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ADAPTIVE INFORMATION SHARING FACTORS IN FEDERATED KALMAN FILTERING

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Abstract: This paper presents an adaptive determination method of the information sharing factors employed in the federated Kalman filtering algorithm. This approach is based on generalised eigenvalue decomposition of the covariance matrix of the estimated errors associated with individual sensors. The paper begins with a discussion of the structural features and information sharing principle of the Federated Kalman filtering approach. Following development of the new method, simulation results demonstrate its capability to provide a considerable improvement in robustness to changing plant conditions, at the cost of a minimal loss in accuracy under ideal plant behaviour. *Copyright © 2002 IFAC*

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1. INTRODUCTION

The Kalman filter is a statistical estimator which provides a reliable best estimate and error matrix for problems involving the evaluation of sensor readings corrupted by noise. Kalman filtering can be applied in both a centralised and distributed format, both of which have been reported extensively in literature (Lawrence and Berarducci, 1994).

The centralised Kalman filter has been the subject of extensive research and application, particularly in the area of autonomous or assisted navigation (Grewal and Andrews, 1993; Brown and Hwang, 1994). In chemical engineering, the extended centralised Kalman filter, which uses local linearisation to extend the scope of the Kalman filter to systems described by nonlinear ordinary differential equations, is by far the most widely used estimation technique (Yang and Lee, 1997, Leu and Baratti, 2000; Oisiovici and Cruz, 2000). Nowadays, navigation systems and chemical processes almost always employ many sensors and their respective outputs are combined to provide one or more integrated outputs which are enhanced in some sense with respect to their component individual sensor measurements. Integrated multi-sensor systems have the potential to provide high levels of accuracy and fault tolerance. However, that potential has not been fully realisable via application of classical (i.e., centralised) Kalman filtering techniques. Classical techniques applied to multi-sensor systems can yield severe computational loads when implemented in a strictly optimal fashion. Moreover, the results obtained can be subject to poor accuracy, instability, and even divergence under certain operating conditions (Carlson, 1988; Carlson, 1990; Lawrence and Berarducci, 1994).

For these and other reasons, there has been considerable interest in the development of decentralised Kalman filter architectures (Carlson, 1990; Wei and Schwarz, 1990; Lawrence and Berarducci, 1994, Sasiadek and Hartana, 2000). The decentralised Kalman filters consist of one or more sensor-dedicated local filters, generally operating in parallel, plus a master combining filter. The master filter periodically combines the local filter solutions to form a global solution. As such the decentralised Kalman filter provides an effective method of implementing multi-sensor fusion technology. Among the approaches in developing decentralised Kalman filtering, the federated Kalman filtering method provided by Carlson (1988) is a very efficient example. This approach is based on information-sharing principles and applies a rigorous "conservation of information" principle which yields an estimator that has both a high level of accuracy and is tolerant to faults in the system.

Carlson (1988) suggested four different information-sharing strategies for four different design configurations and indicated that the information-sharing factors play a very important role for the performance of federated Kalman filtering. However, current applications have not provided an all-purpose method to decide the information-sharing factors to suit a variety of conditions. This paper presents a new adaptive determination method of the information sharing factors based on eigenvalue decomposition of the covariance matrix of the estimated error in order to enhance the adaptability of the federated Kalman filtering algorithm.

The following section provides an introduction to the federated Kalman filtering algorithm. This is followed with a description of the proposed adaptive determination method of the information sharing factors based on generalised eigenvalue decomposition. The approach is then applied to a simulation of an integrated navigation system and the results compared with those obtained using previously proposed approaches. Finally the conclusions from this work are provided.

2. FEDERATED KALMAN FILTERING ALGORITHM

The standard Kalman filter, which processes the data from different systems in one step, is referred to as a centralized Kalman filter. Gelb (1974) provides full details of this form of filter, which for space limitations is not repeated here.

Decentralised filtering is a two-stage data processing technique which processes data from multi-data systems. In the first stage, each local processor uses its own data to make a best local estimate. These estimates are combined in a parallel processing mode. The local estimates are then fused by a master filter to make a best global estimate of the state vector of the master system.

