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Design of Delay Dependent Robust Controller for Uncertain Systems with Time Varying Delay

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Abstract: This paper investigates the controller synthesis problem of uncertain systems with time varying delays. A robust controller with delay compensation is proposed, based on Lyapunov function method. The stability criterion of the closed-loop system, which is dependent on the size of the time delay and the size of its derivative, is derived in the form of linear matrix inequalities (LMI). Examples show that the results using the method in this paper are less conservative than most existing results by other methods. *Copyright © 2002 IFAC*

Keywords: delay, Lyapunov methods, stability, stabilization, uncertainty.

1. INTRODUCTION

The study of stability and stabilization of time delay systems has attracted considerable attention over the past several decades because of their practical applications (Boyd.S., *et al*., 1994)-(Zheng Feng., *et al*., 1995). In these works, the derived results can be classified into two categories: delay independent results (Boyd.S., *et al*., 1994)(M.S.Mahmoud., *et al*., 1999)(M.S.Mahmoud., *et al*., 2001(Kim J.H., *et al*., 1996)(Choi H.H., *et al*., 1996) and delay dependent results (Cao Y.Y., et al., 1998)(Cao Y.Y., *et al*., 2000)(Carlos E. De Souze., *et al*., 1999)(M.S.Mahmoud, *et al*., 2001)(Li Xi, *et al*., 1997). Generally, the delay dependent results are less conservative than the delay independent ones when the time delay is small.

Recently, a number of research works focused on the study of delay dependent methods via memoryless controller for uncertain systems with time delay. When time delay is time varying or constant, some memoryless controller design methods (Cao Y.Y., et al., 1998)(Cao Y.Y., *et al*., 2000)(Carlos E. De Souze., *et al*., 1999)(M.S.Mahmoud, *et al*., 2001)(Li Xi, *et al*., 1997) were proposed, based on the Lyapunov function method and first order transformation (Gu Keqin., *et al*., 2001). To reduce the conservatism of the existing results, Gu (2000) used the discretized Lyapunov functional approach to propose a new design method of a robust controller. The given controller can stabilize the original system with larger maximum allowed value of time delay than the existing ones by other methods. However, only systems with polytopic uncertainty and constant delay were addressed in (Gu Keqin., *et al*., 2000). It is difficult to extend the method in (Gu Keqin., *et al*., 2000) to systems with norm-bounded uncertainties and time varying delays. To study stabilization of time delay systems, a memoryless controller and a memory controller were proposed. See (Cao Y.Y., et al., 1998)(Cao Y.Y., *et al*., 2000)(Carlos E. De Souze., *et al*., 1999)(M.S.Mahmoud, *et al*., 1999)(M.S.Mahmoud, *et al*., 2001)(Li Xi, *et al*., 1997)(Gu Keqin., *et al*., 2000)(Gu Keqin., *et al*., 2001)(Kim J.H., *et al*., 1996)(Sophie Tarbouriech, *et al*., 1999)(Choi H.H., *et al*., 1996) for the memoryless case and (Young Soo Moon, *et al*., 2001)(Zheng Feng, *et al*., 1995) for the memory case. Although the proposed memoryless controllers are easy to implement, they often tend to be more conservative, especially when the past information on the system can be employed. By using past state or past input information, delay dependent controllers were designed in (Young Soo Moon, *et al*., 2001)(Zheng Feng, *et al*., 1995)and were shown by examples to be less conservative than memoryless controllers. The shortcoming of the methods (Young Soo Moon, *et al*., 2001)(Zheng Feng, *et al*., 1995) is that the time delay must be assumed to be known and constant.

In this paper, we investigate the problem of delay dependent robust controller design for systems with norm-bounded uncertainties and time varying delays. To obtain a transformed system, a neutral model transformation and first-order transformation (M.S.Mahmoud, *et al*., 2001) are employed simultaneously. Unlike the memoryless controllers (Cao Y.Y., et al., 1998)(Cao Y.Y., *et al*., 2000)(Carlos E. De Souze., *et al*., 1999)(M.S.Mahmoud, *et al*., 2001)(Li Xi, *et al*., 1997)(Gu Keqin, *et al*., 2000), the given controller provides feedback of the current state and past state information. The advantages of our method are two. First, more information on the state is used to implement the controller. Seconds, the time delay can be time varying and the exact value of the time delay is not required to be known. The derived stability criterions are expressed in terms of LMI, which can be effectively solved by using various optimization algorithms (Boyd.S., *et al*., 1994).

