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INPUT RECONSTRUCTION BY MEANS OF SYSTEM INVERSION: APPLICATION TO FAULT DETECTION AND ISOLATION *

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Abstract. — This paper attempts to look at the fundamental problem of fault detection and isolation (FDI) in nonlinear systems. Using the idea of input reconstruction by means of dynamic inversion the authors first discuss the properties of input (or fault) observability in linear systems. The extension of the results to nonlinear systems as well as the mathematical conditions of the calculation of the inverse system, which provides the inverse in finite algorithmic steps, are given. The applicability of the inversion process to fault reconstruction in nonlinear systems is demonstrated. *Copyright © 2002 IFAC.*

Key words. — Fault detection and isolation, Input observability, Input reconstruction, Inverse system, Linear and non-linear systems.

1. INTRODUCTION

In the solution of the problem of fault detection and isolation (FDI) the principle of analytical redundancy can be used when direct measurements from the system are not available. One method to infer the component fault status and analytically detect the existence of a fault is to look for anomalies in the plant's output relative to a model-based estimate of that output. Plant models, however, are generally incomplete and inaccurate. Moreover, the fault detection and isolation algorithms often assume the presence of a particular failure mode. These plant dynamics and failure mode modeling errors can either cause a high false alarm rate, or make it difficult to detect the faults. Any robust detection and isolation method that is designed to overcome the problems associated with these modeling errors must be able to distinguish between model uncertainties and fault signals in order to avoid excessive false alarms or missed detections.

One possible approach to robustness relies on the use of models that describe the behaviour of the plant more precisely. This often leads to varying structure, time dependent or nonlinear models whose successful treatment depends on the development of new, more complex theories. To start with nonlinear system models, however, may lead to difficulties not only from the point of view of theoretical complexity but also realizability. Beside of this, one of the underlying problems with the application of nonlinear approaches is that most of the standard results established in linear system theory must be relinquished, even though they comprise the basis of our understanding

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of dynamical systems. Nevertheless, it has been widely recognized already that the application of nonlinear system models is much more a matter of necessity than pure mathematical virtuosity.

In this paper, we approach the FDI problem for nonlinear systems by using the idea of dynamic inversion. The idea is basically relies on the concept of system inversion studied by Silverman (1969) for LTI and also considered by Isidori (1995) for nonlinear systems. It will be shown that, by using this idea, linear and nonlinear problems can be treated in the same theoretical framework and the nonlinear solutions can be given through the generalization of the inversion algorithm originally developed for LTI systems.

2. PROBLEM FORMULATION

Consider the general state space model of the dynamical system subject to multiple, possible simultaneous faults

$$\dot{x}(t) = f(x,u) + \sum_{i}^{m} g_i(x,u)\nu_i$$
$$y(t) = h(x,u) + \sum_{i}^{m} \ell_i(x,u)\nu_i \qquad (1)$$

where f, g, h, ℓ are analytic functions of time with $x(t) \in \mathcal{X} \subset \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$, $y(t) \in \mathbb{R}^m$, the vector valued state, input and output variables of the system, respectively. $\nu(t)$ is the fault signal $(\nu_1, \ldots, \nu_m)^T$ whose elements $\nu_i : [0, +\infty) \to \mathbb{R}$ are arbitrary functions of time. Note that the fault signals ν_i can represent both actuator and sensor faults, in general. The goal is to detect the occurrence of the components ν_i independently from each other and identify the fault component which specifically occurred.

In model-based FDI the fault detection and isolation problem can be characterized as a two step procedure containing: (i) detection and isolation of faults on the basis of the residual signal generated by a filter or detector, (ii) isolation of the fault which is accomplished by using a special logic or hypothesis testing to evaluate the situation.

In our approach we focus on problem area (i) only and going to construct a detector, *i.e.,* another dynamic system with outputs ν and inputs $\vartheta = (\vartheta_y, \vartheta_u)$ that contains the measurements of the signals u, y and possible their time derivatives or integrals. This detector can be thought of in the most general form as

$$\dot{\zeta}(t) = \varphi(\zeta, y, \dot{y}, \ldots, u, \dot{u}, \ldots),$$
$$\nu(t) = \omega(\zeta, y, \dot{y}, \ldots, u, \dot{u}, \ldots) \qquad (2)$$

with the state variable $\zeta(t)$ assuming φ, ω are arbitrary analytic functions of time. The filter reproduces the fault signal at its output that is zero in normal operation of the system, while it differs from zero if a particular fault happened.

