

PERFORMANCE ANALYSIS AND EVALUATION OF AR AND PAR ALGORITHMS FOR PREDICTION OF CYCLOSTATIONARY SIGNALS

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Abstract: In this paper, the performance of Auto-Regressive (AR) and Periodic Auto-Regressive (PAR) algorithms when used to predict cyclostationary signals is analyzed and evaluated. Both analytical and computer simulation results indicate that when predicting cyclostationary signals, the PAR predictor significantly outperforms the AR predictor at the expense of higher computational complexity. Various trade-offs between performance improvement and the knowledge of certain signal characteristics as well as computational efficiency are thoroughly investigated. For implementation purposes, a new adaptive algorithm for realizing the PAR predictor is proposed and its performance has been evaluated by means of computer simulations. *Copyright © 2002 IFAC*

Keywords: Signal Processing, Cyclostationary Signals, Auto-Regressive Predictor, Periodic Auto-Regressive Predictor, Software Defined Radio, 3G UMTS

1. INTRODUCTION

In order to provide improved multimedia services and to enable the global and seamless roaming for mobile users, the 3rd-Generation Universal Personal Telecommunication Systems (3G UMTS) must achieve a high degree of flexibility and adaptation even at the physical level (Wiesler, 1999). The radio transceivers with software-defined functionality in every architectural level, commonly referred to as Software Defined Radio (SDR) transceivers, are considered as fundamental components of the proposed 3G UMTS (Wolmarans, 2000). Compared to more conventional transceivers (Zangi, 1999), such a SDR transceiver requires a very wideband and very high speed Analog-to-Digital Converter (ADC). Furthermore, in order to handle the significant Radio Frequency (RF) power differences over the required wide RF bandwidth, the ADC requires impracticably high resolution to satisfy the Signal-to-Noise Ratio (SNR) requirement for the weakest signal in the receiving band (Salkintzis, 1999). As a result, the inherent inability of the commercially available ADC to perform well for such emerging wireless telecommunication systems is one of major technical challenges which could hinder the application of the SDR techniques in the base station of the 3G UMTS (Zangi, 1999). In (Nie, 1999), we have proposed a novel digitization method, which we refer to as *Adaptive Prediction and Cancellation Digitization* (APCD), and which can significantly reduce the high dynamic range at the input of the ADC. This dynamic range reduction is achieved by applying appropriate signal prediction techniques to remove the strong correlation contained in the received signal. In the same reference, it has been shown that,

by using the APCD method in conjunction with Auto-Regressive (AR) and Periodic Auto-Regressive (PAR) prediction algorithms, the stringent ADC resolution requirements for 3G UMTS applications can be relaxed significantly.

Since the SNR improvements offered by the APCD method are mainly determined by the prediction gain achieved by the signal prediction techniques, how to design a prediction algorithm to achieve higher prediction gain becomes an interesting and important research topic. For any prediction algorithm, its performance heavily depends on the statistical characteristics of the signal to be predicted. It has been widely recognized that most RF signals, such as for example GMSK, BPSK and QPSK signals, encountered in 3G UMTS are cyclostationary signals. This means that for certain values of the lag parameter \mathbf{t} , their autocorrelation functions are periodic functions, and can be mathematically expressed as:

$$r_x(t, \mathbf{t}) \stackrel{\Delta}{=} E[x(t)x^*(t + \mathbf{t})] = r_x(t + \Gamma, \mathbf{t}) \quad (1)$$

where $\Gamma \neq 0$ (Gardner, 1994). It should be pointed out that although the basic concepts of the AR and the PAR models are well-known and have been studied extensively in the past (Haykin, 1996 and Gardner, 1990), to the best of our knowledge, the performance analysis of the AR and the PAR algorithms when used to predict cyclostationary signals has not been published in the open technical literature. In this paper we will analyze and compare the performance of the AR and the PAR algorithms when used to predict cyclostationary signals. Moreover, the analysis results will be verified by means of computer simulations.

The organization of the paper is as follows. After this introduction, Sections 2 will describe the AR predictor and analyze its performance degradation. Section 3 will introduce the PAR predictor and analyze its performance improvement over the AR predictor. The various performance evaluation results obtained by computer simulations together with detailed discussion are summarized in Section 4. Finally the conclusions of the paper are contained in Section 5.