Notation: R^n denotes the n-dimensional Euclidean space, $R^{n\times m}$ is the set of $n\times m$ real matrices, I is the identity matrix, $\|\cdot\|$ stands for the induced matrix 2-norm. The notation $X>0$ (respectively, $X\geq 0$), for $X\in R^{n\times n}$ means that the matrix X is a real symmetric positive definite (respectively, positive semi-definite). C_0 denotes the set of all continuous functions, from $[-\bar{\tau},\ \ 0]$ to R^n.

2. SYSTEM DESCRIPTION AND MAIN RESULTS

Consider the following uncertain system with time varying delay

$$\dot{x}(t) = [A + \Delta A(t)]x(t) + [A_1 + \Delta A_1(t)]x(t - \tau(t)) + [B + \Delta B(t)]u(t)\,, \quad (1)$$

$$x(s) = \phi(s)\,,\ s \in [-\bar{\tau},\quad 0], \quad (2)$$

where $x(t) \in R^n$ and $u(t) \in R^m$ are the system state and the control input, respectively. $\tau(t)$ represents the time delay which is continuously differentiable and satisfies $0 \le \tau(t) \le \bar{\tau}$ and $\dot{\tau}(t) \le d < 1$. $\phi(t) \in C_0$ is the initial function. A , A_1 and B are constant matrices of appropriate dimensions. $\Delta A(t)$, $\Delta A_1(t)$ and $\Delta B(t)$ denote the parameter uncertainties which satisfy

$$[\Delta A(t) \quad \Delta A_1(t) \quad \Delta B(t)] = DF(t)[E_1 \quad E_2 \quad E_3]$$

where D , E_1 , E_2 and E_3 are known matrices and $F(t)$ is unknown time varying matrix which satisfies $\|F(t)\| \le 1$.

The following Lemmas will be used in our main result.

Lemma 1(Carlos E. De Souza, et al., 1999): For any $x, \quad y \in R^{n \times n}$ and for any positive symmetric definite matrix $P \in R^{n \times n}$,

$$2x^T y \le x^T P^{-1} x + y^T P y\,.$$

(2). Let A , D, E and F represent real matrices of appropriate dimensions with $\|F\| \le 1$. Then we have:

(a). For any scalar $\varepsilon > 0$,

$$DFE + E^T F^T D^T \le \varepsilon^{-1} DD^T + \varepsilon E^T E\,.$$

(b). For any matrix $P = P^T > 0$ and scalar $\varepsilon > 0$ such that $\varepsilon I - EPE^T > 0$,

$$(A + DFE)P(A + DFE)^T \le$$
$$APA^T + APE^T(\varepsilon I - EPE^T)^{-1} EPA^T + \varepsilon DD^T\,.$$

(c). For any matrix $P = P^T > 0$ and scalar $\varepsilon > 0$ such that $P - \varepsilon DD^T > 0$,

$$(A + DFE)^T P^{-1}(A + DFE) \le A^T (P - \varepsilon DD^T)^{-1} A + \varepsilon^{-1} E^T E\,.$$

Lemma 2(Dong Yue and Sangchul won, 2001): Consider an operator $D(.) : R^n \to R^n$ with $D(y(t)) = y(t) + \hat{B} \int_{t-h}^{t} y(s)ds$, where $y(t) \in R^n$ and $\hat{B} \in R^{n \times n}$. For a given scalar δ, where $0 < \delta < 1$, if there exists a positive definite symmetric matrix P such that

$$\begin{bmatrix} -\delta P & h\hat{B}^T P \\ hP\hat{B} & -P \end{bmatrix} \le 0\,,$$

then the operator $D(y(t))$ is stable.

Next, we give our main result.

Theorem 1: Suppose that scalars $\bar{\tau} > 0$ and $d < 1$ are given. Then the system (1) with the control

$$u(t) = YX^{-1}[x(t) + \int_{t-\frac{\bar{\tau}}{2}}^{t} A_1 x(s)ds] \quad (3)$$

is asymptotically stable for any $\tau(t)$ satisfying $0 \le \tau(t) \le \bar{\tau}$ and $\dot{\tau}(t) \le d < 1$, if there exist positive definite matrices X , X_k $(k = 1,2,3)$, Y_i $(i = 1,2,3,4)$ and matrix $Y \in R^{m \times n}$ and positive scalars ε_j $(j = 1,2,..,7)$ such that

