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ROBUST ILC DESIGN IS STRAIGHTFORWARD FOR UNCERTAIN LTI SYSTEMS SATISFYING THE ROBUST PERFORMANCE CONDITION

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Abstract: This paper demonstrates that the design of a robust feedback-based Iterative Learning Control (ILC) is straightforward for uncertain linear time invariant (LTI) systems satisfying the robust performance condition. It is shown that once a controller is designed to satisfy the well known robust performance condition, a convergent updating rule involving the performance weighting function can be directly obtained. It is also shown that for a particular choice of this weighting function, one can achieve a perfect tracking.

Keywords: Iterative learning control, robust control, robust performance condition.

1. INTRODUCTION

Recently, iterative learning control has been generating a considerable amount of interest in the automatic control community [2]. This technique applies to systems that operate repeatedly over a given time-interval. Basically, if we apply the same control law at each operation, the tracking errors will obviously be repeatedly the same. The main idea behind ILC techniques is to take advantage of the previous operations in order to adjust the control input to be applied to the system in the upcoming operations. This allows the controller to perform progressively better with every new operation in order achieve accurate tracking after a certain number of iterations. The ILC control scheme was initially developed as a feed-forward action applied directly to the open-loop system (see, for example (Arimoto *et al.*, 1984), (Kurek and Zaremba, 1993) and (Moore *et al.*, 1992)). However, this control scheme may generate harmful effects if the open-loop system is unstable or an inappropriate initial control law is chosen. To overcome this drawback, several feedback-based ILC schemes have been proposed in the literature, e.g., (DeRoover, 1996), (Moon *et al.*, 1998). To the best of our knowledge, all existing feedback-based ILC algorithms in the literature are based upon the design of the ILC filters and the feedback controller separately. In this paper, we show that once a feedback controller is designed to guarantee the robust performance condition, there is no need to design the ILC filters. Those filters can be directly obtained from the feedback controller and the performance weighting function appearing in the robust performance condition. Consequently, we are benefiting from the robust performance at the first iteration, when the ILC is not effective, and guaranteeing the convergence of the iterative process. We believe that the connection between

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[2] see the survey papers (Moore *et al.*, 1992) and (Moore, 1998) for more details

the ILC convergence condition and the well known robust performance condition established in this paper, will allow the ILC designer to benefit from the wide range of tools from robust control theory, such as loop shaping, model matching, H_∞ and μ-synthesis approaches (Balas *et al.*, 1998; Doyle *et al.*, 1990; Zhou *et al.*, 1996), to solve ILC problems. For the sake of simplicity, single-input single-output plants are considered, but the result can be easily generalized to multivariable systems. Finally, simulation results are given to illustrate the effectiveness of the proposed approach.

2. MAIN RESULT

Consider the feedback system in Figure 1, where the plant G is described in the following multiplicative uncertain form

$$G = (1 + \Delta W_2)G_n, \tag{1}$$

where G_n is the nominal plant model, W_2 is a known stable transfer function, and Δ is an unknown stable transfer function satisfying $\|\Delta\|_\infty \leq 1$. We assume that the reference signal $y_d(t)$ is bounded.

In the sequel the Laplace variable s will be omitted when this does not lead to any confusion. To derive our main result, we need the following lemma

Lemma 1. :(Doyle *et al.*, 1990)
Consider the feedback system in Figure 1, with G as described in (1). The robust performance condition is then

$$\|W_2T\|_\infty < 1 \;\; and \;\; \left\|\frac{W_1S}{1+\Delta W_2T}\right\|_\infty < 1,$$

which is equivalent to

$$\||W_1S| + |W_2T|\|_\infty < 1,$$

where W_1 and W_2 are known stable transfer functions, $S = \frac{1}{1+CG_n}$ is the sensitivity function and $T = 1-S$ the complementary sensitivity function.