2. AUTO-REGRESSIVE (AR) PREDICTOR

As most RF signals to be digitized by the SDR receiver are modulated real signals, which become real discrete signals after sampling, here we will focus our analysis on such real discrete signals. For this family of signals, their autocorrelation function can be expressed as:

$$r_{xx}(n, m) \stackrel{\Delta}{=} E[x(n)x(n+m)] = r_{xx}(n+P, m) \quad (2)$$

where $P \neq 0$. Because $r_{xx}(n, m)$ is a periodic function with a period of P , it can be decomposed into the sum of a series of sinusoidal waves with frequencies of k/P , where $k \in \{0, 1, \dots, P-1\}$ ¹, and k/P are referred to as the cycle frequencies of the cyclostationary signal $x(n)$. To facilitate our analysis, a mathematically more convenient expression for Eq. 2 is used as follows:

$$r_{x_i x_j}(l) \stackrel{\Delta}{=} E[x_i(n)x_j(n+l)], \forall \text{ integer } l \quad (3)$$

where

$$x_k(n) \stackrel{\Delta}{=} x(nP+k). \quad (4)$$

In many applications, for mainly reasons of simplicity, when no prior knowledge about the statistical characteristics of the signal to be predicted is available, the signal is assumed to be a stationary signal. Hence, the AR predictor is a simple but effective signal prediction algorithm (Haykin, 1996). However, since the AR predictor is based upon the stationary model, its performance may degrade when it is used to predict cyclostationary signals.

2.1 Algorithm Description

A L^{th} -order AR predictor can be expressed as:

$$y_{AR}(n) = W_{AR}^T X(n) \quad (5)$$

where $W_{AR} = [w_{AR1}, w_{AR2}, \dots, w_{ARL}]^T$ are the prediction coefficients, $X(n) = [x(n-1), x(n-2), \dots, x(n-L)]^T$ are the L past samples of the signal to be predicted, and T denotes transposition. When $x(n)$ is a cyclostationary signal, it is easy to show that the mean square of the prediction error $e_{AR}(n)$ is also a periodic function, i.e.

$$E[e_{AR}^2(n)] = E[(x(n) - y_{AR}(n))^2] = E[e_{AR}^2(n+P)]. \quad (6)$$

¹ Unless otherwise noted, from here on we assume that $k \in \{0, 1, \dots, P-1\}$.

It is mathematically convenient to use the following notations: $\overline{X}_k(n) \stackrel{\Delta}{=} X(nP+k)$, $\overline{e}_{AR-k}(n) \stackrel{\Delta}{=} e_{AR}(nP+k)$, $R_k = E[X_k(n)X_k^T(n)]$, and $P_k = E[x_k(n)X_k(n)]$. To achieve the highest prediction gain, W_{AR} should minimize J_{AR} where

$$J_{AR} = E\left[\sum_{k=0}^{P-1} e_{AR-k}^2(n)\right] = \sum_{k=0}^{P-1} \{E[x_k^2(n)] + W_{AR}^T R_k W_{AR} - 2P_k^T W_{AR}\}. \quad (7)$$

Setting $\partial J / \partial W_{AR} = 0$, the following optimal solution for W_{AR} can be obtained:

$$W_{AR}^{op} = \left[\sum_{k=0}^{P-1} R_k\right]^{-1} \left[\sum_{k=0}^{P-1} P_k\right], \quad (8)$$

and the minimum value for J_{AR} is given by

$$J_{AR}^{\min} = \sum_{k=0}^{P-1} E[x_k^2(n)] - \left[\sum_{k=0}^{P-1} P_k^T \left[\sum_{k=0}^{P-1} R_k\right]^{-1} \left[\sum_{k=0}^{P-1} P_k\right]\right]. \quad (9)$$

Furthermore, we note that the highest prediction gain for $x(n)$ is given by

$$G_{PAR}^{\max} = \frac{\sum_{k=0}^{P-1} E[x_k^2(n)]}{\sum_{k=0}^{P-1} E[x_k^2(n)] - \left[\sum_{k=0}^{P-1} P_k^T \left[\sum_{k=0}^{P-1} R_k\right]^{-1} \left[\sum_{k=0}^{P-1} P_k\right]\right]}. \quad (10)$$