$$\begin{bmatrix} \Sigma & -\frac{\bar{\tau}}{2}(A + A_1)A_1 Y_1 & \Omega_1^T \\ -\frac{\bar{\tau}}{2} Y_1 A_1^T (A + A_1)^T & -\frac{\bar{\tau}}{2} Y_1 & \Omega_2^T \\ \Omega_1 & \Omega_2 & \Xi \end{bmatrix} < 0 \quad (4)$$

$$\begin{bmatrix} -Y_2 & Y_2 A^T & Y_2 E_1^T \\ AY_2 & -X_1 + \varepsilon_5 DD^T & 0 \\ E_1 Y_2 & 0 & -\varepsilon_5 I \end{bmatrix} < 0 \quad (5)$$

$$\begin{bmatrix} -Y_3 & Y_3 A_1^T & Y_3 E_2^T \\ A_1 Y_3 & -X_2 + \varepsilon_6 DD^T & 0 \\ E_2 Y_3 & 0 & -\varepsilon_6 I \end{bmatrix} < 0\,, \quad (6)$$

$$\begin{bmatrix} -Y_4 & YB^T & YE_3^T \\ BY & -X_3 + \varepsilon_7 DD^T & 0 \\ E_3 Y & 0 & -\varepsilon_7 I \end{bmatrix} < 0\,, \quad (7)$$

where

$$\Sigma = X(A + A_1)^T + (A + A_1)X + BY + Y^T B^T +$$
$$(\varepsilon_1 + \frac{\bar{\tau}}{2}\varepsilon_2 + \varepsilon_3 + \varepsilon_4)DD^T +$$
$$\frac{\bar{\tau}}{2} A_1 (X_1 + X_2 + X_3) A_1^T + \frac{\bar{\tau}}{2} Y_4$$

$$\Omega_1^T = \begin{bmatrix} XE_2^T & \frac{\bar{\tau}}{2} X & \frac{\bar{\tau}}{2} X & \frac{\bar{\tau}}{2} X & 0 & XE_1^T & Y^T E_3^T \end{bmatrix}$$

$$\Omega_2^T = \Big[-\frac{\bar{\tau}}{2} Y_1 A_1^T E_2^T \quad -\frac{\bar{\tau}^2}{4} Y_1 A_1^T \quad -\frac{\bar{\tau}^2}{4} Y_1 A_1^T$$
$$-\frac{\bar{\tau}^2}{4} Y_1 A_1^T \quad -\frac{\bar{\tau}}{2} Y_1 A_1^T E_1^T \quad 0 \quad 0 \Big]$$

$$\Xi = diag\Big((1-d)\varepsilon_3 I \quad \frac{\bar{\tau}}{2} Y_1 \quad \frac{\bar{\tau}}{2} Y_2 \quad \frac{\bar{\tau}}{2}(1-d) Y_3$$
$$\frac{\bar{\tau}}{2}\varepsilon_2 I \quad \varepsilon_1 I \quad \varepsilon_4 I \Big)\,.$$

Proof: Define a neutral transformation as

$$z(t) = x(t) + \int_{t-\bar{\tau}/2}^{t} A_1 x(s)ds\,. \quad (8)$$

Using (8), we design a controller as

$$u(t) = Kz(t) \quad (9)$$

where K is a constant matrix that will be designed later.

Taking the time derivative of $z(t)$ and combining (1), (9) and Leibniz-Newton formula, we obtain the following transformed system

$$\dot{z}(t)=(A+A_1)x(t)+\Delta A(t)x(t)+\Delta A_1(t)x(t-\tau(t))$$
$$-A_1\int_{t-\tau(t)}^{t-\bar{\tau}/2}\{(A+\Delta A(s))x(s)+(A_1+\Delta A_1(s))x(s-\tau(s))$$
$$+(B+\Delta B(s))Kz(s)\}ds \quad +(B+\Delta B(t))Kz(t)\,, \qquad (10)$$