The detector should satisfy a number of requirements. It should distinguish among different failure modes ν_i, *e.g.,* between two independent faults in two particular actuators. Moreover, it is aimed to completely decouple the faults from the effect of disturbances and also from the input signals. Note that for LTI systems the filter (2), accomplishing these requirements, traditionally serves as a robust residual generator which assign the fault effects and the disturbances into disjoint subspaces in the detector output space.

There are various ways to follow in generating residuals. Traditional methods are based on the error dynamics of a state observer, see *e.g.,* the geometric design approach initiated by Massoumnia (1986) for LTI systems. The same idea was used by Edelmayer et al. (1997) for LTV, moreover, Hammouri et al. (1999) and De Persis and Isidori (2001) for bilinear and nonlinear systems, respectively. The parity space approaches were discussed in Gertler (1998), the unknown input observer in Chen and Patton (1998) the multiple model and the generalized likelihood ratio approaches in Basseville and Nikiforov (1993), just to mention a few. These approaches are used in a number of situations differing in the assumptions on noise, disturbances, robustness properties and in the specific design methods. For comparison, see some representations in the literature like Mangoubi (1998) and Mangoubi and Edelmayer (2000).

It will be shown in this paper that (robust) residual generation can be viewed as an input reconstruction process what addresses the problem of designing a filter which, on the basis of input and output measurements, returns the unknown inputs (failure modes and disturbance signals) by utilizing the inverse representation of the system, see the explanation of Fig. 1. This idea has first appeared in Szigeti et al. (2001) for LTI systems. One of the advantages of this approach is that

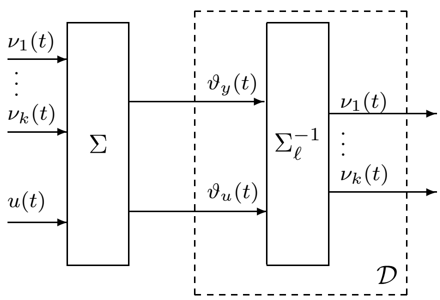

Fig. 1. *Input reconstruction and the idea of system inversion: Σ is the plant, $\mathcal{D}$ is the detector which, most conveniently, can be obtained as the (left) inverse Σ_ℓ^{-1} of the original system*

the extension of the idea to nonlinear systems is possible. Note that this generalization usually cannot be made for the fault detection methods developed to most LTI systems. One special case of the concept with the application to nonlinear systems was published in Szigeti et al. (2000).

It can be seen that, for the reconstruction of unmeasured fault signals at the output of a detector, the property of input observability should be an important quality of the system. Basic issues of input observability for linear systems were discussed earlier such as *e.g.* in Hou and Patton (1998). In this paper we not only review these preliminary results briefly, but bring notice to new properties and give the generalization of the concepts to nonlinear problems.

2.1 *Input (fault) observability for LTI systems*

Input or fault observability of linear dynamical systems are closely related to its invertibility. In order to show this important property consider the representation of the LTI system as

$$\begin{aligned} \dot{x} &= Ax + Bu, \\ y &= Cx + Du. \end{aligned} \tag{3}$$

Definition 1. Consider the dynamical system in (3). If for any two identical outputs $y_1 = y_2$ the corresponding inputs are equal ($u_1 = u_2$) then the system is called input observable.

Remark 1. For any known initial condition $x(0) = \xi$ with $\xi \in \mathbb{R}^n$ input observability implies left invertibility of (3).

Proposition 1. Consider the pairs (u_1, ξ_1), (u_2, ξ_2). Then $y_1 = y_2$ iff $u_1 = u_2$ and ξ_1 and ξ_2 are indistinguishable.

Proof. Assume that system (3) is input observable and consider the representation for the input output pairs (u_1, y_1), (u_2, y_2) as

$$\begin{aligned} \dot{x} &= Ax + Bu_1, \quad x(0) = \xi_1 \\ y_1 &= Cx + Du_1, \\ \dot{x} &= Ax + Bu_2, \quad x(0) = \xi_2 \\ y_2 &= Cx + Du_2. \end{aligned} \tag{4}$$

Subtracting the state and output responses and denoting the state residual $x_1 - x_2$ by $\tilde{x}$ we get:

$$\begin{aligned} \dot{\tilde{x}} &= A\tilde{x} + B(u_1 - u_2) \\ \tilde{x}(0) &= \xi_1 - \xi_2 \\ y_1 - y_2 &= C(x_1 - x_2) + D(u_1 - u_2) = 0. \end{aligned}$$

