-2 UEP lie into the boundary between two or more attraction regions. If the attraction regions of the Dynamical Method are compared with the solution regions of Newton-Raphson method, then it is possible to see that the possibilities to find a specific type-1 UEP by means of the Dynamical Method will be greater than using the Newton-Raphson method.

5. THE EBCU METHOD

The Dynamical Method (DM) can be used instead of Newton-Raphson Method (NRM) under one of the following conditions:

Condition 1. The solution founded by NRM, corresponds to an equilibrium point that is not a type-1 UEP. It means that the obtained solution is not the true CUEP.

Condition 2. The solution founded by NRM (δ^e) corresponds to a type-1 UEP, but the energy function of the gradient system evaluated at δ^e ($V_{pot}(\delta^e)$) is less than its value at SEP ($V_{pot}(\delta_s)$). It means that δ^e is not the true CUEP.

Taking in consideration the above conditions, the algorithm shown on Fig. 5, is proposed. This algorithm conserves the fundamental idea behind BCU method since it contains the BCU method and additional steps in which the DM is used. DM is required only when the BCU method was not able to find the true CUEP. The resulting algorithm was named EBCU method (Extended BCU method) since it is an extension of the BCU method.

6. NUMERICAL RESULTS

The proposed EBCU method was applied to several PES. In this section it is shown the results obtained when the EBCU method was applied first to the 3-machine WSCC system (Anderson, 1994), secondly to the modified 4-machine WSCC system (Fouad, *et. al.*, 1981), then the 16-machine New England system (Rogers, 2000), and finally the 50-machine IEEE system (IEEE, 1992).

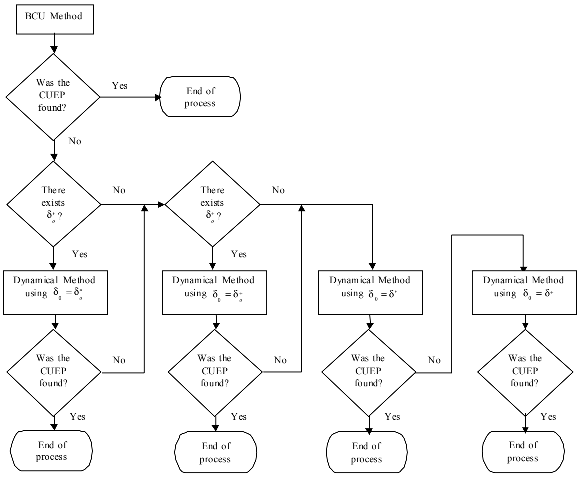

Fig. 5. EBCU method algorithm.

The 4 above cited PES were used in order to compare the performance of the BCU and EBCU methods. Both methods have been applied under several operation and failures condition (3-phase short circuits that are cleared tripping a transmission line), presented in Table 1. The operation condition of each system was modified through the change of their generation and load power level. The 100% case represents the base case of each system. The performance of the BCU and EBCU methods was tested using a total of 1332 simulations.

Table 1. Simulations used to compare the performance of the BCU and EBCU methods

System	Power level			Applied failures	
	Bottom	***Top***	***Steps***	***Per step***	***Total***
3-machine	30%	260%	24	12	288
4-machine	30%	170%	15	14	210
16-machine	30%	130%	11	22	264
50-machine	75%	105%	15	38	570

Figures 6 and 7 show two bar graphs that resume the obtained results. In Fig. 6 there are shown 3 result categories. The first one indicates the number of cases, which was solved successfully with BCU method. The second category indicates the number of cases, which fails to solve by BCU method, and it was solved successfully using EBCU method. Finally, the third category indicates the number of cases, which was not solved.

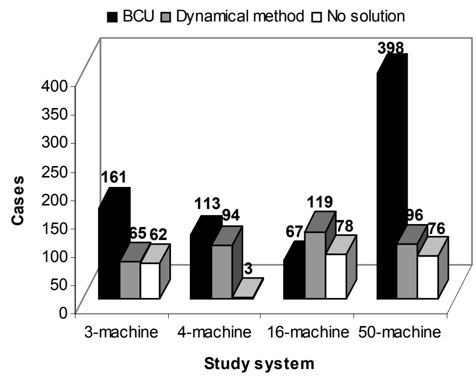

Fig. 6. Performance of BCU and Dynamical methods.

Figure 6 shows that the performance of these methods depends on the system where they are used. For example, the only 28.76% of 3-machine and 19.43% of 50-machine systems' solved cases, was required the use of EBCU method, while 45.41% of 4-machine and 63.97% of 16-machine systems' solved cases, was required the use of EBCU method. In the same way, the percentage of the considered cases that have not been solved depends on the PES.