Construct a Lyapunov functional as

$$V(x_t)=V_1(x_t)+V_2(x_t)\,, \qquad (11)$$

where $x_t(s)=x(t+s)$, $s\in[-\tau,\quad 0]$,

$$V_1(x_t)=z^T(t)Pz(t)$$

$$V_2(x_t)=\int_{t-\tau(t)}^{t}x^T(s)Tx(s)ds+\int_{t-\bar{\tau}/2}^{t}\int_{s}^{t}x^T(v)Qx(v)dvds\oplus$$
$$\int_{t-\bar{\tau}/2}^{t-\tau(t)}\int_{s}^{t-\tau(t)}x^T(v)Gx(v)dvds+\frac{\bar{\tau}}{2}\int_{t-\tau(t)}^{t}x^T(s)Gx(s)ds\oplus$$
$$\int_{t-\bar{\tau}/2}^{t-\tau(t)}\int_{s}^{t-\tau(t)}x^T(v-\tau(v))Wx(v-\tau(v))dvds$$
$$+\frac{\bar{\tau}}{2}\int_{t-\tau(t)}^{t}x^T(s-\tau(s))Wx(s-\tau(s))ds$$
$$+\frac{\bar{\tau}}{2(1-d)}\int_{t-\tau(t)}^{t}x^T(s)Wx(s)ds$$
$$\oplus\int_{t-\bar{\tau}/2}^{t-\tau(t)}\int_{s}^{t-\tau(t)}z^T(v)Rz(v)dvds+\frac{\bar{\tau}}{2}\int_{t-\tau(t)}^{t}z^T(s)Rz(s)ds\,, \qquad (12)$$

where $\oplus$ is "+" as $\tau(t)<\frac{\bar{\tau}}{2}$ and "-" as $\tau(t)\geq\frac{\bar{\tau}}{2}$. *P*, *T*, *Q*, *G* ,*W* and *R* are symmetric positive definite matrices.

It is easy to show that as $\tau(t)\geq\frac{\bar{\tau}}{2}$

$$\int_{t-\bar{\tau}/2}^{t-\tau(t)}\int_{s}^{t-\tau(t)}x^T(v)Gx(v)dvds\leq\frac{\bar{\tau}}{2}\int_{t-\tau(t)}^{t}x^T(s)Gx(s)ds\,,$$
$$\int_{t-\bar{\tau}/2}^{t-\tau(t)}\int_{s}^{t-\tau(t)}z^T(v)Rz(v)dvds\leq\frac{\bar{\tau}}{2}\int_{t-\tau(t)}^{t}z^T(s)Rz(s)ds$$

and

$$\int_{t-\bar{\tau}/2}^{t-\tau(t)}\int_{s}^{t-\tau(t)}x^T(v-\tau(v))Wx(v-\tau(v))dvds$$
$$\leq\frac{\bar{\tau}}{2}\int_{t-\tau(t)}^{t}x^T(s-\tau(s))Wx(s-\tau(s))ds\,,$$

thus, $V_2(x_t)$ is positive definite. Moreover, $V_1(x_t)$ and $V_2(x_t)$ are continuously twice differentiable in x and once in t.

Next, to prove the negative of time derivative of $V(x_t)$, we will consider two cases.

First, we consider the case when $\tau(t)<\frac{\bar{\tau}}{2}$.

We rewrite (10) as

$$\dot{z}(t)=(A+A_1)x(t)+\Delta A(t)x(t)+\Delta A_1(t)x(t-\tau(t))$$
$$+A_1\int_{t-\bar{\tau}/2}^{t-\tau(t)}\{(A+\Delta A(s))x(s)+(A_1+\Delta A_1(s))x(s-\tau(s))$$
$$+(B+\Delta B(s))Kz(s)\}ds \quad +(B+\Delta B(t))Kz(t)\,. \qquad (13)$$

Taking the time derivative of $V_1(x_t)$ with respect to time t and combining (8), we obtain

$$\dot{V}_1(x_t)=2z^T(t)[P(A+A_1)+PBK]z((t)$$
$$-2z^T(t)P(A+A_1)A_1\int_{t-\bar{\tau}/2}^{t}x(s)ds$$
$$+2z^T(t)PDF(t)E_1z(t)-2z^T(t)PDF(t)E_1B\int_{t-\bar{\tau}/2}^{t}x(s)ds$$
$$+2z^T(t)PDF(t)E_2x(t-\tau(t))\ +2z^T(t)P$$
$$A_1\int_{t-\bar{\tau}/2}^{t-\tau(t)}\{(A+\Delta A(s))x(s)+(A_1+\Delta A_1(s))x(s-\tau(s))$$
$$+(B+\Delta B(s))Kz(s)\}ds\ +2z^T(t)PDF(t)E_3Kz(t)\,. \qquad (14)$$

