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ON NONLINEAR INNER SYSTEMS AND CONNECTIONS WITH CONTROL THEORY

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Abstract: This paper extends some results involving linear inner systems to the nonlinear case. In this regard, the arithmetic of nonlinear inner systems is developed further and some new connections with nonlinear spectral and all-pass factorization and control theory are discussed. In particular, explicit formulas for the (state space) realizations of nonlinear inner systems in terms of the components of extremal spectral factors is provided. Relationships between inner systems and process control, geometric control and H_∞-control are also discussed.

Keywords: Nonlinear Inner Systems; Spectral Factorization; Process, Geometric, and H-infinity Control.

1. INTRODUCTION

In the linear setting, inner systems have been discussed in relation to invariant subspaces (Fuhrmann and Gombani, 1998; Fuhrmann and Gombani, 2000) (all-pass and spectral) factorization problems (Finesso and Picci, 1982; Ferrante, *et al.*, 1993; Petersen and Ran, 2001a,b,c,d) and control theory (Ferrante, *et al.*, 1993; Fuhrmann, 1995; Fuhrmann and Gombani, 1998; Fuhrmann and Gombani, 2000; Petersen and Ran, 2001a,b,c,d). In recent contributions on inner-outer factorization for nonlinear systems and the related subject of the spectral factorization problem (Ball and Petersen, 2002; Ball, *et al.*, 2002; Petersen and van der Schaft, 2001; Petersen and van der Schaft, 2002) it has become apparent that a more extensive characterization of nonlinear inner systems and its connection with nonlinear control theory is required. In particular, in Ball and Petersen (2002) a description of an inner system as part of an inner-outer factorization was given in terms of the smooth solution of a certain type of Hamilton-Jacobi equation. Here the system was realized in a manner which proves useful for the extension of the arithmetic of nonlinear inner systems and their factorization. In this paper, recent results (Ball and Petersen, 2002; Petersen and van der Schaft, 2001) are used to derive properties of nonlinear inner systems and consider some connections with control theory (Ball, *et al.*, 2001).

Throughout this paper, the assumption is made that the smooth nonlinear system is of the form

$$\Sigma : \begin{cases} \dot{x} = a(x) + b(x)u, \ u \in \mathbf{R}^m \\ y = c(x) + d(x)u, \ y \in \mathbf{R}^p \end{cases} \quad (1)$$

where $a : \mathbf{R}^n \to \mathbf{R}^n$, $b : \mathbf{R}^n \to \mathbf{R}^{n\times m}$, $c : \mathbf{R}^n \to \mathbf{R}^p$ and $d : \mathbf{R}^m \to \mathbf{R}^{p\times m}$ is a smooth function (at least C^1). Suppose that $p \geq m$ and that $d(x)$ is injective for all x, where $x = (x_1, \ldots, x_n) \in \mathbf{R}^n$ are local coordinates for the n-dimensional state space manifold $\mathcal{M}$, with globally asymptotically stable equilibrium $x_0 = 0$ for $u = 0$ (so $a(x_0) = 0$ and $c(x_0) = 0$). From this it follows that

$E(x) := d(x)^T d(x)$ is invertible for each x. The Hamiltonian extension of Σ (where Σ is given as in (1)) has the form

$$\begin{cases} \dot{x} &= a(x) + b(x)u, \\ \dot{p} &= -[\frac{\partial a}{\partial x} + \frac{\partial b}{\partial x}(x)u]^T p \\ & \quad -\frac{\partial^T c}{\partial x}(x)u_a - u^T \frac{\partial^T d}{\partial x}(x)u_a, \\ y &= c(x) + d(x)u, \\ y_a &= b^T(x)p + d^T(x)u_a, \end{cases} \tag{2}$$

