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INTERCEPT0R MISSILE GUIDANCE - A MATURE SCIENCE OR A NEW CHALLENGE?

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Interceptor missiles designed against aircraft had substantial advantage in speed and maneuverability. Simple guidance concepts and technological progress yielded satisfactory performance, making a mental effort to develop new concepts unnecessary. The Gulf War revealed the threat of tactical ballistic missiles. Missile defense systems, using conventional guidance, demonstrated hit-to-kill accuracy against non-maneuvering targets. Similar performance against future maneuvering targets needs innovative guidance and estimation concepts. This goal can be achieved only if estimation errors against maneuvering targets are minimized. The paper compares the homing performance achieved by using different estimators and raises new ideas for improved estimator design. *Copyright © 2002 IFAC*

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1. INTRODUCTION.

Historically, the target of an interceptor missile was a manned aircraft, against which the missile had substantial advantage in speed, maneuverability and agility. Due to the aircraft vulnerability miss distances of the order of a few meters, compatible with the lethal radius of the missile warhead, were considered admissible. In these circumstances guidance concepts could remain simple, emphasizing heuristics and ease of implementation. Proportional navigation (PN) has been a classical example of this approach, dominating missile guidance for several decades. Later, optimal control theory became fashionable. Following this trend, missile guidance laws were developed using linearized kinematical model and their optimization was based on a linear quadratic optimal control concept (Cottrell, 1971). The optimal control concept requires information on the current and future target maneuvers. In most cases constant target maneuvers were assumed for the sake of simplicity. The information on the target maneuver, not measurable directly, has to be obtained from an estimator. Such guidance law implementation has been based on the *Separation Theorem* (Wohnam, 1968). The progress in the technology lead to improved homing performance and made the mental effort to develop innovative guidance concepts to achieve further improvements unnecessary. Thus, in contrast to the impressive technological progress the concepts of guidance law development remained conservative. For this reason, missile guidance was considered a mature discipline in the late eighties.

The 1991 Gulf War found the free world unprepared and introduced an urgent need to intercept a new type of target, namely the tactical ballistic missile (TBM). Successful interception of a TBM requires a very small miss distance or even a direct hit. The threat motivated an intensive development of several ballistic missile defense (BMD) systems. All of them were designed by using state of art technology, but conventional guidance and estimation concepts. Against non-maneuvering targets flying on predictable ballistic trajectories these systems demonstrated "hit-to-kill" performance.

However, in the future the threat of highly maneuvering TBMs is anticipated. Reentering ballistic missiles fly at very high speeds and their atmospheric maneuvering potential is comparable to that of the interceptors. Since non-maneuvering targets can be easily intercepted, the designer of future anti-surface missiles will make use of their inherent maneuver potential. Not having information on the interceptor, these anti-surface missiles will have to maneuver randomly. Extensive simulation studies (Shinar and Zarkh, 1994; Shinar and Shima, 1995; Shinar *et al.* 1998) showed that currently used guidance and estimation techniques are unable to achieve the required homing accuracy. The new challenge of intercepting highly maneuvering targets with a “hit-to-kill” accuracy requires innovative guidance and estimation concepts.

A new guidance law, motivated by this challenge, was developed recently. It is based on the integration of two *non-orthodox* ideas (Shinar and Shima, 2000). The first one is the formulation of the interception of a maneuvering target as a *zero-sum pursuit-evasion game*

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(Isaacs, 1965). The game solution provides simultaneously the *optimal pursuer strategy* (the missiles guidance law), the *optimal evader strategy* (the "worst" target maneuver) and the *value* of the game (the miss distance guaranteed to both players, if they use their respective optimal strategies). A detailed comparison study, performed more than 20 years ago (Anderson, 1981), clearly stated the superiority of interceptor guidance laws derived from a differential game formulation over those obtained using optimal control theory. Unfortunately, in spite of the impressive results of this comparison study, based on extensive simulations, differential game guidance laws have not been adopted by the missile industry.