Using Lemma 1 and noting that $\frac{\bar{\tau}}{2}-\tau(t)\leq\frac{\bar{\tau}}{2}$, we have

$$2z^T(t)PDF(t)E_1z(t)\leq z^T(t)[\varepsilon_1PDD^TP+\varepsilon_1^{-1}E_1^TE_1]z(t), \qquad (15)$$

$$-2z^T(t)PDF(t)E_1A_1\int_{t-\bar{\tau}/2}^{t}x(s)ds\leq$$
$$\frac{\bar{\tau}}{2}z^T(t)\varepsilon_2PDD^TPz(t)$$
$$+\int_{t-\bar{\tau}/2}^{t}x^T(s)\varepsilon_2^{-1}A_1^TE_1^TE_1A_1x(s)ds\,, \qquad (16)$$

$$2z^T(t)PDF(t)E_2x(t-\tau(t))\leq z^T(t)\varepsilon_3PDD^TPz(t)$$
$$+x^T(t-\tau(t))\varepsilon_3^{-1}E_2^TE_2x(t-\tau(t))\,, \qquad (17)$$

$$2z^T(t)PDF(t)E_3Kz(t)\leq$$
$$z^T(t)\varepsilon_4PDD^TPz(t)+z^T(t)\varepsilon_4^{-1}K^TE_3^TE_3Kz(t), \qquad (18)$$

$$2z^T(t)PA_1\int_{t-\bar{\tau}/2}^{t-\tau(t)}(A+\Delta A(s))x(s)ds\leq$$
$$\frac{\bar{\tau}}{2}z^T(t)PA_1X_1A_1^TPz(t)$$
$$+\int_{t-\bar{\tau}/2}^{t-\tau(t)}x^T(s)(A+\Delta A(s))^TX_1^{-1}(A+\Delta A(s))x(s)ds\,, \qquad (19)$$

$$2z^T(t)PA_1\int_{t-\bar{\tau}/2}^{t-\tau(t)}(A_1+\Delta A_1(s))x(s-\tau(s))ds \le$$

$$\frac{\bar{\tau}}{2}z^T(t)PA_1X_2A_1^TPz(t)$$

$$+\int_{t-\bar{\tau}/2}^{t-\tau(t)}x^T(s-\tau(s))(A_1+\Delta A_1(s))^TX_2^{-1}(A_1+\Delta A_1(s))x(s-\tau(s))ds, \quad (20)$$

$$2z^T(t)PA_1\int_{t-\bar{\tau}/2}^{t-\tau(t)}(B+\Delta B(s))Kz(s)ds \le$$

$$\frac{\bar{\tau}}{2}z^T(t)PA_1X_3A_1^TPz(t)$$

$$+\int_{t-\bar{\tau}/2}^{t-\tau(t)}z^T(s)K^T(B+\Delta B(s))^TX_3^{-1}(B+\Delta B(s))Kz(s)ds. \quad (21)$$

Defining

$$\Sigma_1=(A+A_1)^TP+P(A+A_1)+$$
$$PBK+K^TB^TP+(\varepsilon_1+\frac{\bar{\tau}}{2}\varepsilon_2+\varepsilon_3+\varepsilon_4)PDD^TP$$
$$+\varepsilon_1^{-1}E_1^TE_1+\varepsilon_4^{-1}K^TE_3^TE_3K$$
$$+\frac{\bar{\tau}}{2}PA_1(X_1+X_2+X_3)A_1^TP$$

and combining (14)-(21), we obtain

$$\dot{V}_1(x_t)\le z^T(t)\Sigma_1z(t)$$
$$-2z^T(t)P(A+A_1)A_1\int_{t-\bar{\tau}/2}^{t}x(s)ds$$
$$+x^T(t-\tau(t))\varepsilon_3^{-1}E_2^TE_2x(t-\tau(t))$$
$$+\int_{t-\bar{\tau}/2}^{t}x^T(s)\varepsilon_2^{-1}A_1^TE_1^TE_1A_1x(s)ds$$
$$+\int_{t-\bar{\tau}/2}^{t-\tau(t)}x^T(s)(A+\Delta A(s))^TX_1^{-1}(A+\Delta A(s))x(s)ds$$
$$+\int_{t-\bar{\tau}/2}^{t-\tau(t)}x^T(s-\tau(s))(A_1+\Delta A_1(s))^TX_2^{-1}(A_1+\Delta A_1(s))x(s-\tau(s))ds$$
$$+\int_{t-\bar{\tau}/2}^{t-\tau(t)}z^T(s)K^T(B+\Delta B(s))^TX_3^{-1}(B+\Delta B(s))Kz(s)ds. \quad (22)$$

