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ROBUST MULTI-MODEL CONTROL OF A SYNCHRONOUS SERVOMOTOR USING SIMULATED ANNEALING ALGORITHM

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Abstract: The purpose of this paper is to present a linear quadratic simultaneous multi-model stabilizing approach based on simulated annealing algorithm. This method is developed to design robust control laws for AC drives. A specific application to a synchronous self-controlled motor is detailed. The main objective here is design a robust controller with predetermined structure. Experimental results are given showing the efficiency of the proposed approach. *Copyright © 2002 IFAC*

Keywords: Robust control, Simultaneous stabilization, Quadratic control, Simulated annealing, Synchronous motor.

1. INTRODUCTION

AC synchronous servomotors are commonly used for precision positioning applications in place of DC drives. Efficient control laws are needed to guarantee stability and performance requirements. These constraints impose a good knowledge of the system's parameters. However, due to varying conditions such as temperature, magnetic saturation,… etc, the sought control laws must take into account the uncertainties in the model.
Multi-model robust control techniques can be tackled using several approaches such as H_∞ methods (Banerjee, 1995), (Courties et al., 1999), Pareto game approach (Tarvainen, 1986) or fuzzy fusion (Ksouri et al., 1999)
The purpose of this paper is to present a robust multi-model control design method based on simulated annealing optimization algorithm. The proposed approach allows the design of control laws having a predefined structure, which stabilize a set of linear models. The simulated annealing algorithm leads to an optimal solution, if it exists, guarantying the simultaneous stability and performance for all considered models.
The paper is structured as follows: in section 2, the AC drive is described and a linear model is proposed along with the parameters uncertainties. The synchronous motor is associated with a three phases PWM inverter. In section 3, the robust control design approach is presented; it is based on solving simultaneously several linear quadratic problems. A unique static feedback control law is obtained by iterative optimization: either using an α-stabilization method or simulated annealing. The structure of our controller is justified in section 4 where the multi-model approach gives different design possibilities. Experimental results and robustness analysis are presented in section5. Concluding remarks make up section 6.

2. MODELING OF THE AC DRIVE SYSTEM

We consider a 2.2 kW synchronous self-controlled servomotor with permanent magnets. The AC drive is fed by a three phase PWM inverter (see in Figure 1).

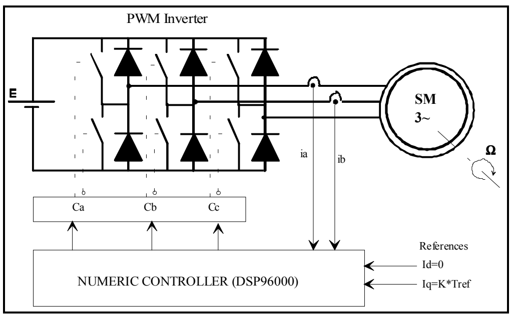

Figure 1: Experimental system

A state model can be obtained as follows (Leonard, 1985): for the electrical part, the differential equations describing the relations between the rotor and stator fluxes, currents and voltages are written first en the statoric frame (or (a,b,c) frame). In a second step, Park transformation is used to obtain an image of these equations in a frame rotating synchronously with the rotor (or (d,q) frame). This leads to:

$$\begin{cases} \dfrac{di_d}{dt} = -\dfrac{R}{L_d}.i_d + p.\dfrac{L_q}{L_d}.\Omega.i_q + \dfrac{V_d}{L_d} \\ \dfrac{di_q}{dt} = -\dfrac{R}{L_q}.i_d - p.\dfrac{L_d}{L_q}.\Omega.i_d - p.\phi_f.\Omega + \dfrac{V_q}{L_q} \\ \dfrac{d\Omega}{dt} = p.\dfrac{L_d - L_q}{J}.i_q.i_d + p.\dfrac{\phi_f}{J}.i_q - \dfrac{f}{J}.\Omega - \dfrac{\Gamma_r}{J} \end{cases} \quad (1)$$

When Ω, the mechanical speed of the rotor, is taken as constant (i.e. when Ω is varying slowly with respect to the dynamics of the currents), equation (1) can be reduced to the following linear state model.