The second innovative feature is the relationship between the optimal guidance law and of the estimation process. Realistic interceptor guidance problems are characterized by bounded controls, saturated state variables and non-Gaussian random disturbances. In such problems, the *Separation Theorem* is not valid. Thus, the comfortable design practice based on these principles yields (at the best) suboptimal results. It has been outlined long ago by Witsenhausen (1971) that in stochastic control problems, where the validity of the *Separation Theorem* cannot be proven, the estimator can be designed independently of the control law, but the optimal control law depends on the conditional probability density of the estimated state variables. In other words, in the derivation of the guidance law the results of the estimation process have to be explicitly taken into account. It has been observed (Shinar *et al.* 1998) that the greatest error source in the interception scenario of maneuvering targets is the inherent delay in estimating time varying target maneuvers, due to the non zero convergence time of the estimation process. Based on this observation the estimation process of the target maneuver was modeled as a pure information delay. Using this deterministic model a delayed information pursuit-evasion game was formulated and solved (Shinar and Glizer, 1999) and based on its solution a guidance law was synthesized and succeeded to reduce the detrimental effect of the estimation delay. (Shinar and Shima, 2000)

The integration of these elements, allowing also time varying velocity profiles and maneuver limits, in a new interceptor guidance law (Shima *et al.,* 2001), provided a significant homing accuracy improvement. Nevertheless, it still fell short to guarantee a “hit-to-kill” because of the only partial compensation of the estimation delay and the non zero residual estimation error.

The current new challenge is further improvement in guidance accuracy. There seem to be several options to achieve this ambitious goal. The objective of this paper is to outline a new research effort aimed to achieve a robust “hit-to-kill” homing by identifying an improved estimation scheme for terminal guidance against randomly maneuvering targets.

The structure of the paper is the following. In the next section the interception end game of a maneuvering target is formulated and modeling assumptions are made. It is followed by a brief description of the perfect information game solution. In section 3 the effects of imperfect information are discussed and the homing performance of a delay compensating guidance law in a planar scenario with two types of estimators is presented. The obstacles achieving further improvement and new ideas to overcome them are discussed in section 4. Due to the limited admissible length of the paper many important details had to be omitted. The interested reader can find these details in the references.

2. PROBLEM STATEMENT

2.1 Scenario description

The endo-atmospheric interception of a reentering maneuverable TBM is a complex scenario. The initial trajectory of the reentry vehicle is ballistic, but it can perform evasive maneuvers on its way to its surface target. The maneuverability of the TBM is a function of its velocity and altitude, both time varying. The objective of the BMD system is to intercept and destroy the TBM at a sufficiently high altitude guaranteeing that its (possibly non conventional) warhead does not affect the defended target. After the detection of the reentry vehicle by a ground-based radar, an interceptor missile is launched against it. During the first phase of interception the missile is guided by the ground-based fire control system. As soon as the onboard seeker of the interceptor detects its target, the homing end game starts. Although the TBM has no information on the interceptor, it can perform random evasive maneuvers to make the interception difficult. The seeker of the interceptor measures the relative position of its target, but these measurements are corrupted by noise. Using the noisy measurements the interceptor has to estimate not only the correct position of the target but also its relative velocity and acceleration. The interceptor’s guidance law has to be based on these estimates. The interception end game can be formulated either as an imperfect information pursuit-evasion game, or equivalently as a terminal control problem with bounded controls and noisy measurements, against unknown bounded random disturbances.

2.2 Deterministic model

First, a deterministic model of the scenario, assuming noise free measurements, is given. This model is based on the following assumptions:

(A-1) Both missiles can be represented by point-masses with linear control dynamics.
(A-2) The relative end-game trajectory can be linearized around the initial line of sight.
(A-3) The velocity profiles of both missiles on a nominal trajectory are known and can be expressed as the function of time.

(A-4) The maximum lateral acceleration of each missile is known as a function of time.
(A-5 The maneuvering dynamics of both missiles are approximated by first order transfer functions.
(A-6) The information structure is perfect.