2.2 Algorithm Performance

For any RF signal with cyclostationary characteristics, it has been well established that although both its carrier signal and its equivalent baseband signal can contribute to its cyclostationary characteristics, as compared to the contribution of the carrier signal, the contribution of the equivalent baseband signal can almost be ignored (Garnder, 1994). Thus, we can reasonably assume that the equivalent baseband signal is a stationary signal. Furthermore, according to *Wold's Decomposition Theorem* (Haykin, 1996), any stationary signal can be decomposed into an AR process of an appropriate order, and a deterministic process, i.e. the process can be completely determined by its own past. Since theoretically the deterministic process does not affect the performance of the predictors, we can omit it and represent any stationary signal with an AR process when we analyze the performance of the AR predictor. As a result, in order to obtain an analytical performance measure for the highest prediction gain G_{PAR}^{\max} of the AR predictor employed to predict RF signals, we have used the following generalized cyclostationary signal:

$$x_c(n) = \sum_i u_i(n) \cos\left(\frac{pK_i}{P_i} n + f_i\right) \quad (11)$$

where $u_i(n)$ are AR processes, f_i are constants taking values between 0 and 2π , and $K_i/P_i = 2f_{ci}/f_s$, where f_{ci} are the frequencies of the carrier signals. According to Nyquist's sampling theorem, $f_s > 2f_{ci}$, thus K_i/P_i takes values between 0 and 1.

For the simplicity of the mathematical analysis, but without loss of generality, in this paper, $x_c(n)$ has been further simplified to $x_G(n)$ as following, which only has one carrier signal and one 1^{st} -order AR process:

$$x_G(n) = u(n) \cos\left(\frac{pK}{P}n + f\right) \quad (12)$$

where $u(n)$ can be expressed as:

$$u(n) = Cu(n-1) + v(n) \quad (13)$$

where $-1 < C < 1$ and $v(n)$ is a stationary, zero mean, memory-less Gaussian random process with $E[v^2(n)] = P_v$. Thus, the autocorrelation function of $x_G(n)$ is given by:

$$E[x_G(n)x_G(n+m)] = \frac{C^m P_v}{2(1-C^2)} \left\{ \cos\left[\frac{pK}{P}(2n+m) + 2f\right] + \cos\left(\frac{pK}{P}m\right) \right\}. \quad (14)$$

Clearly, when $K/P \neq 0$, $x_G(n)$ is a cyclostationary signal which has only one cycle frequency of K/P . Furthermore, when $K/P = 0$ then $x_G(n) = u(n)$. It has been shown in (Haykin, 1996) that because $u(n)$ is a 1^{st} -order AR process, the optimal signal predictor for $u(n)$ is a 1^{st} -order AR predictor given by:

$$y_u(n) = Cx_u(n-1). \quad (15)$$

In the same reference it is also shown that increasing the order of the AR predictor will not increase the highest prediction gain of $u(n)$, G_{pu}^{\max} , which is:

$$G_{pu}^{\max} = \frac{1}{1-C^2}. \quad (16)$$

However, when $K/P \neq 0$, based upon Eqs. 8-10, the highest prediction gain of $x_G(n)$, G_{pAR}^{\max} will decrease seriously as compared with G_{pu}^{\max} . Meanwhile, increasing the order of the AR predictor will increase G_{pAR}^{\max} . For example, if the 1^{st} -order AR predictor is employed to predict $x_G(n)$, according to Eq. 8, the optimal predictor is given by:

$$y_{GAR-1}(n) = C \cos\left(\frac{pK}{P}\right) x_G(n-1), \quad (17)$$

and according to Eq. 10, the highest prediction gain G_{pAR-1}^{\max} is given by:

$$G_{pAR-1}^{\max} = \frac{1}{1-C^2 \cos^2\left(\frac{pK}{P}\right)}. \quad (18)$$

If the 2^{nd} -order AR predictor is employed to predict $x_G(n)$, according to Eq. 8, the optimal predictor is given by:

$$y_{GAR-2}(n) = \frac{C \cos\left(\frac{pK}{P}\right) (1-2C^2 \cos^2\left(\frac{pK}{P}\right) + C^2)}{1-C^2 \cos^2\left(\frac{pK}{P}\right)} x_G(n-1) + \frac{-C^2 \sin^2\left(\frac{pK}{P}\right)}{1-C^2 \cos^2\left(\frac{pK}{P}\right)} x_G(n-2) \quad (19)$$

and according to Eq. 10, the highest prediction gain

G_{pAR-2}^{\max} is given by:

$$G_{pAR-2}^{\max} = \frac{1-C^2 \cos^2\left(\frac{pK}{P}\right)}{1-2C^2 \cos^2\left(\frac{pK}{P}\right) + 2C^4 \cos^2\left(\frac{pK}{P}\right) - C^4}. \quad (20)$$

From Eqs. 18 and 20, as a function of K/P , the highest prediction gains of $x_G(n)$ for the 1^{st} - and the 2^{nd} -order AR predictors are calculated and shown in Fig. 1.