It follows from the definition that $(u_1 - u_2) = 0$. Then, by differentiation

$$\begin{aligned} y_1 - y_2 = Ce^{At}(\xi_1 - \xi_2) &= 0 \\ CA^i e^{At}(\xi_1 - \xi_2) &= 0, \end{aligned}$$

for $i = 0, \ldots, n-1$. For $t = 0$ we get:

$$CA^i(\xi_1 - \xi_2) = 0.$$

If $y_1 = y_2$ then $u_1 = u_2$ and $(\xi_1 - \xi_2)$ is unobservable (*i.e.,* ξ_1 and ξ_2 are indistinguishable).

On the other hand, if $u_1 = u_2$ and $\xi_1 = \xi_2$ then, by the Cauchy formula, it follows that $y_1 = y_2$. ∎

Proposition 2. Consider the following restriction of the input set:

$$\Omega_o = \{u \in \Omega : \ u(0) = 0, \dot{u}(0) = 0, \ldots, \\ \ldots, u^{(n-1)}(0) = 0\}.$$

Pondering system (3) over the input set Ω_o, left invertibility and input observability are equivalent.

Proof. On the one hand, Remark 1 is true for the restriction Ω_o, *i.e.,* for this case, input observability implies left invertibility. On the other, let us suppose that the system is left invertible. For $y = 0$ and arbitrary x_o we can write

$$\begin{aligned} 0 &= Cx + Du, \\ 0 &= CAx + CBu + D\dot{u}, \\ &\vdots \\ 0 &= CA^{n-1}x + CA^{n-2}Bu + \ldots \\ &\qquad + CBu^{(n-2)} + Du^{(n-1)}, \end{aligned}$$

and for $t = 0$ we get

$$\begin{aligned} 0 &= Cx(0), \\ 0 &= CAx(0), \\ &\vdots \\ 0 &= CA^{n-1}x(0) \end{aligned}$$

which means that $x(0)$ is unobservable. Hence, by $y = 0$, left invertibility implies $u = 0$ which means input observability. ∎

Remark 2. Note that if we work with fault detection problems all derivatives of the fault signals in the diagnostic system models would be zero for $t = 0$ since the condition $\nu(t) = 0$ is always supposed for $t \geq t_o > 0$. This means that the residual system is invertible if and only if it is input observable.

2.2 *Input observability for nonlinear systems*

For generalization of the principles of input observability and left invertibility to more general classes of nonlinear systems we need to consider the following simple properties: Consider the system Σ on Fig 2/a, which is given in algebraic state space representation. Σ is said to be left invertible (that is to say it has a left inverse) if there exist a corresponding system representation on Fig 2/b and a differential algebraic polynomial

$$P(u, \dot{u}, \ldots, y, \dot{y}, \ldots)$$

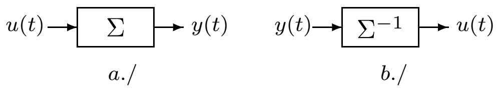

Fig. 2. *The system* Σ *and its inverse representation* Σ^{-1}

such that the composition, shown on Fig 3, will result the identity for each pair (u, y) satisfying $P \neq 0$. For certain classes of nonlinear state space representations one can find algorithms (and also sufficient or necessary conditions) of invertibility, such as *e.g.*, in Isidori (1995).

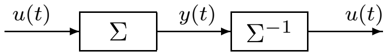

Fig. 3. *Composition of systems* Σ *and* Σ^{-1} *resulting the system of identity*

As the main result of this paper a novel method is proposed that can be viewed as the extension of the procedure described by Isidori. This provides the inverse system Σ^{-1} in some finite $k < m$ algorithmic steps. The construction of the procedure is discussed in the next section.

2.3 *Inversion to fault reconstruction*

Consider the nonlinear system

$$\dot{x} = f(x) + \sum_{i=1}^{m} g_i(x)\nu_i, \qquad u \in \mathbb{R}^m,\ y \in \mathbb{R}^m$$
$$y = h(x), \tag{5}$$

where, in order to make the further notations simpler, neither the known input u nor any other disturbance signals are included in the representation.