(Crouch and van der Schaft, 1987), where $u, y_a \in \mathbf{R}^m$ and $u_a \in \mathbf{R}^p$. Imposing the interconnection law $u_a = y$ in (2), it follows that the Hamiltonian system is of the form

$$\Phi = [D\Sigma]^T \circ \Sigma : \begin{cases} \dot{x} &= \frac{\partial H}{\partial p}(x,p,u) \\ \dot{p} &= -\frac{\partial H}{\partial x}(x,p,u) \\ y_a &= \frac{\partial H}{\partial u}(x,p,u) \end{cases} \tag{3}$$

with Hamiltonian function $H(x,p,u)$ given by

$$H(x,p,u) = p^T[a(x) + b(x)u] + \frac{1}{2}[c(x) + d(x)u]^T[c(x) + d(x)u]. \tag{4}$$

Here the state space is $T^*\mathcal{M}$, inputs $u \in \mathbf{R}^m$, outputs $y_a \in \mathbf{R}^m$. In this case Σ is known as a spectral factor of Φ. Moreover, we make the assumption that the not necessarily invertible spectral system Φ in (3) is **weakly coercive** if its spectral factors are at least one-sided invertible. In addition, from Petersen and van der Schaft 2002 it is possible to find explicit formulas for spectral factors that are minimum and maximum phase. A minimal realization of the stable, minimum phase spectral factor Σ_- is given by

$$\Sigma_-: \begin{cases} \dot{x} &= a(x) + b(x)u \\ y &= c(x) + b(x)^T(P_x^-(x)^T + P_x(x)^T) \\ & \quad +d(x)u \end{cases} \tag{5}$$

where P^- is the smooth solution of the Hamilton-Jacobi equation

$$P_x^-(x)[a(x) - b(x)E^{-1}(x)d^T(x)c(x)] - \frac{1}{2}P_x^-(x)b(x)E^{-1}(x)b(x)^T P_x^-(x)^T = 0, \tag{6}$$

with $P(0) = 0$ and stability side condition

$$a(x) - b(x)E^{-1}(x)d^T(x)c(x) - b(x)E^{-1}(x)d^T(x)b(x)^T P_x^-(x)^T \tag{7}$$

is Lyapunov stable. A minimal realization of the stable, maximum phase spectral factor Σ_+ is given by

$$\Sigma_+: \begin{cases} \dot{x} &= a(x) + b(x)u \\ y &= c(x) + b(x)^T(P_x^+(x)^T + P_x(x)^T) \\ & \quad +d(x)u \end{cases} \tag{8}$$

where P^+ is the smooth solution of the Hamilton-Jacobi equation

$$P_x^+(x)[a(x) - b(x)E^{-1}(x)d^T(x)c(x)] - \frac{1}{2}P_x^+(x)b(x)E^{-1}(x)b(x)^T P_x^+(x)^T = 0, \tag{9}$$

with $P(0) = 0$ and antistability side condition

$$a(x) - b(x)E^{-1}(x)d^T(x)c(x) - b(x)E^{-1}(x)d^T(x)b(x)^T P_x^+(x)^T \tag{10}$$

is antistable.

From Petersen and van der Schaft, 2001 it is possible to compute a nonsquare, stable nonlinear system as

$$R: \begin{cases} \dot{x} &= a(x) + b(x)u \\ y &= \begin{pmatrix} c(x) + b(x)^T P_x(x)^T \\ \widetilde{c}(x) \end{pmatrix} \\ & \quad + \begin{pmatrix} d(x) \\ 0 \end{pmatrix} u \end{cases} \tag{11}$$

where $\widetilde{c}$ satisfies the equation

$$\frac{1}{2}c(x)^T c(x) - P_x(x)[a(x) - b(x)E^{-1}(x)d^T(x)c(x)] + [c(x)^T + P_x(x)b(x)]E^{-1}(x)b(x)^T P_x(x)^T + \widetilde{c}(x)^T \widetilde{c}(x) = 0. \tag{12}$$