$$\begin{bmatrix}\frac{di_d}{dt}\\ \frac{di_q}{dt}\end{bmatrix} = \begin{bmatrix} -\frac{R}{L_d} & p\Omega\frac{L_q}{L_d} \\ -p\Omega\frac{L_d}{L_q} & -\frac{R}{L_q}\end{bmatrix}\cdot\begin{bmatrix} i_d\\ i_q\end{bmatrix} + \begin{bmatrix}\frac{1}{L_d} & 0\\ 0 & \frac{1}{L_q}\end{bmatrix}\cdot\begin{bmatrix}V_d\\ V_q\end{bmatrix} + \begin{bmatrix}0\\ -p\Omega\frac{\Phi_f}{L_q}\end{bmatrix} \qquad (2)$$

Where (i_d,i_q) represent the components of the currents (model states) and (V_d,V_q) the component of the voltage (model inputs) in the (d,q) frame.

It should be noted that the parametric of such a model (R, L_d and L_q) cannot be measured with precision and are subject to wide variations due to temperature (for the winding resistance) and magnetic saturation (for the inductances).

The parametric variations modify the closed loop response of the system. This can be easily seen in Figure 2 where the poles of the system are shown for a 50% variation in the measured values of the parameters.

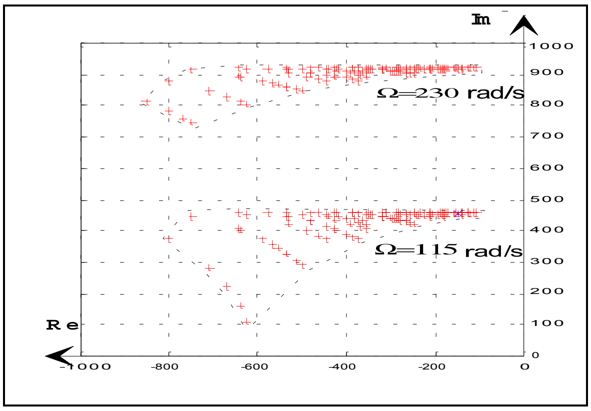

Figure 2: Pole of the system

In our application, 27 models are defined by considering ±50% variation in each measured parameter values i.e.:

$$\begin{aligned} &R \in \{0{,}3\ \Omega;\ 0{,}6\ \Omega;\ 0{,}9\ \Omega\} \\ &L_d \in \{7.10^{-4}\ H;\ 1{,}4.10^{-3}\ H;\ 2{,}1.10^{-3}\ H\} \\ &L_q \in \{1{,}4.10^{-4}\ H;\ 2{,}8.10^{-3}\ H;\ 4{,}2.10^{-3}\ H\} \end{aligned} \qquad (3)$$

The three phases PWM inverter is modeled, in the (d,q) referential, as a gain. The two statistical time delays of the PWM: half of the average switching period T and one more time delay of one sampling period (computation time) are taken into account through a delay margin.

3. MULTI-MODEL LINEAR QUADRATIC SIMULTANEOUS CONTROL ALGORITHM

A classical approach to the stabilization problem of a linear time-invariant system is the Linear Quadratic "L.Q." method, which consists of minimizing a quadratic energetic function of the states and inputs of the system.

The Linear Quadratic Simultaneous "L.Q.S." stabilization is an efficient approach (Anderson and Moor, 1989), (Bonnassieux, 1998) providing stability and robustness with respect to parametric uncertainties.

3.1. Problem formulation

Consider the M Linear Time models Invariant

$$\begin{aligned} &\dot{x}_1(t) = A_i.x_i(t) + B_i.u_i(t) \\ &y_i(t) = C_i.x_i(t) \end{aligned} \qquad i = 1\ldots M \qquad (4)$$

Where:

$$A_i \in \Re^{N_i*N_i},\quad B_i \in \Re^{N_i*N_u},\quad C_i \in \Re^{N_y*N_i}$$

We suppose that:

- (A_i, B_i) is stabilizable and (C_i, A_i) is detectable
- C_i has full rank,

The synthesis problem of an L.Q.S. regulator is defined as follows:

A static output feedback control law has to be found:

$$\forall i, i = 1,\ldots,M,\ u_i(t) = -K_{opt}.y_i(t),\quad K_{opt} \in \Re^{N_u*N_y} \qquad (5)$$

Which:

- Asymptotically stabilizes the M linear time invariant models,
- minimizes the closed-loop quadratic function:

$$I_0(K_{opt}) = \sum_{i=1}^{M}\gamma_i E\left\{\int_0^{+\infty}(x_i(t)^T Q_i x_i(t) + u_i(t)^T R_i u_i(t))dt\right\} \qquad (6)$$

Where $\gamma_i>0$ are scalars and Q_i, R_i are weighting matrices satisfying $Q_i>0$, $R_i>0$ for each i (i=1...M). "E" being the mathematical expectation.

The initial condition vectors x_{i0} are zero mean random vectors with:

$$\forall i, (1 = 1,\ldots,M)\quad E\{X_{0i}.X_{0i}^T\} > 0 \qquad (7)$$

We can note that the following criterion can also be written as:

$$I_0(K_{opt}) = \sum_{i=1}^{M}\gamma_i \mathrm{trace}(P_i.V_i) \qquad (8)$$

Where P_i is the symmetric real non-negative definite solution of the set of Lyapunov equations:

$$\begin{aligned} &(A_i - B_i.K_{opt}.C_i)^T.P_i + P_i.(A_i - B_i.K_{opt}.C_i) \\ &+ Q_i + C_i^T.K_{opt}{}^T.R_i.K_{opt}.C_i = 0 \end{aligned} \qquad (9)$$

3.2. The LQS Algorithms

The "L.Q.S." algorithm is an iterative method based on non-linear optimization It is well known (Blondel et al., 1994)] that there is no theoretical result guaranteeing the existence of static output or full state feedback control stabilizing simultaneous M linear models as given in (3). We propose in this paper two approaches witch converge to a optimal stabilizing solution K_{opt} (if it exists).

3.2.1. "α–stabilization" algorithm

The main problem in a multi-model stabilizing algorithms is to find an initial regulator which stabilizes all considered models. To avoid such difficulty "α-Stabilization" technique is used to shift the eigenvalues of unstable models such that the algorithm could start with a zero regulator. Figure 3 give the different stages of the proposed algorithm, which could be summed up as:

a- Initialize the algorithm by, $K = 0_{Nu*Ny}$

b- Determine the scalar α for which the regulator K asymptotically stabilizes the M models (Schmittendorf, 1989) $(A_i+\alpha I, B_i, C_i)$:

$$\alpha = -\max_{\substack{1\le j\le N_i\\1\le i\le M}}(\mathrm{Re}(\lambda_j(A_i - B_i K C_i))) - \varepsilon \quad (10)$$

With $0 < \varepsilon << 1$

Indeed, α translates the closed-loop eigenvalues in the left haft plant.

c- A local minimum of the $I_0(K)$ criterion is find by the Simplex method (Nelder and Mead, 1965). This leads to a new regulator K_m which α_m stabilizes the M systems with $\alpha_m<\alpha$ (Bourles, 1986.)

d- Go to step (b) and repeat this algorithm till α=0. Then The regulator K_{opt} stabilizes the original models (A_i, B_i, C_i)

This algorithm has been successfully tested on several examples (Bonnassieux, 1998). However, it rather slow to converge and it could stop at a local minimum.

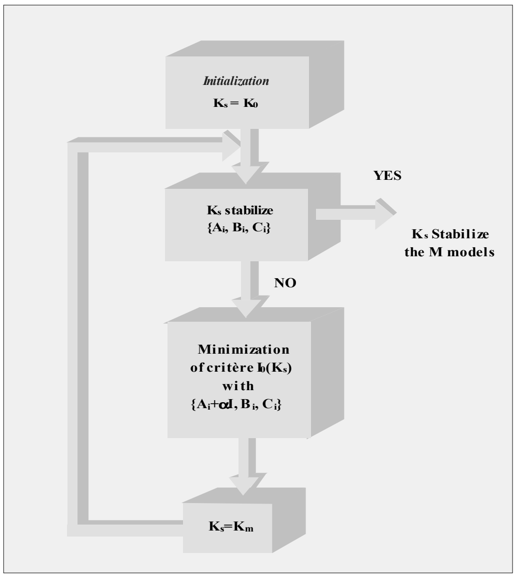

Figure 3 :α-stabilization algorithm

3.2.2. "Simulated annealing" algorithm

Another approach can be used to define the initial stabilizing regulator: the simulated annealing. This is a Monte Carlo approach for minimizing such multivariate functions.