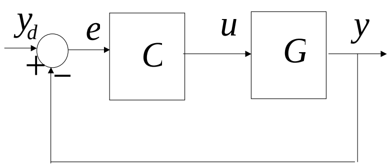

Fig. 1. Feedback system

Now, assume that one is able to design a controller $C(s)$ guaranteeing the robust performance condition

$$\||W_1S| + |W_2T|\|_\infty < 1. \tag{2}$$

If the system in Figure 1 is to be operated repeatedly, the application of the same control input at every operation will lead to the same tracking error over and over. The main idea in ILC techniques is to add another iteratively updated control input v_k to the feedback control variable u_k, as shown in Figure2, in order to ensure that the tracking error $e_k(t)$ converges to a small neighborhood of zero when k tends to infinity, for all t within a given time-interval. The subscript k is introduced to designate the variable at the k^{th} operation.

If the controller $C(s)$ has been already designed to guarantee robust performance condition (2) for the closed loop system, then the design of the iterative updating rule for v_k is straightforward and is given by

$$\begin{aligned} V_{k+1}(s) &= W_1(s)\left(V_k(s) + C(s)E_k(s)\right) \\ &= W_1(s)\left(V_k(s) + U_k(s)\right), \end{aligned} \tag{3}$$

with $V_1(s) = 0$. Where $W_1(s)$ is the performance weighting function involved in the robust performance condition (2), and $E_k(s)$, $V_k(s)$, $U_k(s)$ are respectively the Laplace transforms of $e_k(t)$, $v_k(t)$ and $u_k(t)$. The iterative rule (3) in time domain becomes

$$v_{k+1}(t) = \int_0^t w_1(t-\tau)(v_k(\tau) + u_k(\tau))d\tau,$$

where $w_1(t) = \mathcal{L}^{-1}\{W_1(s)\}$.

If the robust performance condition is satisfied, then the control scheme in figure 2 guarantees the boundedness and the convergence of the tracking error to a fixed value in the sense of the $\mathcal{L}_2$-norm when k tends to infinity. Moreover, the tracking error converges to zero if $W_1 = 1$.

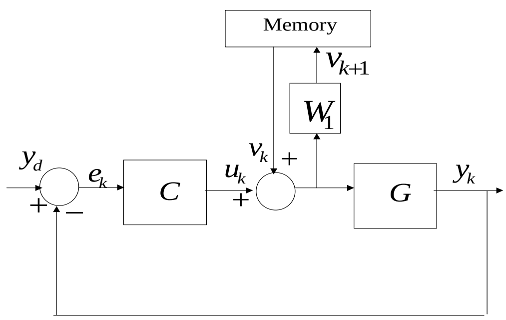

Fig. 2. Feedback-based ILC

Summarizing, we have the following theorem

Theorem 2. Consider the iterative control scheme in figure 2 with the iterative rule (3).
If there exists $C(s)$ such that the robust performance condition (2) is satisfied then the tracking error is bounded and converges uniformly to

$$\begin{aligned} e^*(t) &= \lim_{k\to\infty} e_k(t) \\ &= \mathcal{L}^{-1}\left(\frac{1-W_1}{1-W_1+CG_n(1+\Delta W_2)}Y_d\right). \end{aligned} \tag{4}$$

when $k \to \infty$, in the sense of the $\mathcal{L}_2$-norm.

Proof. From figure 2, the tracking error at the k^{th} iteration is given by

$$\begin{aligned} E_k(s) &= Y_d(s) - Y_k(s) \\ &= \frac{Y_d(s)}{1 + C(s)G(s)} - \frac{G(s)V_k(s)}{1 + C(s)G(s)}. \end{aligned} \quad (5)$$

Hence, the tracking error at the $(k+1)^{th}$ iteration is given by

$$E_{k+1} = \frac{Y_d}{1+CG} - \frac{GV_{k+1}}{1+CG}. \quad (6)$$

Using (3),(5) and (6), we get

$$E_{k+1} = \left(W_1 - \frac{CGW_1}{1+CG}\right) E_k + \frac{1 - W_1}{1+CG} Y_d. \quad (7)$$

Using (1), we get

$$\begin{aligned} E_{k+1} &= \left(\frac{W_1}{1 + CG_n(1 + \Delta W_2)}\right) E_k \\ &+ \frac{1 - W_1}{1 + CG_n(1 + \Delta W_2)} Y_d. \end{aligned} \quad (8)$$