The relative degree of (5) is the integer r_i derivatives of $y_i = h_i(x)$, *s.t.*,

$$\begin{aligned} &(i) \quad L_{g_j} {L_f}^k h_i(x) = 0 \quad \text{for} \quad 0 \leq k < r_i - 1 \\ &(ii) \quad \exists_j \quad L_{g_j} L^{r_i - 1} h_i(x) \neq 0. \end{aligned} \tag{6}$$

If the relative degree r_i does not exist *i.e.*,

$$\forall j, k \qquad L_{g_j} {L_f}^k h_i(x) = 0,$$

then r_i equals to $+\infty$.

It can be seen, that the i^{th} output derivatives have the forms:

$$y^{(i)} = L_f^k h_i(x), \quad k = 0, 1, \ldots, r_i - 1,$$
$$y^{(r_i)} = L_f^{r_i} h_i(x) + \sum_{j=1}^{m} L_{g_j} L_f^{(r_i - 1)} h_i(x)\nu_j.$$

Let the vector relative degree $r^1 \in \mathbb{Z}^m$ of (5) be defined as

$$r = (r_1, \ldots, r_m). \tag{7}$$

If the matrix $A(x)$ defined as

$$A(x) = \begin{bmatrix} L_f^{(r_1 - 1)} h_i(x) \\ \vdots \\ L_f^{(r_m - 1)} h_i(x) \end{bmatrix} [g_1(x), \ldots, g_m(x)] \tag{8}$$

is nonsingular, then the inverse of the system can be computed from

$$\begin{bmatrix} {y_1}^{(r_1)} \\ \vdots \\ {y_m}^{(r_m)} \end{bmatrix} = \begin{bmatrix} L_f^{r_1^1} h_1(x) \\ \vdots \\ L_f^{r_m^1} h_m(x) \end{bmatrix} + A(x) \begin{bmatrix} \nu_1 \\ \vdots \\ \nu_m \end{bmatrix} \tag{9}$$

see, Isidori (1995), and this will be referred as a *1-step* algorithm to obtain an inverse. The non singularity of $A(x)$, however is a strong requirement that restricts the possible use of this algorithm. In the rest of this paragraph an extension of this approach will be elaborated. The idea is to construct new output functions and use their derivatives leading to a procedure that generates the inverse in some finite steps. Note that this idea has already appeared in Szigeti et al. (2001).

Suppose now that the matrix $A_1(x) = A(x)$ constructed in this first step is well defined, *i.e.,* each pseudo relative degree is finite, but $A_1(x)$ is singular. Denote the vector relative degree associated to A_1 by $\rho^1 = (\rho_1^1, \rho_2^1, \ldots, \rho_m^1) = r$.

Suppose that $\max_x \operatorname{rank} A_1(x) = d_1$ and the first d_1 rows are linearly independent. Then, there exist a matrix $F_1(x) \in \mathbb{R}^{(m-d_1)xm}$, rank $F_1(x) = (m - d_1)$, with entries $F_{ij}(x)$, $i = 1, 2, \ldots, m - d_1, j = 1, 2, \ldots, m$ that are polynomial functions in $L_{g_j} L_f^{r_i - 1} h_i(x)$ such that

$$F_1(x) A_1(x) = 0. \tag{10}$$

Using the following vectorial notations

$$y^{(r)} = (y_1^{(r_1)}, y_2^{(r_2)}, \ldots, y_m^{(r_m)})^T,$$
$$L_f^r h(x) = (L_f^{r_1} h_1(x), \ldots, L_f^{r_m} h_m(x))^T,$$

one can write

$$F_1(x)(y^{(r)} - L_f^{(r)} h(x)) = 0.$$

These equations will be considered later as additional new output relations. Denote the projection of $\mathbb{R}^m$ onto $\mathbb{R}^{d_1}$ of the first d_1 coordinates, by $P_1 = P_{d_1} : \mathbb{R}^m \to \mathbb{R}^{d_1}$ *i.e.,*

$$P_1(y_1, y_2, \ldots, y_m) = (y_1, y_2, \ldots, y_{d_1}).$$

Then the new output relations will be defined as

$$\begin{bmatrix} P_1 y - P_1 h(x) \\ F_1(x)(y^{(r)} - L_f^{(r)} h(x)) \end{bmatrix} = 0. \tag{11}$$

Next calculate the derivatives of all components of these new output relations up to the inputs appear. This way one can define a second set of relative degrees, *i.e.,* a new *pseudo* vector relative degree denoted by

$$\rho^2 = (\rho_1^2, \ldots, \rho_{d_1}^2, \rho_{d_1+1}^2, \ldots, \rho_m^2).$$

It is clear that the first d_1 elements of ρ^2 are identical to those of ρ^1, since the first d_1 rows of (11) are identical to the original ones in (8).