Additionally, Figure 6 shows that 219 simulations were not solved of a total of 1332. Some of them were not solved because the corresponded post-fault system does not have a SEP. Another were not solved because the post-fault SEP is far away from the pre-fault SEP, and the remaining simulations were not solved because the founded type-1 equilibrium point does not satisfy the above written conditions. These kinds of troubles happen when the generation and load power level or stress level is high.

Since EBCU method include the original BCU combining with the Dynamical method, it is able to find the true CUEP in more or equal cases than the original BCU. This is shown in Figure 7.

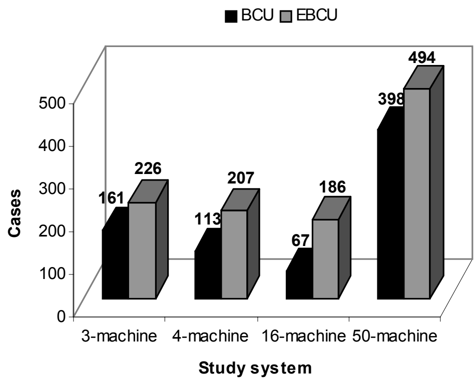

Fig. 7. Performance of BCU and EBCU methods.

7. CONCLUSIONS

In this paper the conditions under which the BCU fails to find the real CUEP for a specific failure, are derived, and the EBCU method to find the Controlling UEP (CUEP), is proposed. This method uses the Dynamical Method that pretends to be a back up of classical algebraic equation systems solution methods. It is shown that proposed method has a better performance than the original BCU method. This was demonstrated through the analysis of a total of 1332 failures applied to 4 different power electrical systems.

REFERENCES

Anderson, P. M. (1994). *Power System Control and Stability*, IEEE Press, USA.

Chiang, H. D., F. F. Wu and P. P. Varaiya (1987). Foundation of Direct Methods for Power Systems Transient Stability Analysis. *IEEE Trans. on Circuits and Systems,* **vol. 34**, pp. 712–728.

Chiang, H. D., M. W. Hirsh and F. F. Wu (1988). Stability Regions of Nonlinear Autonomous Dynamical Systems. *IEEE Trans. on Automatic Control,* **vol. 33**, pp. 16-27.

Chiang, H. D., F. W. Wu and P. P. Varaiya (1994). A BCU Method for Direct Analysis of Power System Transient Stability. *IEEE Trans. on Power Systems,* **vol. 9**, pp. 1194–1208.

Chiang, H. D. (1995). The BCU Method for Direct Stability Analysis of Electric Power Systems: Theory and Applications. *Systems and Control Theory for Power Systems IMA Vol. 64 in Mathematics and its Applications*, Springer-Verlag.

Fouad, A. A., K. C. Kruempel, K. R. C. Mamandur, S. E. Stanton, V. Vittal, M. A. Pai (1981), *Transient Stability Margin as a Tool for Dynamic Security Assessment Tech. Rep. EL-1755*, EPRI Palo Alto, CA.

Fouad, A. A., V. Vittal (1992). *Power System Transient Stability Analysis Using the Transient Energy Function Method*. Prentice Hall, New Jersey.

IEEE committee report, Task Force of The Dynamic System Performance Subcommittee (1992). Transient Stability Test Systems for Direct Stability Methods. *IEEE Trans. on Power Systems*, **vol. 7**, pp. 37-44.

Kakimoto, N., Y. Ohsawa and M. Hayashi (1978). Transient Stability Analysis of Electric Power System via Lure-type Lyapunov Function, parts I and II. *Trans. IEE of Japan,* **vol. 98**, pp. 516.

Liu, Chih-Wen and J. S. Thorp (1997). A Novel Method to Compute the Closest Unstable Equilibrium Point for Transient Stability Region Estimate in Power Systems. *IEEE Trans. on Circuits and Systems – I Fundamental Theory and Applications,* **vol. 44**, pp. 630-635.

Rogers, Graham (2000). *Power System Oscillations*. Kluwer Academic Publishers, USA.

Struggs, J. T., L. Mili (2001). Dynamic gradient method for PEBS detection in power system transient stability assessment. *Elsevier Science Electrical Power & energy Systems*, **vol. 23**, pp. 155-165.

Zaborsky, J., G. Huang, B. Zheng and T. Leung (1988). On the Phase Portrait of a Class of Large Nonlinear Dynamical Systems Such as the Power System. *IEEE Trans. on Automatic Control,* **vol. 33**, pp. 4-15.