Federated Kalman filtering is a decentralised filtering algorithm with a two-level structure as shown in Fig 1. The difference between the federated Kalman filter and other decentralised filters is that the former contains an information sharing process. During this process, the total system information is divided among the local filters based on an information-sharing principle. The basic concepts of information-sharing also include that it can perform local time propagation and measurement update processing (adding local sensor information) and it can recombine the updated local information into a new total sum.
The advantages of information sharing, as implemented by the new federated filtering technique, are these:

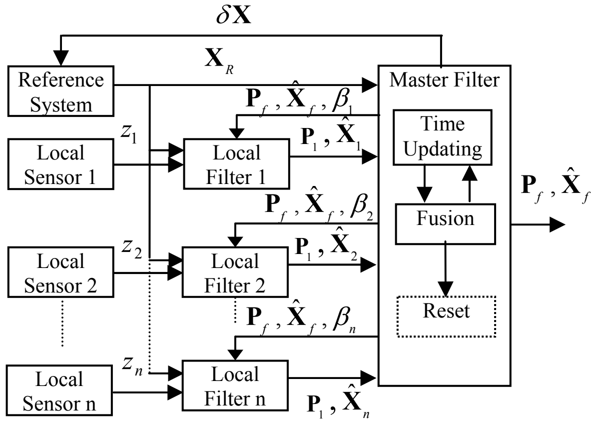

Fig. 1 Federated Kalman filtering architecture

- Using parallel operation of local filters to increase the total amount of data passing through the filtering systems;
- Using local filters for data compression to further increase the amount of data processing;
- Maintaining multiple component solutions usable as back-ups to improve overall system reliability;
- Using a theoretically correct formulation to reduced system development, test and maintenance costs.

The remainder of this section will illustrate how the federated filter applies information-sharing principles to a system with n local filters (LFs) and one master filter (MF), as illustrated in Figure 1.

Consider the following system:

$$x(k) = \Phi(k,k-1)x(k-1) + G(k-1)w(k-1) \quad (1)$$

Where $x(k) \in R^n$ is the state vector, ϕ is the state transition matrix, G is the process noise distribution matrix, and $w(k-1) \in R^r$ is the system noise with covariance matrix Q_{k-1}.

If there are N measurements from separate external local sensor subsystems, then accordingly N local filters are needed to implement the local Kalman filtering in parallel.

The discrete i-th ($i = 1,2,\cdots,N$) local subsystem can be described as:

$$\begin{aligned} x_i(k) &= \Phi_i(k,k-1)x_i(k\text{-}1) + G_i(k-1)w_i(k-1) \\ z_i(k) &= H_i(k)x_i(k) + v_i(k) \end{aligned} \quad (2)$$

Where, $z_i \in R^{m_i}$ is the i-th measurement; $x_i \in R^{n_i}$ is the i-th state vector; H_i is the sensor i measurement observation matrix, $v_i \in R^{m_i}$ is the i-th measurement noise with covariance matrix $R_i(k)$.

The initial state estimate $x_i(0)$ and the sequential values *w(k)* are uncorrelated, as per the following error statistics.

$$E\{x_i(0)\} = x_{i0}\,,\; E\{x_i(0)x_i^T(0)\} = P_{i0} \quad (3)$$

$$E\{w_i(k)x_i^T(0)\} = 0\,,\; E\{v_i(k)x_i^T(0)\} = 0 \quad (4)$$

Because the LF solutions are statistically independent, the measurement noises are uncorrelated as described in equation (5).

$$E\left\{v_i(k)v_m^T(k)\right\} = E\left\{v_i(k)v_m^T(j)\right\} = 0 \quad (i \neq m) \quad (5)$$

Let the full (centralized) filter solution be represented by the covariance matrix P_f and the state vector $\hat{x}_f$; the local filter i solution by P_i and $\hat{x}_i$; and the master filter solution by P_m and $\hat{x}_m$.

Now, if the LF and MF solutions are statistically independent, they can be optimally combined by the following additive information algorithm, where the inverse covariance is the "information matrix":

$$P_f^{-1} = P_1^{-1} + \ldots + P_n^{-1} + P_m^{-1} \quad (6)$$

$$P_f^{-1}\hat{x}_f = P_1^{-1}\hat{x}_1 + \cdots + P_n^{-1}\hat{x}_n + P_m^{-1}\hat{x}_m \quad (7)$$

The key to the federated filtering method is to construct individual LF and MF solutions so they can be combined or recombined at any time by the above algorithm. In particular, the construction avoids the need to maintain local/local or local/master cross-covariances. The procedure for doing so is the essence of the information-sharing approach.