Define now the matrix $A_2(x)$ such that its first d_1 rows are the same as those rows of $A_1(x)$, but the remaining $m - d_1$ rows are selected from the derivatives of the new output relations. These will have the form:

$$A_2(x,y)_{d_1+k,j} = \sum_{i=1}^{m} (L_{g_j} L_f^{r_k^2-1} F_{ki}(x)(y_i^{(r_i^1)} h_i(x))) -$$

$$-F_{ki} L_{g_j}(x) L_f^{r_i^1+r_k^2-1} h_i(x)$$

where

$$d_2 = \mathrm{rank} A_2(x) \geq \mathrm{rank} A_1(x) = d_1.$$

If $d_1 = d_2 < m$ holds then the system is not invertible. If $d_2 = m$ then the input functions can be obtained in this step from the equation analogous to (9) as

$$\sum_{l=0}^{r^2} \binom{r^2}{l} L_f^l F(x) \otimes (y^{(r^1+r^2-l)} - L_f^{r^1+r^2-l} h(x)) +$$

$$+A_2(x, y^r)\nu = 0 \qquad (12)$$

where the operator $\otimes$ is the Kronecker product, and the procedure stops. The vector relative degree can be written as

$$r^2 = (r_1^2, \ldots, r_{d_1}^2, r_{d_1+1}^2, \ldots, r_m^2),$$

where, for $i = 1, \ldots, m$

$$r_i^2 = \rho_i^2, i = 1, \ldots, d_1; \quad r_{d_1+i}^2 = \rho_{d_1+i}^1 + \rho_{d_1+i}^2.$$

In (12) the following vector notations were used

$$\binom{r^2}{l} = \left[\binom{r_1^2}{l_1}, \ldots, \binom{r_m^2}{l_m}\right], \quad l = (l_1, \ldots, l_m).$$

Remark 3. Assuming the technical hypothesis that for a given k and r_k

$$F_{ki} L_{g_j}(x) L_f^{r_i^1+r_k^2-1} h_i(x) \neq 0,$$

$$L_{g_j} L_f^{r_k^2-1} F_{ki}(x) = 0 \quad \forall i, j,$$

then the definition of A_2 will be replaced by

$$A_2(x)_{d_1+k,j} = -F_{ki} L_{g_j}(x) L_f^{r_i^1+r_k^2-1} h_i(x).$$

If $A_2(x, y^{(r_1)})$ (or $A_2(x)$, resp.) is not invertible but $\mathrm{rank}\, A_2 = d_2 < m$, then it is possible to select its linearly independent rows. Assume that the first d_2 rows are linearly independent (if not, one can permute the rows) and it is possible to define an $(m - d_2) \times m$-dimensional matrix $F_2(x, y^{(r_1)})$ (or $F_2(x)$, resp.) analogously to F_1 in (10). The algorithm continues by defining new output equations analogously to (11). Assume now, that the above algorithm terminates in k steps, *i.e.,* when $d_k = m$. Then the relative degree will be defined as follows.

Definition 2. The (vector) relative degree computed by the above algorithm is the ordered set of integers:

$$r = (r_1^1, \ldots, r_{d_1}^1; r_{d_1+1}^2, \ldots, r_{d_2}^2; \ldots; r_{d_{k-1}+1}^k, \ldots, r_m^k).$$

where for $k \geq 2$,

$$r_i^1 = r_1, i = 1, \ldots, d_1, \quad r_i^j = \sum_{l=1}^{j} \rho_i^j,$$

$$d_j \leq i \leq dj + 1, \quad 2 \leq j \leq k.$$

It can be noticed that the relative degree defined above is not unique, since it depends on the order of selection of the independent original and new output relations. It satisfies, however, that

$$r_1^1 + \ldots r_m^k \leq n. \qquad (13)$$

Remark 4. This relative degree plays the same role as the one defined in (6) in constructing canonical (or normal) forms for the inverse dynamics. The basic difference in the structure of normal forms described *e.g.,* in Chapter 5 of Isidori (1995), when using the coordinates $\Phi(x) = (dh_1, \ldots, L_f^{r_1-1} h_1; \ldots; h_m, \ldots, L_f^{r_m-1} h_m; \ldots, \phi_n)$ is that in our case the output components and their derivatives appear in the state transform. This implies that the normal equations are not explicit, they can, however, be transformed into a matrix pencil form

$$Q(\Phi, y, \dot{y}, \ldots)\dot{\Phi} = CF(\Phi),$$

where $CF(\Phi)$ is a symbol for the usual nonlinear canonic forms consisting of m blocks $(\Phi_2^i, \ldots, \Phi_{rj-1+i^j}, \ldots)$. If the above algorithm generates matrices $A_1(x), A_2(x), \ldots, A_k(x)$, which are functions of the state only, then the matrix pencil Q will depend only on x, *i.e.,* $Q = Q(x)$. If (13) is satisfied with equality, then the system has no zero dynamics as expected.