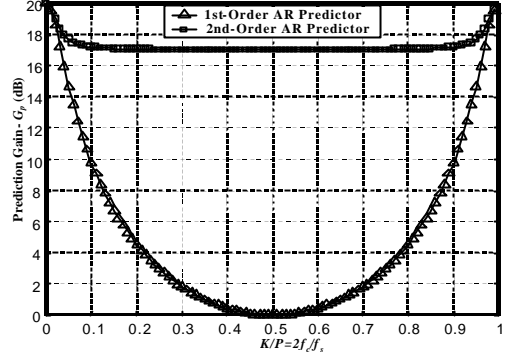


Figure 1. Performance Degradation of the AR Predictor

In this figure, because we are more interested in the relation between K/P and G_{pAR}^{\max} , C is fixed at 0.995, and thus according to Eq. 16, $G_{pu}^{\max} = 20$ dB. From the results presented in Fig. 1, it is clear that firstly, the factor of $\cos(pnK/P + f)$ will significantly decrease the highest prediction gain achieved by the AR predictor, especially when $K/P = 0.5$. For that case, as $E[x_G(n)x_G(n-1)] = 0$, the optimal 1^{st} -order AR predictor is given by $y_{ARG1}(n) = 0$, which means that the 1^{st} -order AR predict cannot predict $x_G(n)$ at all. Secondly, with the existence of $\cos(pnK/P + f)$, increasing the order of the AR predictor can effectively increase the highest prediction gain of $x_G(n)$, G_{pAR}^{\max} . However, it appears logical that G_{pAR}^{\max} is upperbounded by G_{pu}^{\max} .

2.3 Algorithmic Implementation

One well-known adaptive algorithm to implement the AR predictor is the Least Mean Square (LMS) algorithm, which can be mathematically described as follows (Haykin, 1996):

$$y_{ARA}(n) = W_{ARA}(n)^T X(n) \quad (21)$$

$$e_{ARA}(n) = x(n) - y_{ARA}(n) \quad (22)$$

$$W_{ARA}(n+1) = W_{ARA}(n) + 2\mathbf{m} e_{ARA}(n) X(n) \quad (23)$$

where \mathbf{m} referred to as adjustment step, is a small positive constant. In (McLernon, 1991) it has been proven that for cyclostationary signals, when \mathbf{m} is small, $W_{ARA}(n)$ will converge to W_{AR}^{op} in the mean:

$$\lim_{\substack{n \rightarrow \infty \\ \mathbf{m} \rightarrow 0}} E[W_{ARA}(n)] = \left[\sum_{k=0}^{P-1} R_k \right]^{-1} \left[\sum_{k=0}^{P-1} P_k \right]. \quad (24)$$

3. PERIODIC AUTO-REGRESSIVE (PAR) PREDICTOR

When certain characteristics about the signal to be predicted are known in advance, such as for example, the modulation scheme, Baud rate or carrier frequency, more appropriate signal prediction algorithms that can exploit these known signal characteristics should achieve better performance. The PAR predictor, which is based upon the PAR model (Garnder, 1994), is a more accurate, yet more complicated algorithm to predict cyclostationary signals than the AR model.