A typical procedure for the specification of a federate Kalman filter can be described as follows:

1). Divide the full (global) filter solution P_f, $\hat{x}_f$ and the common process noise covariance Q_f so that the $i = 1,\ldots,n$ LFs and the MF each receive non-negative fractions β_i, β_m of the total information:

$$\hat{x}_i(k) = \hat{x}_f(k) \quad (8)$$

$$P_i^{-1}(k) = P_f^{-1}(k)\beta_i \quad (9)$$

$$Q_i^{-1}(k) = Q_f^{-1}(k)\beta_i \quad (10)$$

Where β_i, $(i = 1,\ldots,n,m)$ are information-sharing factors. The "conservation of information" principle dictates that the information-sharing factors β_i sum to unity:

$$\sum_{i=1}^{N,m}\beta_i = 1 \quad (11)$$

2). Time propagation process is executed independently for each local filters and master filter:

$$\begin{aligned}&P_i(k+1,k) = \Phi_i(k+1,k)P(k)\Phi_i^T(k+1,k)\\&+ G_i(k)Q_i(k)G_i^T(k)\\&\hat{x}_i(k+1,k) = \Phi_i(k+1,k)\hat{x}_i(k)\\&i = 1, 2, \cdots\cdots, n, m\end{aligned} \quad (12)$$

3). Each local filter uses the measurement from the corresponding local sensor to update:

$$\begin{aligned}&\hat{x}_i(k+1) = \hat{x}_i(k+1,k) + K_i(k+1)\cdot\\&[Z_i(k+1) - H_i(k+1)\hat{x}_i(k+1,k)]\end{aligned} \quad (13)$$

$$\begin{aligned}&K_i(k+1) = P_i(k+1,k)H_i^T(k+1)\cdot\\&[H_i(k+1)P_i(k+1,k)H_i^T(k+1) + R_i(k+1)]^{-1}\end{aligned} \quad (14)$$

$$P_i(k+1) = [I - K_i(k+1)H_i(k+1)]P_i(k+1,k) \quad (15)$$

Where $i = 1,\ldots,n$. Note that there is no measurement information available to update the master filter, therefore:

$$P_m(k+1) = P_m(k+1,k) \quad (16)$$

4). The above results are combined by the fusion algorithm (6) to yield the correct total solution, i.e., the solution that would be achieved by a single centralized filter processing all of the $i = 1,\ldots,n$ sensor measurement sets:

$$\begin{aligned}&P_f^{-1}(k+1) = P_1^{-1}(k+1) + P_2^{-1}(k+1) + \cdots\cdots +\\&P_n^{-1}(k+1) + P_m^{-1}(k+1)\end{aligned} \quad (17)$$

$$\begin{aligned}&\hat{x}_f(k+1) = P_f(k+1)[P_m^{-1}(k+1,k)\hat{x}_m(k+1,k) +\\&\sum_{i=1}^{n}P_i^{-1}(k+1)\hat{x}_i(k+1)]\end{aligned} \quad (18)$$

3. AN ADAPTIVE DETERMINATION METHOD OF INFORMATION SHARING FACTORS BASED ON EIGENVALUE DECOMPOSITION

Information-sharing factors play a very important role in determining the performance of the federated Kalman filter. Choosing different information-sharing factors can change the performance of the filter and thus satisfy different requirements in various conditions.

Carlson (1988) presented four different approaches to specifying the information-sharing factors according to different strategies. These approaches are: Fusion-Reset mode, No-Reset mode, Zero-Reset mode and Rescale Mode respectively. Each mode derives from different performance criteria that suggest a different-sharing strategy. The advantages and disadvantages of each of the approaches are discussed in his paper (Carlson 1988). Unfortunately, there is still no effective method for determining the information-sharing factors to suit a particular application. In practical applications, the conditions of the subsystems can alter greatly over time subject to disturbances. Fixed information sharing factors cannot reflect these changes, resulting in deterioration of the performance of the associated Kalman filters. The research carried out in this paper is aiming to address this issue.

When the information-sharing factors are set as $\beta_1 = \beta_2 = \cdots\cdots = \beta_N = 1/n$, it has been observed from simulation results that the global estimation $\hat{x}_f$ provides the best estimate accuracy when there is no disturbed changes of the local systems (Fang, 1998). The higher the value of a particular β_i value, the larger the contribution that this *i-th* local filter makes to the overall estimate. Therefore, the performance of the federated Kalman filter is closer to the performance of the $i-th$ local filter. During the filter design procedure, it is desired that the total performance of the federated Kalman filter is as close as possible to the performance of the optimal local filter. That is to say to give bigger information-sharing factors to the local filters of the more accurate subsystems, and smaller β_i to the local filters of the less accurate subsystems to reduce their respective influences on the global estimation accuracy. Therefore, adaptive determination of the information-sharing factors according to the estimation accuracy of different subsystems could reflect the change of the estimation accuracy so as to reduce the influences of the faults in subsystems and accuracy degradation.