Due to the linearization (A-2), the original three-dimensional scenario can be decoupled into two planar engagements in perpendicular planes. This decoupling results in a significant simplification of the mathematical analysis.

In Fig. 1 a schematic view of the two-dimensional end-game geometry is shown, where the X-axis is aligned with the initial line of sight while the Y-axis is perpendicular to it. Note that the respective velocity vectors of the missiles are generally not aligned with the reference line, but the angles ϕ_P and ϕ_E are small. Tough the approximations $\cos(\phi_i) \approx 1$, $\sin(\phi_i) \approx \phi_i$, $(_{i=P,E})$, are uniformly valid, the longitudinal accelerations of the missiles have non negligible components normal to the line of sight.

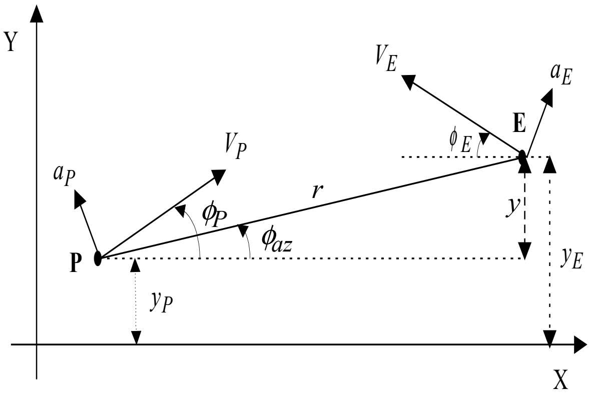

Fig. 1. Planar interception geometry

Based on (A-2) and (A-3) the final time of the interception can be computed for any given initial conditions of the end game, allowing to define the time-to-go ($t_{go} = t_f - t$), which becomes the independent variable of the problem. The assumptions (A-1)–(A.6) allow casting the problem in the canonical form of linear games, from which a reduced order game can be obtained. For planar interception geometry there is only a single state variable, the *zero effort miss distance* denoted as Z, well known in missile guidance analysis.

2.3 Perfect information game solution

The performance index of the deterministic game, to be minimized by the interceptor (*pursuer*) and maximized by the maneuvering TBM (*evader*), is the miss distance. The solution of such a game (Gutman, 1979; Shinar, 1981) results in the decomposition of the reduced game space into two regions of different strategies, as it can be seen in Fig. 2 .

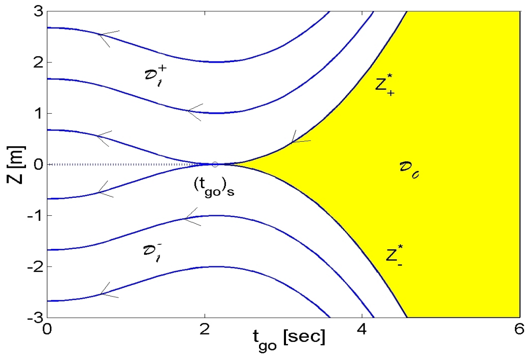

Fig. 2. Decomposition of the reduced game space.

These regions are separated by the pair of optimal boundary trajectories denoted respectively by Z^*_+ and Z^*_-, reaching tangentially the Z=0 axis at $(t_{go})_s$. where $(t_{go})_s$ is the non zero root of the equation $dZ/dt_{go} = 0$. (θ is the t_{go} divided by the *pursuer*'s time constant.)