3.1 Algorithm Description

In general, an $L \times M^{\text{th}}$ -order PAR predictor can be described as below:

$$y_{PAR}(n) = W_{PAR}^T \Phi(n) X(n) \quad (25)$$

where

$$W_{PAR} = [W_{PAR1}^T, W_{PAR2}^T, \dots, W_{PAR2M+1}^T]^T, \quad (26)$$

$$W_{PARi} = [w_{PARi,1}, w_{PARi,2}, \dots, w_{PARi,L}]^T, \quad (27)$$

$i = 1, 2, \dots, 2M+1,$

$$\Phi(n) = [\Lambda(1), \Lambda(\cos(2p \frac{K_1}{P} n)), \Lambda(\sin(2p \frac{K_1}{P} n)), \dots, \Lambda(\cos(2p \frac{K_M}{P} n)), \Lambda(\sin(2p \frac{K_M}{P} n))]^T. \quad (28)$$

In the above equations, $\Lambda(z)$ is a L^{th} -order diagonal matrix with z as its diagonal elements, and $K_1/P, \dots, K_M/P$ are the cycle frequencies of $x(n)$. Since $x(n)$ is now assumed to be a cyclostationary signal, we can easily find that the mean square of the prediction error $e_{PAR}(n)$ is a periodic function, i.e.

$$E[e_{PAR}^2(n)] = E[(x(n) - y_{PAR}(n))^2] = E[e_{PAR}^2(n+P)]. \quad (29)$$

Thus, to facilitate our analysis, we rewrite Eq. 28 as

$$\Phi_k = [\Lambda(1), \Lambda(\cos(2p \frac{K_1}{P} k)), \Lambda(\sin(2p \frac{K_1}{P} k)), \dots,$$

$$\Lambda(\cos(2p \frac{K_M}{P} k)), \Lambda(\sin(2p \frac{K_M}{P} k))]^T \text{ and Eq. 29 as}$$

$e_{PAR-k}(n) = e_{PAR}(nP+k)$. To achieve the highest prediction gain, W_{PAR} should let:

$$J_{PAR} = E[\sum_{k=0}^{P-1} e_{PAR-k}^2(n)] = \sum_{k=0}^{P-1} \{E[x_k^2(n)] + W_{PAR}^T \Phi_k R_k \Phi_k^T W_{PAR} - 2P_k^T \Phi_k^T W_{PAR}\} \quad (30)$$

be minimized. Setting $\partial J / \partial W_{PAR} = 0$, it is straightforward to get the following optimal solution for W_{PAR} :

$$W_{PAR}^{op} = [\sum_{k=0}^{P-1} \Phi_k R_k \Phi_k^T]^{-1} [\sum_{k=0}^{P-1} P_k \Phi_k], \quad (31)$$

the minimum value for J_{PAR} :

$$J_{PAR}^{\min} = \sum_{k=0}^{P-1} E[x_k^2(n)] -$$

$$[\sum_{k=0}^{P-1} P_k \Phi_k]^T [\sum_{k=0}^{P-1} \Phi_k R_k \Phi_k^T]^{-1} [\sum_{k=0}^{P-1} P_k \Phi_k] \quad (32)$$

and the highest prediction gain for $x(n)$:

$$G_{PAR}^{\max} = \frac{\sum_{k=0}^{P-1} E[x_k^2(n)]}{\sum_{k=0}^{P-1} E[x_k^2(n)] - [\sum_{k=0}^{P-1} P_k \Phi_k]^T [\sum_{k=0}^{P-1} \Phi_k R_k \Phi_k^T]^{-1} [\sum_{k=0}^{P-1} P_k \Phi_k]} \quad (33)$$

3.2 Algorithm Performance

When the PAR predictor described by Eq. 31 is employed to predict $x_G(n)$ (see Eq. 12), the optimal $1 \times 1^{\text{st}}$ PAR predictor is given by:

$$y_{GPAR-1 \times 1}(n) = C \{ \cos(\frac{pK}{P}) + \sin(\frac{pK}{P}) [\sin(\frac{2pK}{P} - 2f)] \cos(\frac{2pK}{P} n) - \cos(\frac{2pK}{P} - 2f) \sin(\frac{2pK}{P} n) \} x_G(n-1) \quad (34)$$

and based upon Eq. 33, the highest prediction gain is given by:

$$G_{GPAR-1 \times 1}^{\max} = \frac{1}{1 - \frac{C^2}{2} [\cos^2(\frac{pK}{P}) + 1]} \quad (35)$$

From Eq. 35, when C is still fixed at 0.995, as a function of K/P , the highest prediction gains of $x_G(n)$ for the $1 \times 1^{\text{st}}$ PAR predictors are calculated and are shown in Fig. 2 together with the previously obtained result when the 1^{st} -order AR predictor was analyzed.