Moreover, if P is any solution of the Hamilton-Jacobi equation (12) then there exists a map X such that

$$\widetilde{c}(x) = -X_x(x)^T P_x(x)^T \tag{13}$$

2. COLUMN AND ROW RIGID SYSTEMS

Concepts that are related to inner systems are the idea of *column rigid* and *row rigid* systems. Consider the Hamiltonian system $\Theta_c^* \Theta_c$ (Hamiltonian extension with $u_a = y$) with Hamiltonian

$$H(x,p,u) = p^T[a(x) + b(x)u] + \frac{1}{2}[c(x) + u]^T[c(x) + u]. \tag{14}$$

Consider the observability function P^o defined as the solution of $P_x^o(x)a(x) + \frac{1}{2}c(x)^T c(x) = 0.$ and define new coordinates $\overline{p} = p - P_x^o(x)$. Then

$$H(x,\overline{p},u) = \overline{p}^T[a(x) + b(x)u] + u^T[b(x)^T P_x^o(x)^T + c(x)] + \frac{1}{2}u^T u. \tag{15}$$

Now, if P^o satisfies $P^o_x(x)b(x) + c(x)^T = 0$ then the submanifold $\overline{p} = 0$ is an invariant manifold, and the system $\Theta_c^*\Theta_c$ restricted to this manifold is given by the static input-output identity map $u \longmapsto y_a = u$. In this case the system Θ_c is said to be **column rigid**.

Next, consider the Hamiltonian system $\Theta_r\Theta_r^*$ (Hamiltonian extension with $u = y_a$) with Hamiltonian

$$H(x,p,u_a) = p^T a(x) + \frac{1}{2}p^T b(x)b(x)^T p \quad (16)$$
$$+p^T b(x)u_a + c(x)^T u_a + \frac{1}{2}u_a^T u_a.$$

Consider the controllability function P^c defined as the solution of
$P^c_x(x)a(x) + \frac{1}{2}P^c_x(x)c(x)c(x)^T P^c_x(x)^T = 0$ and define canonical coordinates $\overline{p} = p - P^c_x(x)$. Then

$$\begin{aligned} H(x,\overline{p},u_a) = {} & \overline{p}^T a(x) + \frac{1}{2}\overline{p}^T b(x)b(x)^T\overline{p} \\ & +P^c_x(x)^T b(x)b(x)^T\overline{p} \\ & -P^c_x(x)^T b(x)u_a \qquad (17) \\ & +c(x)^T u_a + \frac{1}{2}u_a^T u_a. \end{aligned}$$

Now, if P^c satisfies $P^c_x(x)b(x) + c(x)^T = 0$ then the submanifold $\overline{p} = 0$ is an invariant manifold, and the system $\Theta_r\Theta_r^*$ restricted to this manifold is given by the static input-output identity map $u_a \longmapsto y = u_a$. In this case the system Θ_r is **row rigid**.

The next proposition tells us that we can express the rigid systems Θ_c and Θ_r in terms of smooth solutions of the Hamilton-Jacobi equations (6) and (9), respectively, and components of the state space formula for (1).

Proposition 1. Suppose $\Phi = [DR]^T \circ R$ as in (3).

The minimal column rigid system satisfying $R = \Theta_c \circ \Sigma_-$ is given by

$$\Theta_c\colon \begin{cases} \dot{x} = a(x) - b(x)E^{-1}(x)d^T(x)c(x) \\ \quad - b(x)E^{-1}(x)b(x)^T \\ \quad \times (P^-_x(x)^T + P_x(x)^T) \\ \quad + E^{-1}(x)d^T(x)b(x)u \\ y_a = \begin{pmatrix} -d(x)E^{-1}(x)b(x)^T P^-_x(x)^T \\ \widetilde{c}(x) \end{pmatrix} \\ \quad + \begin{pmatrix} I \\ 0 \end{pmatrix} u \end{cases} \quad (18)$$

where P^- is the smooth solution of the (6) with stability side condition (7).