One of the regions is a *regular* one, denoted by $\mathcal{D}_1$, where the optimal strategies of the players are of the "bang-bang" type

$$\mathbf{u}^* = \mathbf{v}^* = \text{sign}\ \{Z\} \quad \forall \quad Z \neq 0 \qquad (1)$$

u and **v** are the normalized controls of the *pursuer* (interceptor) and the *evader* (the maneuvering target) respectively. The *value* of the game in this region is a unique function of the initial conditions. The boundary trajectories Z^*_+ and Z^*_- also belong to $\mathcal{D}_1$. Inside the other region, denoted by $\mathcal{D}_0$ and defined by $|Z(t)| < Z^*_+(t)$, the optimal strategies are arbitrary and the *value* of the game is constant, depending on the parameters of the game. Note, that the bang-bang strategies (1) are also optimal in $\mathcal{D}_0$. If the parameters of the game are such that the only root of the equation $dZ/dt_{go} = 0$ is zero, than the *value* of the game in $\mathcal{D}_0$ is also zero. Consequently, $\mathcal{D}_0$ is he *capture zone* of the game, guaranteeing a "hit-to-kill".

Unfortunately, his ideal situation is only theoretical. Implementation of the guidance law based on this game solution, denoted as DGL/1, requires knowledge of all the original state variables, including the target lateral acceleration. Since this variable cannot be directly measured, it has to be estimated based on the available noise corrupted measurements. Thus, the information structure becomes imperfect.

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3. IMPERFECT INFORMATION EFFECTS

3.1 Stochastic model

A realistic interception scenario has two elements of imperfect information: the TBM has no information on the state of the interceptor and the interceptor has noisy measurements the relative position of is target. Due to the lack of information, the TBM must maneuver randomly in order to avoid interception. Based on the perfect information game solution, the *optimal* random target maneuver sequence should have a bang–bang structure. The implementation of such a strategy in a short duration end game is a maximal maneuver to one direction followed by a maximal maneuver to the opposite direction, using a randomly timed switch.

The interceptor measures the line of sight angle and its own acceleration. It can be assumed that the range to the target as well as the closing velocity, obtained by the ground based fire control system and up-linked to the interceptor, are rather accurate. Moreover, own acceleration is also accurately measured. The measurement of the line of sight angle is assumed to be corrupted by a zero-mean gaussian noise of a given variance, well suited to classical stochastic analysis.

Unfortunately, the modeling of the random TBM maneuver is much more difficult. In a stochastic control model it plays the role of the *process noise*, but its statistics is unknown. It is not necessarily zero-mean and its probability distribution is certainly not gaussian. Consequently, the *Separation Theorem* is not valid in this case, as it was confirmed by recent simulation studies (Shima *et al.* 2000).

In the stochastic formulation the performance index can either be the expected value of the miss distance, or the probability of successful interception based on the lethality function of the interceptor's warhead. Another measure is the required lethal radius to guarantee a given prescribed probability (say 0.95) of successful interception.

Due to the non-classical stochastic model, the analytical solution of the imperfect information game, - or its equivalent, the finite horizon terminal control problem with noisy measurements and bounded arbitrary disturbance, - is not known. Moreover, the estimator design, necessary for guidance law implementation, is not a straightforward process. The Kalman filter is well known to be the optimal estimator for linear systems, in the sense of minimum variance. The assumption underlying in the filter design is that the system mathematical model, which includes also the target maneuver, is perfectly known. A discrepancy between the actual and the assumed target maneuver creates large estimation errors. A multiple model adaptive estimator (MMAE) can alleviate this problem at the expense of increased computational load, if it is based on a sufficiently large set of suitably chosen target maneuver models. Early identification of the correct target maneuver model will yield smaller estimation errors and consequently smaller miss distances.

3.2 Estimator model

Analyzing the estimator performance by extensive simulation studies with different types of estimators, noise models and random target maneuver structures it was found that the greatest error source in the interception scenario of maneuvering targets is the inherent delay in estimating time varying target maneuvers. Based on this observation, a rough approximation of the estimation process assumed that the evader's lateral acceleration is a delayed perfect outcome, while the estimation of the other state variables is ideal. This modeling assumption allowed a deterministic analysis.

3.3 Delayed information game solution

If the *pursuer* uses the guidance law derived from the perfect information game solution (Shinar, 1981), the *evader* can take advantage of the estimation delay and achieve a large miss distance by adequate optimal maneuvering (Glizer and Shinar, 2001). Therefore, a new *pursuit-evasion game* had to be formulated and solved.