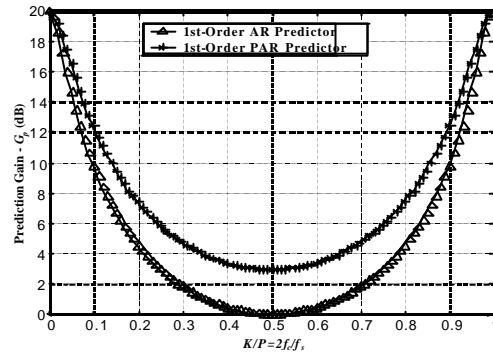


Figure 2. Performance Comparison of the AR and the PAR Predictor

From this figure, it is clear that as compared with the AR predictor, the PAR predictor can effectively increase the highest prediction gain. The maximum improvement of 3 dB is achieved at $K/P = 0.5$, where the 1^{st} -order AR predictor cannot provide any prediction gain. Clearly, similar results can also be derived for the higher order PAR predictors. However, because of the complication of the mathematical analysis, performance evaluation for the higher order PAR predictor will be performed by computer simulations and will be presented in Section 4. As the trade-offs of the performance improvement, the penalty of using the PAR predictor is that the cycle frequencies of the signal to be predicted must be known in advance. Furthermore, as compared to the AR predictor, the PAR predictor requires higher computational effort. For example, in a L^{th} -order AR predictor, the number of prediction coefficients is L , but in a $L \times M^{\text{th}}$ PAR predictor, that

number increases to $L \times (2M+1)$.

3.3 Algorithmic Implementation

Contrary to the AR predictor case, the algorithmic implementation of the previously described PAR predictor is not a trivial issue and to the best of our knowledge this problem has not been addressed in the open technical literature. Perhaps the most straightforward approach to implement the PAR predictor is to calculate W_{PAR}^{op} by using Eq. 31. However, in order to derive P_k , R_k and $\left[\sum_{k=0}^{P-1} \Phi_k R_k \Phi_k^T \right]^{-1}$ which involves much calculation on expected value and matrix inversion, this approach requires significant computational power, especially when higher order PAR predictors are considered. These computational intensive calculations make it very difficult to be used to in the real time signal prediction. In this paper, we will propose an alternative approach which essentially is an extension of the LMS algorithm for the AR predictor described in Eqs. 21-23. The main idea of the new approach is to adaptively calculate W_{PAR}^{op} by using a recursive algorithm, as follows:

$$y_{PARA-k}(n) = W_{PARA-k}^T(n) \Phi_k X_k(n), \quad (36)$$

$$e_{PARA-k}(n) = x_k(n) - y_{PARA-k}(n), \quad (37)$$

$$W_{PARA-(k+1)}(n) = W_{PARA-k}(n) + 2m e_{PARA-k}(n) \Phi_k X_k(n) \quad (38)$$

$$k \in \{0, 1, \dots, P-2\}$$

$$W_{PARA-0}(n+1) = W_{PARA-(P-1)}(n) + 2m e_{PARA-(P-1)}(n) \Phi_{P-1} X_{P-1}(n) \quad (39)$$

where

$$W_{PARA-k}(n) = [W_{PARA-1,k}^T(n), W_{PARA-2,k}^T(n), \dots, W_{PARA-(2M+1),k}^T(n)]^T \quad (40)$$

$$W_{PARA-i,k}(n) = [w_{PARA-i,1,k}(n), w_{PARA-i,2,k}(n), \dots, w_{PARA-i,L,k}(n)]^T \quad (41)$$

$$i = 1, 2, \dots, 2M+1$$

$$w_{PARA-i,j,k}(n) = w_{PARA-i,j}(nP+k), j = 1, 2, \dots, L \quad (42)$$

Clearly, algorithm described by Eqs. 36-39 is not as computational intensive as the one described by Eq. 31, because it only includes matrix addition and multiplication, and no calculation on expected value or matrix inversion is required. Furthermore, when $m \rightarrow 0$, after a sufficiently large number of iterations, i.e. $n \rightarrow \infty$, the algorithm described by Eqs. 36-39 will let $W_{PARA-k}(n)$ converge to W_{PAR}^{op} of Eq. 31 in the mean.²