a- Define the initial regulator $K=0_{Nu*Ny}$ and a high temperature $T_{ini}=10^9$.

b- Define the cost function:

$$\mathrm{Cost\_N} = \max_{\substack{1\le j\le N_i\\1\le i\le M}}(\mathrm{Re}(\lambda_j(A_i - B_i K_s C_i))) \quad (11)$$

The simulated annealing process lowers the temperature by slow stages until the system "freezes" and no further changes occur. At each temperature the algorithm must proceed long enough to minimize the cost function. This is known as thermalization.

c- At each temperature, a new regulator is constructed by imposing a random displacement.

$$K_s=K_s+2*0.1*\xi_{Nu*Ny} \quad (12)$$

ξ_{Nu*Ny} is a matrix with elements $\xi_{i,j}$ where

$-1 < \xi_{i,j} <1 \qquad i=1,\dots,N_u \;\; j=1,\dots,N_y$

- If the cost function of this new state is lower than that of the previous one, the change is accepted unconditionally and the system is updated.
- If the cost function is greater, the new configuration is accepted probabilistically with the condition defined by (13). This is the Metropolis step, the fundamental procedure of simulated annealing

$$\exp\left(\frac{\mathrm{Cost\_N}(j) - \mathrm{Cos\_N}(j+1)}{T}\right) < \xi \quad \forall \xi \in [-1, 1] \quad (13)$$

This procedure allows the system to move consistently towards lower cost function, yet still `jump' out of local minima due to the probabilistic acceptance of some upward moves. If the temperature is decreased logarithmically, simulated annealing guarantees an optimal solution: an optimal L.Q.S. regulator K_{opt}

Moreover the time for convergence is much less that the α-stabilization algorithm. For several examples, the computation time has been divided by a factor 100.

4. REGULATOR SYNTHESIS

4.1. Closed-loop objectives

First, we note that, using the Park model of a synchronous drive, the torque is linear of I_q when the current I_d is zero.

$$\Gamma = p[(L_d - L_q).I_d + \Phi_f].I_q \cong p.\Phi_f.I_q \quad (14)$$

Thus, in the sequel, instead of tracking the motor torque, we will control the current I_q, I_d being tracked to zero.

To design an efficient robust regulation, we define the following robustness and performance criteria:

- Tracking the step reference without any static error,
- Assuring the satisfaction of settling time and overshoot for all parametric variations.
- Achieve a delay margin of greater than 1.5 switching periods.
- Achieve a modulus margin higher than 6 dB to assure robustness with respect to non-linearities.
- Stability robustness of the closed-loop for the system modes despite parametric uncertainties and low switching-sampling frequencies.
- Robustness with respect to the neglected dynamics.

4.2. Topology of the robust controller

Since the controller has to be implemented in real time, its topology must be as simple as possible. The reference tracking (I_q, I_d) criterion imposes an integrator on each error (ε_q, ε_d). The gain K is therefore a 4*2 real matrix as could be seen in Figure 4.

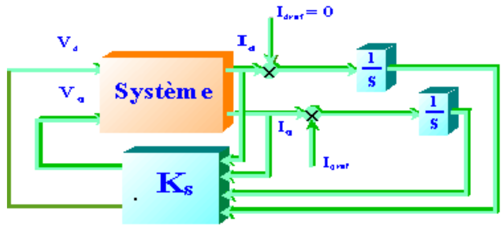

Figure 4: Topology of our controller

4.3. The M parametric models

To satisfy all design criteria, an iterative approach is used to introduce at each step the worst-case model. For our application only three parametric models were necessary to synthesize a stabilizing optimal controller for the whole set of 27 models.