Since $\frac{W_1}{1+CG_n(1+\Delta W_2)} = \frac{W_1 S}{1+\Delta W_2 T}$, equation (8) becomes

$$E_{k+1} = \left(\frac{W_1 S}{1 + \Delta W_2 T}\right) E_k + \frac{1 - W_1}{1 + CG_n(1 + \Delta W_2)} Y_d. \quad (9)$$

Hence,

$$E_k = \left(\frac{W_1 S}{1 + \Delta W_2 T}\right) E_{k-1} + \frac{1 - W_1}{1 + CG_n(1 + \Delta W_2)} Y_d. \quad (10)$$

From (9) and (10), one has

$$E_{k+1} - E_k = \left(\frac{W_1 S}{1 + \Delta W_2 T}\right)(E_k - E_{k-1}). \quad (11)$$

which leads to

$$\begin{aligned} \| \ E_{k+1}(s) - E_k(s)\|_2 &= \|e_{k+1}(t) - e_k(t)\|_2 \\ \leq \left\| \frac{W_1 S}{1 + \Delta W_2 T} \right\|_\infty^{k-1} & \|e_2(t) - e_1(t)\|_2. \end{aligned} \quad (12)$$

Now, under robust performance condition (2), and the fact that y_d is bounded, it is clear that $e_1(t)$ and $e_2(t)$ are bounded. Hence, if

$$\left\| \frac{W_1 S}{1 + \Delta W_2 T} \right\|_\infty < 1, \quad (13)$$

one can conclude that the tracking error converges to $e^*(t) = \mathcal{L}^{-1}\{E^*(s)\}$ given in (4), when k tends to infinity. The limit $E^*(s)$ can be obtained from equation (7), by substituting E_{k+1} and E_k by E^*, and using (1).
According to Lemma 1, condition (13) is guaranteed under the robust performance condition. □

Remark 1:
According to Theorem 2, it is appropriate to take $W_1 = 1$ to ensure zero tracking error when k tends to infinity, and design the controller $C(s)$ satisfying the robust performance condition (2) using the loop shaping, model matching methods (Doyle *et al.*, 1990), or the μ-synthesis approach (Zhou *et al.*, 1996).
If the problem is not solvable[3] with $W_1 = 1$, then take $W_1 \neq 1$, but close to one within the tracking bandwidth, and solve the robust performance condition to obtain the controller $C(s)$.

Remark 2:
As shown in (Tayebi and Zaremba, 2000), one can design $C(s)$ by means of the μ-synthesis approach as follows:
Define the matrix

$$M_1 = \begin{pmatrix} -W_2C_1G_n & W_2C_1G_n \\ -W_1(1 - C_1G_n) & W_1(1 - C_1G_n) \end{pmatrix} \quad (14)$$

where $C_1 = \frac{C}{1+G_nC}$. Hence the upper fractional transformation associated with M_1 and Δ is given by

$$\mathcal{F}_u(M_1, \Delta) = \frac{W_1 S}{1 + \Delta W_2 T}. \quad (15)$$

Note that the ILC convergence condition is nothing else but $\|\mathcal{F}_u(M_1, \Delta)\|_\infty < 1$.
Consider

$$M_2 = \begin{pmatrix} 0 & 0 & W_2 \\ -W_1 & W_1 & -W_1 \\ -G_n & G_n & 0 \end{pmatrix} \quad (16)$$

such that

$$M_1 = \mathcal{F}_l(M_2, C_1), \quad (17)$$

where $\mathcal{F}_l$ denotes the lower fractional transformation (LFT). Now, given W_1, W_2 and G_n, to find C_1 satisfying $\sup_{\omega\in\Re} \mu_{\underline{\Delta}}(M_1(j\omega)) < 1$, one can use the D-K iteration procedure provided in the μ-Analysis and Synthesis Toolbox of Matlab (Balas *et al.*, 1998). Finally, our controller $C(s)$ can be obtained as $C = \frac{C_1}{1-G_nC_1}$.