4. COMPUTER SIMULATION RESULTS AND DISCUSSION

In order to thoroughly evaluate the performance the AR and the PAR predictors offer, we have employed the computer simulation approach. More specifically, we have implemented in software the various

prediction algorithms proposed and analyzed in the previous sections. In particular, the algorithms described by Eqs. 21-23 (for the AR predictor) and by Eqs. 36-39 (for the PAR predictor) with $m=0.0005$ were realized and used to evaluate the overall system performance. This specific value of m was obtained by trial and error. As the signals to be predicted, we have considered two kinds of signals, $x_G(n)$ (see Eq. 12) and BPSK signal corrupted by Additive White Gaussian Noise (AWGN). For all the simulations, unless otherwise noted, the following parameters have been chosen: a) the BPSK signal is shaped by a square root raise cosine filter with the excess bandwidth $\alpha = 0.5$; b) $L=5$, because extensive computer simulations have shown that when $L > 5$, the achieved prediction gain increase becomes very small; c) $R_s / f_s = 0.02$, where R_s is the Baud rate of the BPSK signal.

In order to compared the performance of the AR and the PAR predictors, firstly, we have assumed that the signal to be predicted is $x_G(n)$ with $C = 0.995$. For this case, the following four different predictors have been employed: *i*) the 1st-order AR predictor, *ii*) the 1 \times 1st-order PAR predictor, *iii*) the 2nd-order AR predictor, and *iv*) the 2 \times 1st-order PAR predictor.

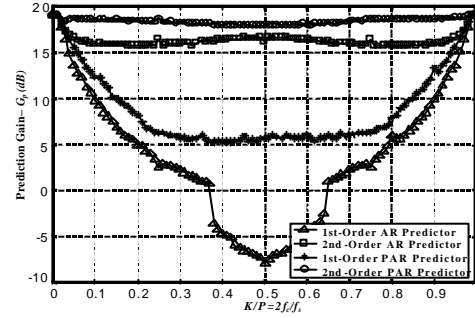


Figure 3. Performance Comparison of the AR and the PAR Predictor - I

Their performance in prediction gain G_p , as a function of K/P , is illustrated in Fig. 3. Comparing the results of Fig. 3 with Figs. 1 and 2, we can conclude that the analytical results about the performance of the AR and the PAR predictor given in Sections 2 and 3 are well verified by the numerical results obtained here.

Secondly, we have assumed that the signal to be predicted is a BPSK signal and an AWGN signal with RF power difference of 30 dB. It has been shown in (Garnder, 1994) that although the BPSK signal has more than one cycle frequencies, the most important cycle frequency is $K/P = 2f_c / f_s$. Thus, in order to simplify the complexity of the simulation, only this cycle frequency is exploited in the PAR predictor. When the 5th-order AR and the 5 \times 1st PAR predictors are employed respectively, their performance in prediction gain G_p , as a function of K/P , is illustrated in Fig. 4. The performance of higher order predictors (e.g. $L=6-10$) is not included in Fig. 4, because extensive computer simulation have shown that higher order predictors

² The proof of this statement is not given since it is considered to be beyond the scope of this paper.

can hardly improve the prediction gain G_p by more than 1 dB. Finally, in the same figure, we have included an upper bound of G_p of $x(n)$ which is numerically obtained by the method presented in (Bernhard, 1998).

Clearly, the obtained results in Fig. 4 indicate that the PAR predictor can achieve much better prediction gains than the AR predictor. Moreover, in Fig. 4, the relatively gap between the prediction gains achieved by the PAR predictor and their upper bounds shows that we still have great potentials to improve the performance of the signal prediction algorithms.

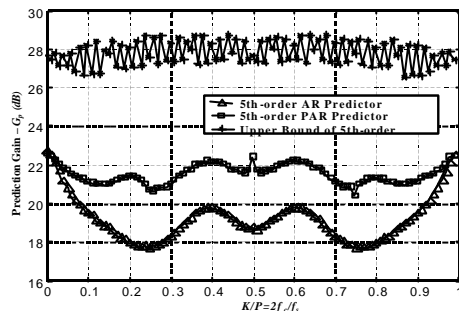


Figure 4. Performance Comparison of the AR and the PAR Predictor - II

5. CONCLUSION

In this paper, by analysis and by means of computer simulations, we have illustrated that as compared with the AR predictor, the PAR predictor is a better algorithm to predict cyclostationary signals. However, as the trade-offs of the performance improvement, certain statistical characteristics about the cyclostationary signals must be known in advanced and higher computational effort is required by the PAR predictor.

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