These three models $\forall i, (1 = 1,2,3)$ $(\mathcal{A}_i, \mathcal{B}_i, \mathcal{C}_i)$ correspond to the following parameter values:

$$\begin{aligned} \text{Model n°1} &= \left[R_s = 0{,}6\Omega \;;\; L_d = 1{,}4\text{mH}, \;;\; L_q = 1{,}4\text{mH}\right] \\ \text{Model n°2} &= \left[R_s = 0{,}3\Omega \;;\; L_d = 0{,}7\text{mH} \;;\; L_q = 4{,}2\text{mH}\right] \\ \text{Model n°3} &= \left[R_s = 0{,}9\Omega \;;\; L_d = 1{,}4\text{mH} \;;\; L_q = 4{,}2\text{mH}\right] \end{aligned} \quad (15)$$

4.3. The performance models

The settling time and the damping ratio are translated by means of two performance models for each parametric model $(\mathcal{A}_i, \mathcal{B}_i, \mathcal{C}_i)$:

- The settling time is imposed when stabilizing the synthesis model $(\mathcal{A}_i+\alpha.I, \mathcal{B}_i, \mathcal{C}_i)$. As we can see in (Figure 5), the impulse response of each output decreases faster than $e^{-\tilde{\alpha}t}$ (the real part of the closed loop eigenvalues is lower than $-\alpha$ (Figure 5)
- The damping ratio is imposed when stabilizing the synthesis model $((j+\beta_i)\mathcal{A}_i, (j+\beta_i)\mathcal{B}_i, \mathcal{C}_i)$ where $\beta_i > 0$ Then, the real part of the closed-loop eigenvalues of the i^{th} system is included on a $(-\xi_i, \xi_i)$ sector, with $\xi_i = \frac{\beta_i}{\sqrt{1+\beta_i^2}}$.

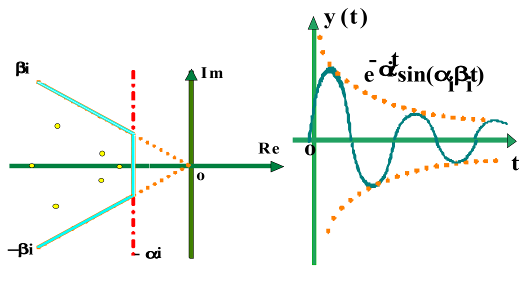

Figure 5: Eigenvalues locus and pulse response of the i^{th} closed loop model.

4.5. The M_i synthesis models

Thus, the LQS method leads to 6 synthesis models (two performance models for each parametric models).

model	γ_i	α_i	β_i
n° 1	1	300	0
n° 1	0.001	0	0.7
n° 2	1	300	0
n° 2	0.001	0	0.7
n° 3	1	300	0
n° 3	0.001	0	0.7

- The γ_i weighting coefficient assures a domination of the settling time ratio over the damping ratio.
- The coefficient α_i assures a settling time near 5 ms.
- The coefficient β_i assures an overshoot lower than 20 %.

4.6. The weighting matrices

For the L.Q.S. method, we choose R_i as the identity matrix. The robustness with respect to neglected dynamics is obtained through the delay margin (Anderson and Moore, 1989). To achieve such requirement the diagonal elements of Q_i matrix associated with the currents are set lower those associated with the regulator dynamics.

$$\forall i = 1, \cdots, M \;;\; Q_i = \begin{bmatrix} 10^{-1}.I_2 & 0_2 \\ 0_2 & 10^2.I_2 \end{bmatrix} \quad (16)$$

5. ANALYSIS AND EXPERIMENTAL RESULTS

5.1. Robustness margins and μ-analysis

To check the efficiency of the designed controller, modulus and delay margins were computed for the 27 parametric models (§II.2).