The solution of this *"delayed information"* game, assuming for sake of simplicity constant velocities and lateral acceleration bounds (Shinar and Glizer, 1999, was based on using the idea of *reachable sets* suggested for such problems (Petrosjan, 1993). It yielded a guidance law, denoted as DGL/C, which partially compensates for the inherent estimation delay (Shinar and Shima, 2000). The deterministic analysis guarantees a substantial reduction of the guaranteed miss distance and a robust guidance performance with respect to the target maneuver structure. However, due to the delay the guaranteed miss distance cannot be zero and it is a function of the estimation delay divided by the time constant of the *evader.*

3.4 Validation

The homing performance of the delay compensating guidance law was tested in planar interception scenarios with noisy measurements using two different estimator structures. The scenario was based on a planar constant speed model, selected for the sake of simplicity and reduced computational load.

The first estimator was a "classical" Kalman filter with a shaping filter (KF/SF), driven by white noise representing random target maneuvers in form of a maximum maneuver with random starting time (Zarchan, 1979). This model could match only the assumed variance of the process. The second type of estimator was a computationally efficient MMAE with 30 models, applying the minimum mean square error (MMSE) weighting method (Shima *et al.* 2000).

For a given noise variance and estimator parameters the "worst" target maneuver and the associated estimation delay (to be compensated) were determined. The homing performance was characterized by the accumulative miss distance distribution obtained against the "worst" target maneuver, based on a set of 100 Monte Carlo simulation runs.

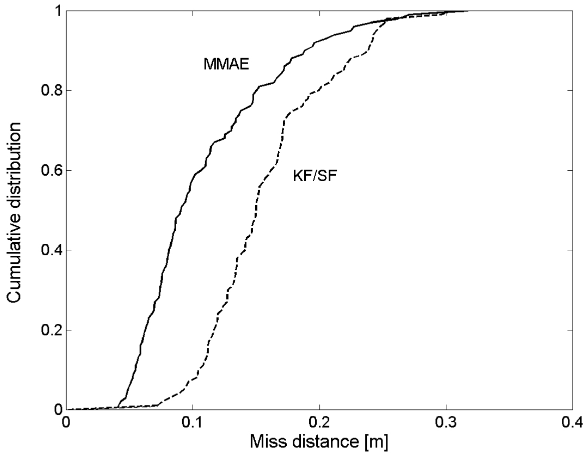

Fig. 3. Homing performance with different estimators

The results of the comparison shown in Fig. 3 indicate the advantage of the MMEA/MMSE (with 30 models) over the KF/SF version. From this figure it seems that distribution is composed of a "deterministic" minimal miss distance, due to the partially compensated delay (as predicted by the solution of the delayed information game and a Rayleigh type distribution characterized by a single parameter associated with the variance of the converged estimation error. Both values are monotonic functions of the noise variance, but depend also on the estimator structure.

An additional validation study performed in a three dimensional generic endo-atmospheric BMD scenario with time-varying velocities and acceleration limits using an advanced KF/SF yielding similar results was presented by Shima *et al.* (2001). Both studies demonstrated the significant improvement obtained by the new guidance law, denoted as DGL/EC, as it can be seen in Fig. 4. However, a "hit-to-kill" homing performance has not been reached because of the only partial compensation of the estimation delay and the non-zero residual estimation error.

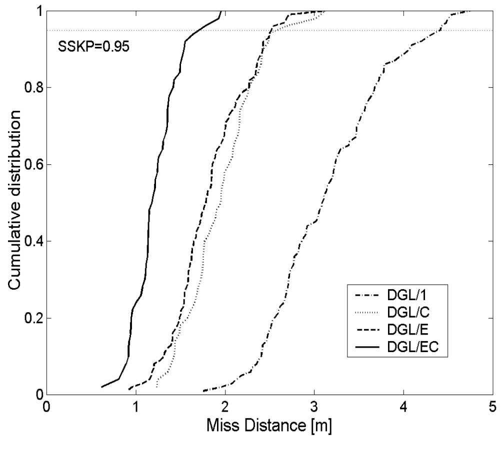

Fig. 4 Cumulative miss distance comparison.