The minimal row rigid system satisfying $\Sigma_+ = \Theta_r \circ R$ is given by

$$\Theta_r\colon \begin{cases} \dot{x} = -a(x)^T + c(x)^T d(x)E^{-1}(x)b(x)^T \\ \quad + (P^+_x(x) + P_x(x))b(x)E^{-1}(x)b(x)^T \\ \quad - \left(P^+_x(x)b(x)E^{-1}(x)d^T(x) \ \ \widetilde{c}(x)^T \right) u \\ y = d(x)E^{-1}(x)b(x)^T + \left(I \ \ 0 \right) u \end{cases} \quad (19)$$

where P^+ is the smooth solution of the (9) with antistability side condition (10).

PROOF. We can compute Θ_c directly from $R \circ \Sigma_-^{-L}$, where R is given by (11) and Σ_-^{-L} is derived from (5).

We note that R is left invertible with R^{-L} being derived from (11). We define $P_r = R \circ \Sigma_+^{-L}$, where Σ_+ is given by (8). Also, we define $\Theta_r = P_r^*$, which shows that Θ_r is rigid. Furthermore, we have to show that $\Sigma_+ = \Theta_r \circ R$. By using the first part with $R = \Theta_c \circ \Sigma_-$ we may conclude that

$$\begin{aligned} \Sigma_+ &= \Theta_r \circ P_r \circ \Sigma_+ \\ &= \Theta_r \circ [R \circ \Sigma_+^{-L}] \circ \Sigma_+ \\ &= \Theta_r \circ \Theta_c \circ \Sigma_- \circ \Sigma_+^{-L} \circ \Sigma_+ \\ &= \Theta_r \circ \Theta_c \circ \Sigma_- \\ &= \Theta_r \circ R \end{aligned}$$

In the linear case, the function Θ_{linear} is said to be *row rigid* if

$$\Theta_{linear}\Theta^*_{linear} = I \ \& \ p \leq m$$

and is *column rigid* if

$$\Theta^*_{linear}\Theta_{linear} = I \ \& \ p \geq m.$$

3. NONLINEAR INNER SYTEMS

Firstly, we provide a general description of a *nonlinear inner system.* We assume that j is any $m \times m$ signature matrix ($j = j^* = j^{-1}$) and J is any $p \times p$ signature matrix ($J = J^* = J^{-1}$).

Definition 2. A nonlinear system Θ is (j, J)-**inner** (or (j, J)**-stable conservative**) if

- the vector field $x \rightarrow a(x)$ is stable (w.r.t. assumed equilibrium point $x = 0$) and
- if there is a nonnegative-valued storage function $P(x)$ with $P(0) = 0$ such that

$$P(x(t_2)) - P((x(t_1)) = \frac{1}{2}\int_{t_1}^{t_2} [\|u(t)\|^2 - \|y(t)\|^2]\, dt \quad (20)$$

over all trajectories $(u(t), x(t), y(t))$ of the system.

Alternatively, Θ is said to be (j, J)-*inner* if it is lossless with respect to the L_2-gain supply rate

$$s(u, y) = \frac{1}{2}u^T ju - \frac{1}{2}y^T Jy.$$

The above characterization of nonlinear (j, J)-inner systems from Ball and Petersen (2002), was achieved within the dissipative systems framework of Hill-Moylan-Willems (Willems, 1972; Hill and Moylan, 1980). Here the dissipation equality in (20) may be derived from a state space-implementation of the L_2-gain condition in the formulation of the nonlinear H_∞-problem. Note that the function defined in (20) may also be thought of as a Lyapunov function (see Hill and Moylan, 1980). If P is assumed to be smooth, the energy balance relation (20) can be expressed in infinitesimal form as

$$\begin{aligned} P_x(x)b(x) + c(x)^T Jd(x) = 0 \\ P_x(x)a(x) + \frac{1}{2}c(x)^T Jc(x) = 0. \\ d(x)^T Jd(x) = j \end{aligned} \tag{21}$$