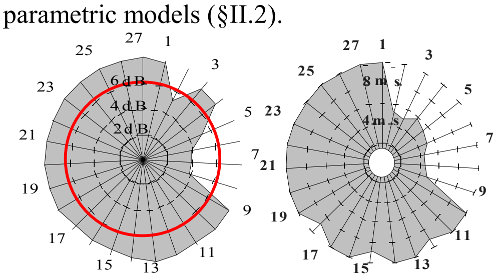

Figure 6: modulus and delay margins

As it can be seen in Figure 6, the modulus margin provides an efficient robustness to non-linearities and is always ≥4 dB. The delay margin (≥4 ms) assures stability robustness despite the PWM and the computation delays.

In a second time, µ-analysis is used for global robustness analysis. Introduced by Doyle in (Doyle,1982), structured singular value, also called µ value, has proved to be a powerful tool for system robustness analysis.

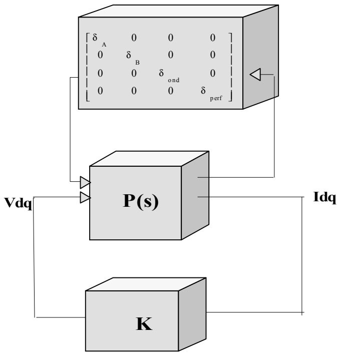

Figure 7: Generalized plant

By linear fractional transformation, we can define for our system and parameters variations, a generalized plant. Where P(s) includes all the known parts of the system and Δ all the uncertainties. Assuming Δ has a particular structure i.e. $\Delta \in E\Delta$ with:

$$E_{\Delta} = \left\{ \begin{array}{l} \Delta = \mathrm{diag}\left(\begin{array}{l} \delta_1 I_{r_1}, \cdots, \delta_r I_{r_r}, \varepsilon_1 I_{c_1}, \cdots, \varepsilon_c I_{c_c}, \\ \Delta_q, \cdots, \Delta_q \end{array} \right) \\ \delta_k \in \mathbf{R}, \varepsilon_k \in \mathbf{C}, \Delta_k \in \mathbf{C}^{r_k x s_k} \end{array} \right\} \qquad (17)$$

Where **R** and **C** are respectively the sets of real and complex numbers.
Structured singular value μ of the system P is defined by

$$\begin{array}{l} \mu(P) = \left(\inf\limits_{\Delta \in E_{\Delta}} \left(\overline{\sigma}(\Delta) : \det(I - \Delta P) = 0 \right) \right)^{-1} \\ \quad = 0 \quad if \quad \det(I - \Delta P) = 0 \ \forall \Delta \in E_{\Delta} \end{array} \qquad (18)$$

μ value is the inverse of the size of the largest uncertainty capable to turn the system unstable. Assuming that Δ is of H_{∞} norm inferior to one, robustness is guarantee if $\mu < 1$.

To build the generalized plant, as defined on Figure 7, we choose 3 pairs of weighting function/uncertainties.

- $\begin{bmatrix} \delta_A & 0 \\ 0 & \delta_B \end{bmatrix}$ Modelized by a multiplicative uncertainty the effect of parametric variations on state matrix A and B.
- The delay of the inverter can take into account by an input multiplicative uncertainty (δ_{ond}) on each voltage V_d and V_q. Then we defined the weighting function $W_{ond}(s)$:

$$W_{ond}(s) = 3{,}2.\frac{1.10^{-3}s}{5.10^{-4}s+1}.I_{2x2} \qquad (19)$$

- The closed-loop system performance can be describe by output multiplicative uncertainties and the weighting function $W_{perf}(s)$:

$$W_{perf}(s) = 1{,}05.\frac{s+1.10^{-3}}{s+1{,}5.10^{-3}}.I_{2x2} \qquad (20)$$

Finally, the following generalized plant is obtained:

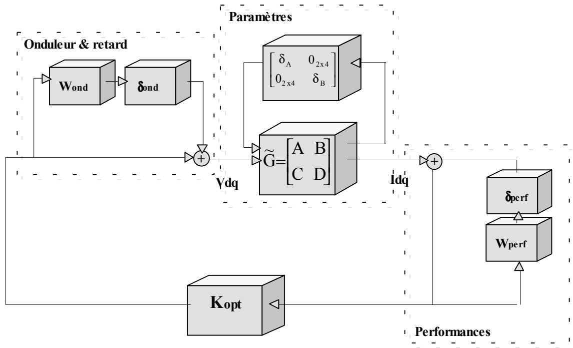

Figure 8: generalized plan with structured uncertainties

On Figure 9 are shown upper and lower bounds of μ for a large scale of angular frequency. Upper bound is less than one, showing that robustness constraints are satisfied.