4. DISCUSSION

Extended survey of the estimation literature (the details are out of the limited scope of this paper) indicated that several attempts were made to solve similar problems, where the *Separation Theorem* is not valid. However, none of these can be directly implemented to the interception end game problem under consideration. The basic mathematical approach of solving the original dual stochastic control problem by dynamic programming seems to be, unfortunately, computationally prohibitive.

The results of the earlier mentioned validation studies clearly indicate that for further improvements in guidance accuracy an improved estimator design is required. It is required to minimize both the estimation delay and the variance of the converged estimation error. The convergence time associated with identifying a rapid target maneuver change is composed of the maneuver detection time, and the estimator's response time. Short detection time comes at the price of high false alarm rate, while short response time requires large bandwidth, generating large estimation errors. Moreover, there is a lower bound of the detection time, based on physical characteristics, such as the signal to noise ratio and target maneuver dynamics (Hexner *et al.*, 2001).

A separate estimator design suggested by Witsenhausen (1971) is based on the assumption that the system mathematical model is perfectly known for the filter design. Unfortunately, this assumption is not valid because the actual target maneuver structure is unknown. The validation studies quoted in this paper were carried out against a bang-bang maneuver of an assumed magnitude with the "worst" timing. In a real situation neither the structure, nor the magnitude of the actual maneuver are certain. For this reason, on-line identification of the correct target maneuver, followed by a rapid convergence of the estimated variables, is of primary importance. The experience with MMAE showed that reliable (converged) estimation starts only after correct model identification. An inherent problem of MMAE is that it can only identify models that are included in the set of hypotheses for the design. Fortunately, the efficient MMAE of Shima *et al,* (2000) allows including a large number of different models with an affordable computational load.

There seem to be several options to achieve the ambitious objective of robust "hit-to-kill" homing accuracy by an improved estimation scheme for terminal guidance against random target maneuvers. The first intuitive idea is to find the optimal balance between the contradictory requirements of fast model identification and small (converged) estimation error. The lower bound of the detection time predicted by Hexner *et al.*, (2001) constitutes an inherent limit in this process. Thus, the optimal balance has to be found between a short estimator's response time and small (converged) estimation error, both depending on the estimator's bandwidth.

Should this optimal bandwidth be the same during the interception end game as in a conventional estimator design? The answer to this question is probably negative. In a very recent study (Shinar and Turetsky, 2002) it was shown that an estimator, designed for unbiased optimal tracking, is not the best for an interception end game. The miss distance sensitivity function of a guidance law to estimation errors is a function of the time-to-go and has a maximum at some short time before reaching the point of closest approach. The information on the time-to-go, used in all the advanced guidance laws, can probably be also used to tailor the optimal estimator for a given guidance law. This approach can be particularly attractive if the variance of the measurement noise is also range dependent. It will require an iterative design process because the guidance law has to take into account the estimator performance. In the frame of an ongoing multi-year research effort many other tentative options (such as robust H∞ estimator, dual control, unknown input detection, temporal multiple model, etc.) are considered at least as building blocks for an innovative estimator design for interception end game guidance.

5.CONCLUSIONS

In this paper the requirements towards achieving a robust "hit-to-kill" homing accuracy against highly maneuvering ballistic missiles expected in the future are discussed. It was shown that following the development of advanced (differential game based) guidance laws, the key for success lies with an innovative design of improved estimators. The question asked in the title of the paper is clearly answered. Missile guidance faces today a major challenge, not yet fully appreciated by parts of the professional community. In spite of the impressive technological progress and demonstrated success to find answers to the threat of tactical ballistic missiles presented by the 1991 Golf War, the future threat of maneuvering reentry vehicles, (possibly carrying non conventional warheads) is still unanswered. The above mentioned multi-year research effort hopes to contribute to the solution of this challenging problem.

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