3. SIMULATION RESULTS

Example 1: Consider the following example given in (Doyle *et al.*, 1990).

$$G_n(s) = \frac{1}{1+s}, \ W1(s) = \frac{1}{s+1}, \ W_2(s) = \frac{0.02s}{0.01s+1}$$

Solving the model matching problem using the spectral factorization and the Nevanlinna-Pick procedure described in (Doyle *et al.*, 1990), we obtain the following controller

$$C(s) = \frac{Q(s)}{1 - G_n(s)Q(s)},$$

[3] The problem is not always solvable as explained in (Doyle *et al.*, 1990), Chapter 6. One necessary condition for robust performance is that $min\{|W_1(j\omega)|, |W_2(j\omega)|\} < 1, \ \forall\omega$

where $Q(s) = \frac{N(s)}{D(s)}$, with

$$\begin{aligned} N(s) &= 0.0523s^4 + 5.9974s^3 + 80.8592s^2 \\ &\quad +402.8644s + 327.9503 \\ D(s) &= 0.0001s^4 + 1.0017s^3 + 17.3984s^2 \\ &\quad +123.7362s + 366.3000. \end{aligned}$$

This controller leads to

$$\| |W_1 S| + |W_2 T| \|_\infty = 0.2030.$$

Figure 3, shows the plot of $|W_1 S| + |W_2 T|$ with respect to frequency.
Considering $y_d(t) = 100sin(\omega t)$, $t \in [0, \frac{2\pi}{\omega}]$, we perform two simulations, with $\omega = 0.1rd/s$ and $\omega = 1rd/s$. In figure 4 and figure 5, one can see the evolution of the Sup-norm of the tracking error with respect to the number of iterations. Figure 6 and figure 7 illustrate the time evolution of the reference trajectory (star) and the output (solid) at different iterations k. We can clearly see that the final tracking error is the smallest with $\omega = 0.1rd/s$, this is due to the fact that W_1 is much closer to one for $\omega = 0.1rd/s$.

Example 2:

$$G_n(s) = \frac{s+1}{s^4 + 14s^3 + 71s^2 + 254s + 120},$$

$$W_1(s) = \frac{1}{0.1s+1}, \quad W_2(s) = \frac{0.02s}{0.01s+1}$$

Using the μ Analysis and Synthesis Toolbox of Matlab (Balas *et al.*, 1998), one can solve the robust performance condition to obtain $C(s) = \frac{N(s)}{D(s)}$, with

$$\begin{aligned} N(s) &= 2.50e-4s^7 + 3.53e5s^6 + 9.44e7s^5 + 1.09e9s^4 \\ &+ 5.33e9s^3 + 1.33e10s^2 + 1.65e10s + 8.24e9, \\ D(s) &= s^7 + 197.86s^6 + 1.86e4s^5 + 6.98e5s^4 \\ &+ 1.89e7s^3 + 9.70e7s^2 + 1.68e8s + 8.97e7. \end{aligned}$$

This controller leads to

$$\| |W_1 S| + |W_2 T| \|_\infty = 0.6556,$$

as shown if figure 10.

We perform a simulation with $y_d(t) = 100sin(0.1t)$, $t \in [0, 20\pi]$. Figure 8 shows the evolution of the tracking error with respect to the iteration number and figure 9 shows the time evolution of the reference trajectory (star) and the output (solid) for $k = 1$, $k = 3$ and $k = 10$.

Example 3:

$$G_n(s) = \frac{24s+70}{s^2+25s+350}, \; W_1(s) = 1, \; W_2(s) = \frac{0.5s+5}{s+100}.$$

Using the μ Analysis and Synthesis Toolbox of Matlab (Balas *et al.*, 1998), one can solve the robust performance condition to obtain the following controller

$$C(s) = \frac{4\;10^6 s + 1.9912\;10^8}{s + 2.4\;10^7},$$

providing

$$\| |W_1 S| + |W_2 T| \|_\infty = 0.6,$$

as shown if figure 13.
We perform a simulation with $y_d(t) = 1 - e^{-2t}$, $t \in [0, 10]$. Figure 11, shows the evolution of the tracking error with respect to the iteration number and figure 12, shows the time evolution of the reference trajectory (star) and the output (solid) for $k = 1$, $k = 3$ and $k = 6$. In this example one can see that the tracking error converges to zero since $W_1 = 1$.