In fact, realizations for nonlinear invertible (j, J)-inner systems may be expressed in terms of smooth solutions of Hamilton-Jacobi equations as follows

$$\Theta : \begin{cases} \dot{x} = a(x) + b(x)u, \\ y = -b(x)^T P_x(x)^T + d(x)u, \end{cases} \tag{22}$$

where $u \in R^m$, $y \in R^p$ and P is a solution of the Hamilton-Jacobi equation

$$\begin{aligned} &P_x(x)[a(x) - b(x)E^{-1}(x)d^T(x)c(x)] \\ &+\frac{1}{2}c^T(x)[I_p - d(x)E^{-1}(x)d^T(x)]c(x) \\ &-\frac{1}{2}P_x(x)b(x)E^{-1}(x)b^T(x)P_x^T(x) = 0, \end{aligned} \tag{23}$$

with $P_x(x) = \left(\frac{\partial P}{\partial x_1}(x), \ldots, \frac{\partial P}{\partial x_n}(x)\right)$ and $P(0) = 0$. The situation in which E is not invertible is dealt with when we discuss the connection between nonlinear (j, J)-inner systems and nonlinear optimal control in Petersen, 2001 (see also Ball, *et al.*, 2001). For ease of calculation, in the sequel, we put $j = J = d(x) = I$.

In this section, we study the embedding of rigid systems in inner ones. Before we proceed with the statement of the first important result in this section, we establish some notation to be used in the sequel. Suppose that we wish to extend a column rigid system Θ_c given by (18) by an appropriate column rigid system $\Theta_c^{'}$. This we do in order to obtain a system $\Theta^{'}$ that is inner. On the other hand, we wish to extend a row rigid system Θ_r given by (19) by an appropriate row rigid system $\Theta_r^{'}$ in order to obtain a inner system $\Theta^{''}$.

Theorem 3. Suppose that P is a smooth solution of the Hamilton-Jacobi equation (23). Then the following hold.
Suppose that Θ_c is a column rigid system as given in (18), that is the interconnection $\Theta_c^*\Theta_c$ can be associated with the identity map. Then there exists a column rigid system $\Theta_c^{'}$ such that the inner extension $\Theta^{'}$ is given by

$$\Theta^{'} : \begin{cases} \dot{x} = a(x) - b(x)E^{-1}(x)d^T(x)c(x) \\ \quad - b(x)E^{-1}(x)b(x)^T \\ \quad \times (P_x^-(x)^T + P_x(x)^T) \\ \quad + \left(-E^{-1}(x)d^T(x)b(x) \;\; X_x^-(x)^T\right)u \\ y = \begin{pmatrix} -d(x)E^{-1}(x)b(x)^T P_x^-(x)^T \\ \widetilde{c}(x) \end{pmatrix} \\ \quad + \begin{pmatrix} I & 0 \\ 0 & I \end{pmatrix} u \end{cases} \tag{24}$$

where P^- is a smooth solution of the (6) with stability side condition (7) and X^- satisfies (13). Inner $\Theta^{'}$ and column rigid Θ_c have the same drift-dynamics vector field a.

Suppose that Θ_r is given as in (19), that is the interconnection $\Theta_r\Theta_r^*$ can be associated with the identity map. Then there exists a row rigid system $\Theta_r^{'}$ such that extension $\Theta^{''}$ given by

$$\Theta^{''} : \begin{cases} \dot{x} = -a(x)^T + c(x)^T d(x)E^{-1}(x)b(x)^T \\ \quad + (P_x^+(x) + P_x(x))b(x)E^{-1}(x)b(x)^T \\ \quad - \left(P_x^+(x)b(x)E^{-1}(x)d^T(x) \;\; \widetilde{c}(x)^T\right)u \\ y = \begin{pmatrix} d(x)E^{-1}(x)b(x)^T \\ X_x^+(x) \end{pmatrix} + \begin{pmatrix} I & 0 \\ 0 & I \end{pmatrix} u \end{cases} \tag{25}$$

is inner, where P^+ is a smooth solution of (9) with stability side condition (10) and X^+ satisfies (13). Inner $\Theta^{''}$ and row rigid Θ_r have the same drift-dynamics vector field a.