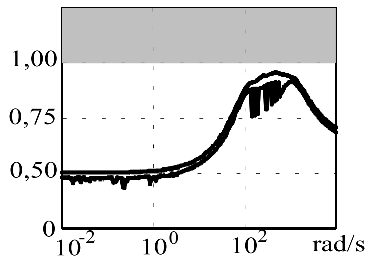

Figure 9: Upper and lower bounds of μ

5.2. The experimental results

The experimental benchmark presented in Figure 10, is composed, for the power part, of a 2.2 kW synchronous servomotor with sinusoidal winding and flux concentrator rotoric structures associated with a three phase MOS inverter (120V, 30A) with 150 ns dead times. Currents are measured by two Hall effect sensors and the mechanical position by a 1024 pt. incremental optical encoder. The control board, built around a 32 bit 40MHz floating point Digital Signal Processor (Motorola DSP96000) and 8 bit Analog-to-Digital converters, Digital-to-Analog converters and 8 bit timers, achieves the PWM MOS-grid signals.

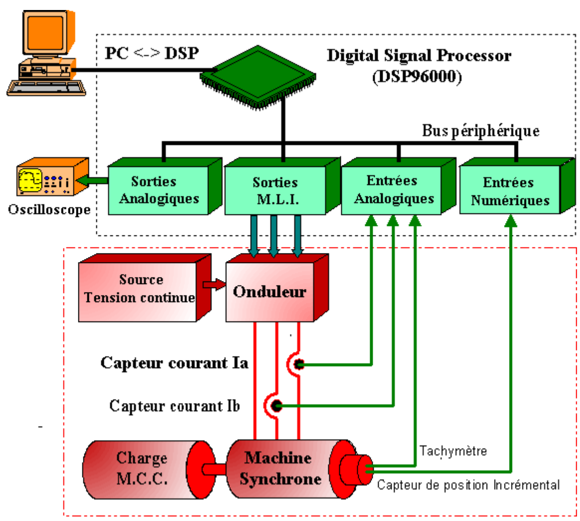

Figure 10: experimental benchmark

Experimental results are shown in Figure 12 & 13. The measured currents are given for two temperatures and two magnetic states. These results prove the good behaviour of the proposed controller.

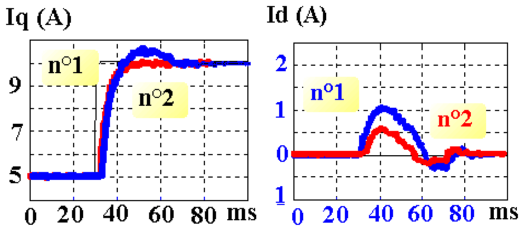

Figure 11: experimental result for currents Iq and Id for 2 thermal states (290°K& 350°K), with magnetic saturation

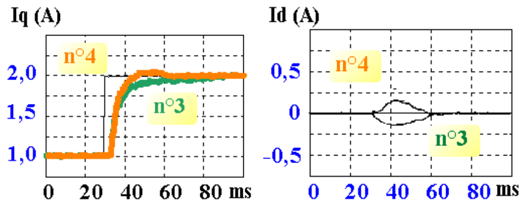

Figure 12: experimental result for currents Iq and Id for 2 thermal states (290°K& 350°K), with linear magnetic state

6. CONCLUSION

The Linear Quadratic Simultaneous method for controlling a synchronous drive associated with a PWM inverter assures good stability, high performance and robustness, despite high parametric variations. The time delay due to the PWM and the computation time has been taken into account.

The simulated annealing algorithm allows a very fast and optimal convergence despite local minima.

To analyze the effects of parameter uncertainties, a generalized model including real uncertainties, suitable for µ-analysis, has been developed. The robustness and performance of this 'L.Q.S.' is checked by µ-analysis. Experimental results show the efficiency of this approach

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