4. CONCLUSION

In this note, we have presented a straightforward derivation of a robust feedback-based iterative learning controller for uncertain LTI systems satisfying the robust performance condition. One of the main objectives of this paper is to establish a connection between the ILC convergence condition and the well known robust performance condition. We believe that this result will allow the ILC designer to benefit from the wide range of tools from robust control theory to solve ILC problems. The filter W_1 appearing in the robust performance condition can be set by the designer according to the ILC performance requirements, i.e., equal to one or close to one within the tracking bandwidth in order to minimize the tracking error when $k \to \infty$. Moreover, the proposed approach guarantees robust performance for the feedback system performing without ILC at the first iteration (i.e., $V_1 = 0$). Consequently, with the design of only one controller $C(s)$, one can guarantee robust performance at the first iteration and the convergence of the iterative process.

REFERENCES

Arimoto, S., S. Kawamura and F. Miyazaki (1984). Bettering operation of robots by learning. *Journal Robot. Syst.* **1**, 123–140.

Balas, G.J., J.C. Doyle, K. Glover, A. Packard and R. Smith (1998). *μ-Analysis and synthesis toolbox*. The Mathworks Inc.

DeRoover, D. (1996). Synthesis of a robust iterative learning controller using an h_∞ approach. *Proc. of the 35th IEEE CDC, Kobe, Japan* pp. 3044–3049.

Doyle, J., B. Francis and A. Tannenbaum (1990). *Feedback control theory*. Macmillan Publishing Co.

Kurek, J.E. and M. Zaremba (1993). Iterative learning control synthesis based on 2-d system theory. *IEEE Trans. on Automatic Control* **1**, 121–125.

Moon, J.H., T.Y. Doh and M. J. Chung (1998). A robust approach to iterative learning control design for uncertain systems. *Automatica* **34(8)**, 1001–1004.

Moore, K.L. (1998). Iterative learning control: An expository overview. *To appear in Applied and Computational Controls, Signal Processing, and Circuits.*

Moore, K.L., M. Dahleh and S. P. Bhattacharyya (1992). Iterative learning control: a survey and new results. *Journal of Robotic Systems* **9(5)**, 563–594.

Tayebi, A. and M.B. Zaremba (2000). Internal model-based robust iterative learning control for uncertain lti systems. *Proc. of the 39th IEEE CDC, Sydney, Australia* pp. 3439–3444.

Zhou, K., J.C. Doyle and K. Glover (1996). *Robust and optimal control.* Prentice Hall, Englewood Cliffs, NJ.

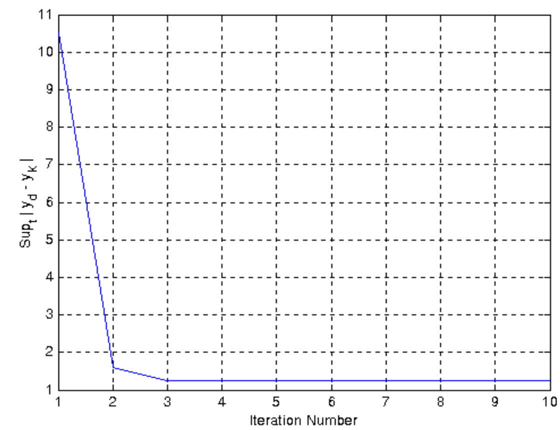

Fig. 5. Example 1, Sup-norm of the tracking error versus the number of iterations, $\omega = 0.1rd/s$

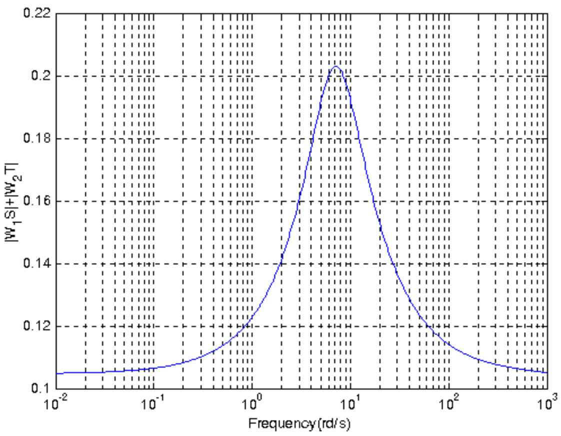

Fig. 3. Example1, plot of $(|W_1S| + |W_2T|)$ versus frequency.