PROOF. In order to prove this result we make use of the description of row rigid and column rigid systems given in Proposition 1 of Section 2.

It is clear that any minimal, inner extension of column rigid Θ_c given by (18) will be of the form (24). Furthermore, by considering (23), (13) and (9) we can check that the vector field $x \to a(x) - b(x)E^{-1}(x)d^T(x)c(x) - b(x)E^{-1}(x)b(x)^T(P_x^-(x)^T + P_x(x)^T)$ is stable and that $P_x^-(x)^T + P_x(x)^T$ satisfies the energy balance relation given in (20).

The proof of the second part can be obtained from the first part by "duality" considerations. Of course, any minimal, inner extension of row rigid Θ_r given by (19) will be of the form (25).

The results determined in this section are generalizations of the results obtained by Fuhrmann and co-workers (Fuhrmann, 1995; Fuhrmann and Gombani, 1998; Fuhrmann and Gombani, 2000) to the nonlinear case.

4. CONNECTIONS BETWEEN NONLINEAR INNER SYSTEMS AND CONTROL THEORY

4.1 *Process Control*

An application of the theory of nonlinear inner systems to chemical process control was discussed in Ball, *et al.* (2001). Here the inner-outer factorization of noninvertible nonlinear systems in continuous time is considered. Our approach is via a nonlinear analogue of spectral factorization which concentrates on first finding the outer factor instead of the inner factor.

4.2 *Geometric Control*

In the linear case, Beurling's Theorem suggests that inner functions are intimately related to the geometry of invariant subspaces in Hardy spaces. In turn, this leads to many geometric relations and is closely related to geometric control theory. For instance, in observation problems the dual of the disturbance decoupling problem (DDP), the simplest application of geometric control theory (Wonham, 1974) is the disturbance decoupling estimation problem (DDEP), studied by Schumacher (Schumacher, 1979). In the problem of disturbance decoupling by observation feedback (PDDOF) one is compelled to study (A, B)- and (C, A)-invariant subspaces simultaneously (Schumacher, 1979; Willems and Commault, 1991). A characterization of these invariant subspaces is given through a study of the operation of output injection. Does one have an analogue of this in the nonlinear case ? Well, duality is not a notion that carries over nicely to a nonlinear setting. However, Isidori (2001) describes a new differential geometric approach to the problem of detection and isolation of faults that does not use the concept output injection explicitly. The nice feature of his work is that important elements of this differential geometry may be exploited to consider a connection between the inner systems discussed in this paper and nonlinear geometric control. This issue is discussed in more detail in Petersen and van der Schaft (2002).

4.3 *Nonlinear H_∞-Control*

A number of papers have shown how a solution of the H_∞-control problem can be obtained from a smooth solution of a Hamilton-Jacobi equation for the state feedback case (van der Schaft, 1992; James, 1993) or (at least locally) from smooth solutions of a coupled pair of Hamilton-Jacobi equations for the measurement feedback case (Isidori and Astolfi, 1992a; Isidori and Astolfi, 1992b; Helton and James, 1999). Moreover, in Ball and van der Schaft, 1996 a similar type of result was established via a (j, J)-inner-outer factorization procedure for the case of disturbance feedforward. This was achieved within the dissipative systems framework of Hill-Moylan-Willems (Willems, 1972; Hill and Moylan, 1980). The aforementioned framework has recently been generalized by James (James, 2001; also James, 1993) to include L^∞ criteria and a mixed L^∞ and integral criteria. The generalized dissipation property is characterized in terms of a partial differential inequality (in the viscosity sense). These new results enables one to make connections with robust control (mixture of L^∞-bounded/integral robust control design). The solutions of the Hamilton-Jacobi equations mentioned above need not be smooth. In fact, there is a theory of viscosity solutions (also sub- and supersolutions) which has been very succesful in characterizing value functions for optimization problems/games and for storage functions. Thus James' work becomes important when studying the storage functions that are associated with inner systems and there relationship with nonlinear H_∞-control.

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