As a result of the above discussion, the information-sharing factor represents the unitary portion of estimation information from the local Kalman filter in the total fusion estimation. So the basic idea here is to change the information-sharing factor according to the performance of the corresponding local filter and hence to change the proportion of the estimation information from the local filter in the global information. The key issue here is to find an index to measure the performance of the local filter which may be used to determine changes to the information sharing factors.

Given the covariance matrix of *i*-th local filter, a conclusion from Carlson (1990) is provided as follows:

The eigenvalues of matrix P in the Kalman filtering equation represent the covariances of their corresponding state vectors or their combination.

This implies that the estimation performance of the local filter to the state vectors or combination of the state vectors can be obtained by analysing the eigenvalue and eigenvectors of the covariance matrix P_i. The bigger the eigenvalue of P_i, the bigger the estimation covariance of the corresponding state vectors or their combination would be and thus the worse would be their filtering performance. On the other hand, the smaller the eigenvalue of P_i, the smaller the estimation covariance of the corresponding state vectors or their combination, therefore the filtering performance would be better.

In federated Kalman filter equations, the covariance matrix of the *i*-th local filter P_i can be decomposed as:

$$P_i = L\Lambda_i L^T \tag{19}$$

Where $\Lambda_i = diag\{\lambda_{i1}, \lambda_{i2}, \cdots, \lambda_{in}\}$; $\lambda_{i1} \sim \lambda_{in}$ is the eigenvalues of P_i; L is the corresponding eigenvectors matrix.

Because the eigenvalues $\lambda_{i1} \sim \lambda_{in}$ of P_i can be positive or negative, a new problem is how to use it to measure the filter performance. One solution is to use $P_i^T P_i$ to replace P_i to perform the eigenvalue decomposition, i.e., to do the generalised eigenvalue decomposition.

$$P_i^T P_i = L' \Lambda_i' (L')^T \tag{20}$$

Where $\Lambda_i' = diag\{\lambda_{i1}', \quad \lambda_{i2}', \cdots \quad \cdots \quad , \lambda_{in}'\}$.

Obviously,

$$\lambda_{ij}' = \lambda_{ij}^2, \quad j = 1,\ 2\ ,\cdots\cdots,\ n \tag{21}$$

The following information-sharing factors are chosen:

$$\beta_i = \frac{tr\Lambda_i'}{tr\Lambda_1' + tr\Lambda_2' + \cdots tr\Lambda_n' + tr\Lambda_m'} = \frac{tr\Lambda_i'}{\sum_{i=1}^{n,m} tr\Lambda_i'} \tag{22}$$

Where $tr\Lambda_i'$ is the trace of matrix Λ_i', defined as:

$$tr\Lambda_i' = \lambda_{i1}' + \lambda_{i2}' + \cdots\cdots + \lambda_{in}' = \lambda_{i1}^2 + \lambda_{i2}^2 + \cdots\cdots + \lambda_{in}^2 \tag{23}$$

It can be proved that the above-mentioned information-sharing factors still satisfy the "conservation of information" principle so that the information sharing factors β_i sum to unity:

$$\sum_{i=1}^{n,m} \beta_i = 1 \tag{24}$$

At the same time, β_i has an explicit physical meaning, i.e., adaptive change of β_i reflects the on-line performance change of the local filter and also adaptively adjusts the proportion of estimation information which local filters contribute to the overall estimation information.

4. CASE STUDY

Kalman filtering is the most common filter design implemented in integrated navigation system. In addition Federated Kalman filtering has been a key technique in these applications. The advantage of the federated filter architecture is obtained through the sharing of the estimation information by the sensor-dedicated local filters. In a typical integrated Inertial Navigation System (INS)/Global Positioning System (GPS)/Terrain Aided Navigation (TAN) navigation system, the INS serves as the reference system for the local filters and the master filter, while the GPS and TAN systems work as local filters. The local filters receive the measurements directly from the sensors, and then provide the error state information to the master filter for recombination. A typical INS/GPS/TAN navigation profile (Wang, *et al.*, 2000) is used in this paper as the case study of the proposed information sharing method. A comparison of the proposed adaptive Kalman filter, the federated Kalman filter and the centralised Kalman filter is made with respect to filter performance by comparing the filter error state performance plots for each filter implementation. The initialisation data of the simulation model is listed in Table 1. The error sources parameters of the simulation model. Their physical meanings and values are listed in Table 2.