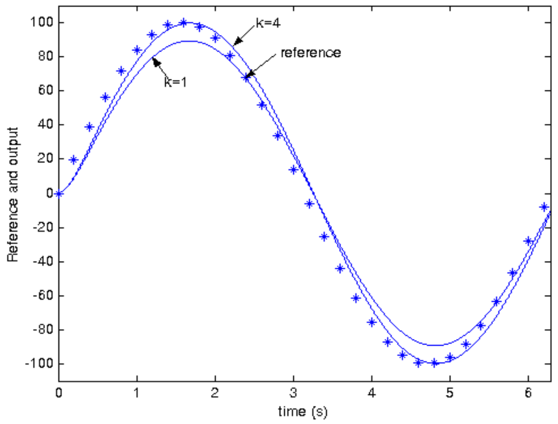

Fig. 6. Example 1, Reference trajectory (star) and output (solid) for $k = 1, 6$, with $\omega = 1rd/s$

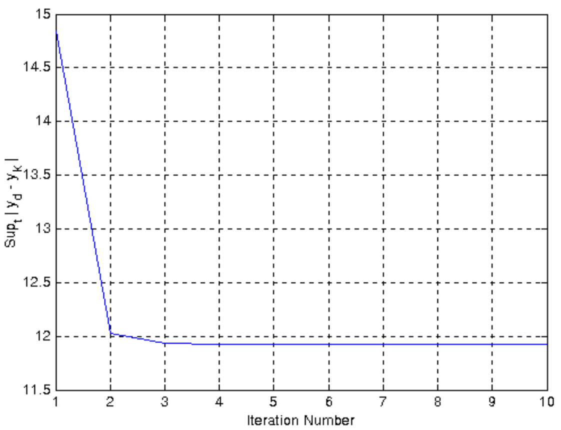

Fig. 4. Example 1, Sup-norm of the tracking error versus the number of iterations, $\omega = 1rd/s$

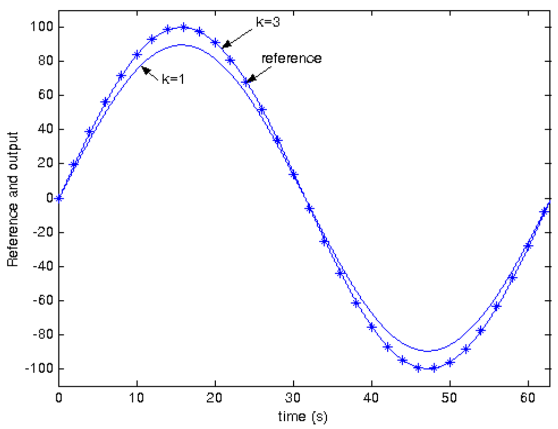

Fig. 7. Example 1, Reference trajectory (star) and output (solid) for $k = 1, 3$, with $\omega = 0.1rd/s$

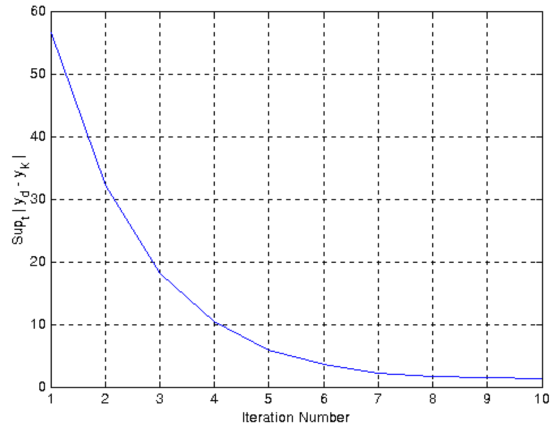

Fig. 8. Example 2, Sup-norm of the tracking error versus the number of iterations.