Example 1. For illustration of the idea, consider the following system representation:

$$\begin{aligned}
\dot{x}_1 &= x_1 + (x_2 - 1)\nu_1, \\
\dot{x}_2 &= x_3 + (x_1 + 1)\nu_1, \\
\dot{x}_3 &= x_2 + (1 + x_1 x_3)\nu_2, \\
y_1 &= x_1, \quad y_2 = x_2, \\
f(x) &= (x_1, x_3, x_2)^T, \\
g_1(x) &= (x_2 - 1, x_1 + 1, 0)^T, \\
g_2(x) &= (0, 0, 1 + x_1 x_3)^T.
\end{aligned}$$

Differentiating the output in the first step ($k = 1$), we get

$$\dot{y}_1 = x_1 + (x_2 - 1)\nu_1,$$
$$\dot{y}_2 = x_3 + (x_1 + 1)\nu_1.$$

It can be seen that the pseudo relative degree is $\rho^1 = (1, 1)$, and the matrix

$$A_1(x) = \begin{bmatrix} x_2 - 1 & 0 \\ x_1 + 1 & 0 \end{bmatrix}$$

is singular. The matrix $F_1(x)$ in (10) can be chosen as

$$F_1(x) = [x_1 + 1, -(x_2 - 1)].$$

In the second step ($k = 2$) define the new output in the form

$$y_3 = (x_1{+}1)\dot{y}_1 - (x_2{-}1)\dot{y}_2 = (x_1{+}1)x_1 - (x_2{-}1)x_3$$

and, by calculating the derivatives we get

$$\begin{aligned}
\dot{y}_1 &= x_1 + (x_2 - 1)\nu_1, \\
\dot{y}_3 &= (2x_1 + 1)(x_1 + (x_2 - 1)\nu_1) - \\
&\quad (x_3 + (x_1 + 1)u_1)x_3 + \\
&\quad (1 - x_2)(x_2 + (1 + x_1x_3)\nu_2) = \\
&\quad (2x_1 + 1)x_1 + (1 - x_2)x_2 - {x_3}^2 + \\
&\quad ((2x_1 + 1)(x_2 - 1) - (x_1 + 1)x_3)\nu_1 + \\
&\quad (1 - x_2)(1 + x_1x_3)\nu_2.
\end{aligned}$$

It follows that the pseudo relative degree is $\rho^2 = (1, 1)$, and the matrix

$$A_2(x) = \begin{bmatrix} x_2 - 1 & (2x_1 + 1)(x_2 - 1) - (x_1 + 1)x_3 \\ 0 & (1 - x_2)(1 + x_1x_3) \end{bmatrix}^T$$

is nonsingular. The relative degree is $r^2 = (1, 2)$. Since the sum of the relative degrees is equal to the state dimension, the inverse has no dynamics and the unknown inputs can be obtained as

$$\nu_1 = \frac{\dot{y}_1 - y_1}{y_2 - 1}, \quad \nu_2 = \frac{\dot{x}_3 - y_2}{1 + y_1x_3},$$

where $x_3 = \dot{y}_2 - (y_1 + 1)(\dot{y}_1 - 1)/(y_2 - 1)$.

3. CONCLUSIONS

In this paper the fault detection and isolation problem for nonlinear systems in view of the fault reconstruction process by means of dynamic system inversion has been discussed. It was shown that a detector relying on the inverse representation of the original system fully reconstruct the failure modes at its output on the basis of standard input and output (sometimes state variable) measurements. The main contribution of this work is an algorithm which can be used for the calculation of the inverse. The procedure can be viewed as a generalization of the *1-step* algorithm proposed by Isidori (1995) for systems represented in canonical normal form. The method proposed by this paper resolves the strong requirement included in this *1-step* algorithm by providing the inverse in some $k > 1$ finite steps thus making the applicability of the method less restrictive in the practice.

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