Table 1 Initialisation data of the simulation model

Parameters	Meaning	Value
L_0, λ_0, h_0	Initial position: latitude, longitude, altitude	$N30^\circ$, $E120^\circ$, 500m
V_{E0}, V_{N0}, V_{U0}	Initial velocity: eastward, northward, skyward	100m/s, 0m/s, 0m/s
$\delta L_0, \delta\lambda_0, \delta h_($	Initial errors of position	$100'', 100'', 5m$
$\delta V_{E0}, \delta V_{N0}, ($	Initial errors of velocity	0.6 m/s, 0.6m/s, 0.6m/s
$\phi_{E0}, \phi_{N0}, \phi_{U0}$	Initial errors of angle	$100'', 100'', 300$

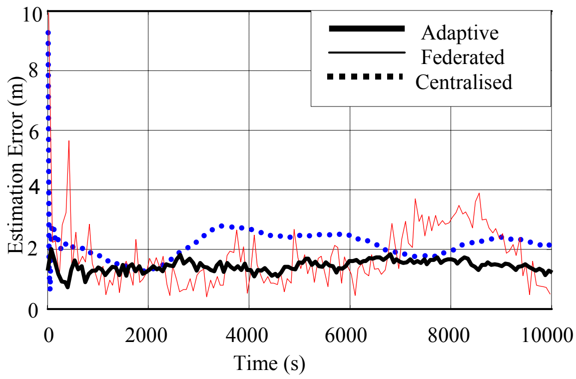

Figure 2 Comparison of positioning errors of INS/GPS/TAN integrated navigation system.

According to the simulation conditions described above an INS/GPS/TAN integrated navigation system was implemented using the adaptive Kalman filtering algorithm described in Section 3. To test the performance of the adaptive Kalman filter, we degrade the GPS receiver accuracy artificially at time 1200s to 2000s by adding a constant value to the incoming position measurement.

The initial information sharing factors are:

$$\beta_m = 0, \quad \beta_1 = 0.7, \quad \beta_2 = 0.3$$

Figure 2 shows the altitude errors of the INS/GPS/TAN integrated navigation system using centralised Kalman filtering, federated Kalman filtering and adaptive Kalman filtering respectively. From the simulation results shown in Figure 2, the proposed adaptive Kalman filter provides comparable estimation accuracy to the optimal centralised Kalman filter when the performance of local filters do not change. When slow performance degradation occurs in the INS/GPS system at time 1200s to 2000s, the estimation errors of the federated Kalman filter increase significantly eventually since it depends on the local estimation information of the degraded GPS system. At the same time, the estimation errors of the centralized Kalman filter increases as the GPS accuracy reduces. By comparison, the proposed adaptive approach controls the influence of faulty sub-systems to the overall estimation and adjusts the information sharing proportion in real-time according to the performance change of the subsystem sensors. Consequently the overall estimation can achieve improved accuracy.

5. CONCLUSIONS

In this paper, the structural features and information sharing principle of federated Kalman filtering have been discussed. To enhance the adaptability of the federated filtering algorithm, an adaptive determination method of the information sharing factors, based on generalised eigenvalue decomposition of the covariance matrix of the estimated error, has been proposed. Simulation results show that the adaptive federated filtering algorithm with adaptive information allocation factors could follow the estimation from the best-performed local filter at any time, thus providing improved accuracy and adaptability over existing approaches.

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Table 2 Error sources parameters of the simulation model

Error Source Parameters	Root-mean-square value	Correlation time (s)
Gyro constant drift $\varepsilon_{bE}, \varepsilon_{bN}, \varepsilon_{bU}$	$0.1^\circ/h, 0.1^\circ/h, 0.1^\circ/h$	
Gyro white noise drift $\omega_{gE}, \omega_{gN}, \omega_{gU}$	$0.01^\circ/h, 0.01^\circ/h, 0.01^\circ/h$	
Gyro first-order Markov drift $\varepsilon_{rE}, \varepsilon_{rN}, \varepsilon_{rU}$	$0.1^\circ/h, 0.1^\circ/h, 0.1^\circ/h$	300s, 300s, 300s
Accelerometer first order Markov zero-offset $\sigma_{aE}, \sigma_{aN}, \sigma_{aU}$	$10^{-4}g, 10^{-4}g, 10^{-4}g$	600 s, 600s, 600s
GPS receiver position error: latitude, longitude, altitude	$20'', 20'', 50m$	100s, 100s, 100s
GPS receiver velocity error (eastward, northward, skyward)	0.3 m/s, 0.3m/s, 0.3m/s	100s, 100s, 100s
Digital map error γ_m	5m	
Terrain linearisation error γ_l	8m	
Electronic Altitude meter error γ_r	5m	
Baric altitude meter error	50m	1000s