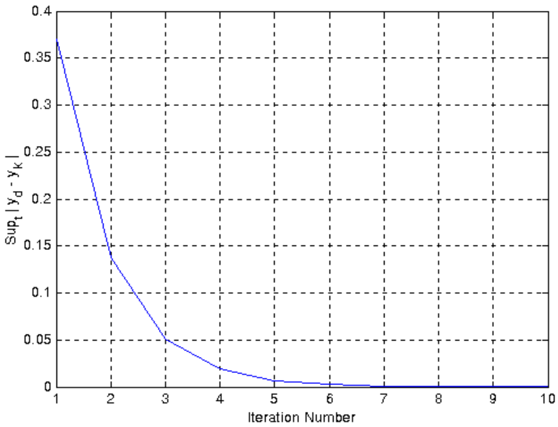

Fig. 11. Example 3, Sup-norm of the tracking error versus the number of iterations.

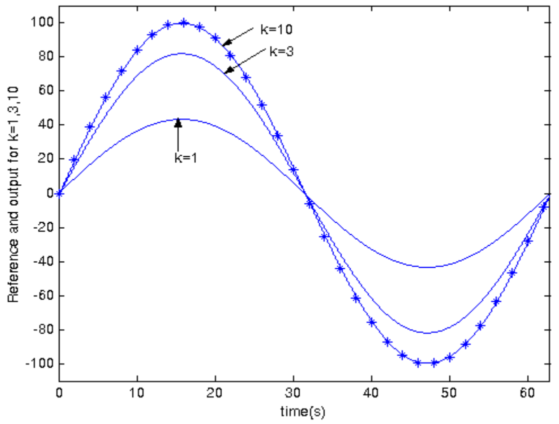

Fig. 9. Example 2, Reference trajectory (star) and output (solid) for $k = 1, 3, 10$.

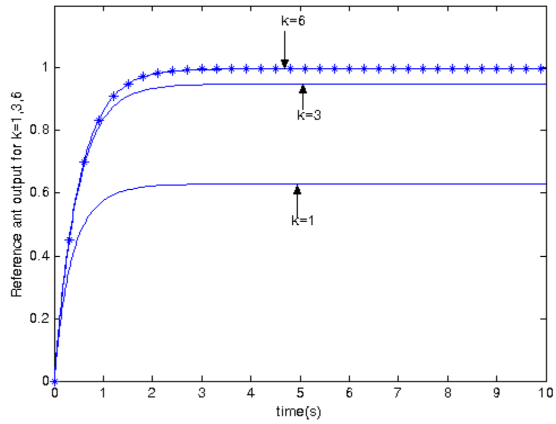

Fig. 12. Example 3, Reference trajectory (star) and output (solid) for $k = 1, 3, 6$.

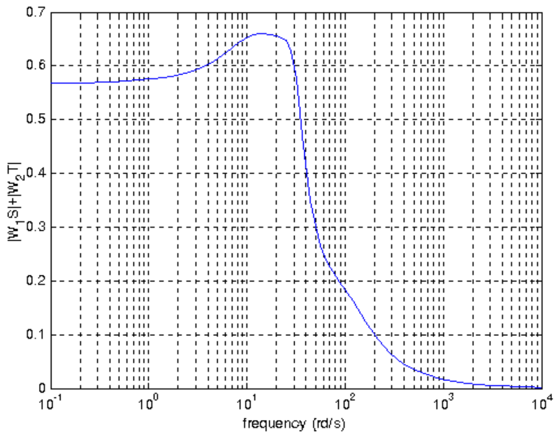

Fig. 10. Example 2, $|W_1 S| + |W_2 T|$ versus frequency.

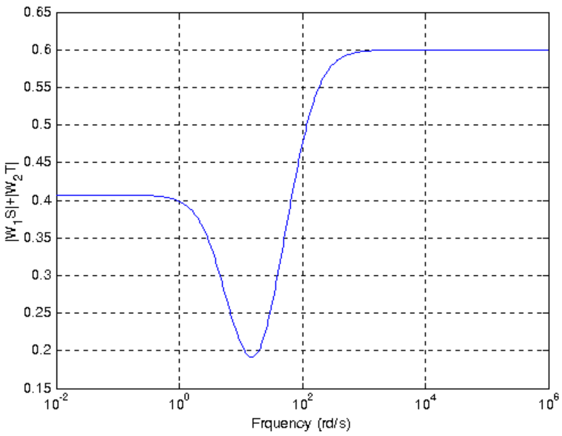

Fig. 13. Example 3, $|W_1 S| + |W_2 T|$ versus frequency.