It is easy to verify that as $\tau(t)<\frac{\bar{\tau}}{2}$,

$$\dot{V}_2(x_t)\le x^T(t)\hat{T}x(t)-(1-d)x^T(t-\tau(t))Tx(t-\tau(t))$$
$$-\int_{t-\bar{\tau}/2}^{t}x^T(s)Qx(s)ds-\int_{t-\bar{\tau}/2}^{t-\tau(t)}x^T(s)Gx(s)ds$$
$$-\int_{t-\bar{\tau}/2}^{t-\tau(t)}x^T(s-\tau(s))Wx(s-\tau(s))ds$$
$$+\frac{\bar{\tau}}{2}z^T(t)Rz(t)-\int_{t-\bar{\tau}/2}^{t-\tau(t)}z^T(s)Rz(s)ds, \quad (23)$$

where $T_1=T+\frac{\bar{\tau}}{2}Q+\frac{\bar{\tau}}{2}G+\frac{\bar{\tau}}{2(1-d)}W$.

In addition, using (8), we can prove that

$$x^T(t)T_1x(t)\le z^T(t)T_1z(t)-2z^T(t)T_1A_1\int_{t-\bar{\tau}/2}^{t}x(s)ds$$
$$+\frac{\bar{\tau}}{2}\int_{t-\bar{\tau}/2}^{t}x^T(s)A_1^TT_1A_1x(s)ds. \quad (24)$$

Combining (22), (23) and (24), we obtain

$$\dot{V}(x_t)=\dot{V}_1(x_t)+\dot{V}_2(x_t)$$
$$\le z^T(t)(\Sigma_1+T_1+\frac{\bar{\tau}}{2}R)z(t)$$
$$-2z^T(t)[P(A+A_1)A_1+T_1A_1]\int_{t-\bar{\tau}/2}^{t}x(s)ds$$
$$-x^T(t-\tau(t))[(1-d)T-\varepsilon_3^{-1}E_2^TE_2]x(t-\tau(t))$$
$$-\int_{t-\bar{\tau}/2}^{t}x^T(s)[Q-\varepsilon_2^{-1}A_1^TE_1^TE_1A_1-\frac{\bar{\tau}}{2}A_1^TT_1A_1]x(s)ds$$
$$-\int_{t-\bar{\tau}/2}^{t-\tau(t)}x^T(s)[G-(A+\Delta A(s))^TX_1^{-1}(A+\Delta A(s))]x(s)ds$$
$$-\int_{t-\bar{\tau}/2}^{t-\tau(t)}x^T(s-\tau(s))[W-(A_1+\Delta A_1(s))^TX_2^{-1}(A_1+\Delta A_1(s))]x(s-\tau(s))ds$$
$$-\int_{t-\bar{\tau}/2}^{t-\tau(t)}z^T(s)[R-K^T(B+\Delta B(s))^TX_3^{-1}(B+\Delta B(s))K]z(s)ds. \quad (25)$$

At this point, we have obtained (25) when $\tau(t)<\frac{\bar{\tau}}{2}$. On the other hand, using a similar analysis method, we can obtain that when $\tau(t)\ge\frac{\bar{\tau}}{2}$,

$$\dot{V}(x_t)=\dot{V}_1(x_t)+\dot{V}_2(x_t)$$
$$\le z^T(t)(\Sigma_1+T_1+\frac{\bar{\tau}}{2}R)z(t)$$
$$-2z^T(t)[P(A+A_1)A_1+T_1A_1]\int_{t-\bar{\tau}/2}^{t}x(s)ds$$
$$-x^T(t-\tau(t))[(1-d)T-\varepsilon_3^{-1}E_2^TE_2]x(t-\tau(t))$$
$$-\int_{t-\bar{\tau}/2}^{t}x^T(s)[Q-\varepsilon_2^{-1}A_1^TE_1^TE_1A_1-\frac{\bar{\tau}}{2}A_1^TT_1A_1]x(s)ds$$
$$-\int_{t-\tau(t)}^{t-\bar{\tau}/2}x^T(s)[G-(A+\Delta A(s))^TX_1^{-1}(A+\Delta A(s))]x(s)ds$$
$$-\int_{t-\tau(t)}^{t-\bar{\tau}/2}x^T(s-\tau(s))[W-(A_1+\Delta A_1(s))^TX_2^{-1}(A_1+\Delta A_1(s))]x(s-\tau(s))ds$$
$$-\int_{t-\tau(t)}^{t-\bar{\tau}/2}z^T(s)[R-K^T(B+\Delta B(s))^TX_3^{-1}(B+\Delta B(s))K]z(s)ds. \quad (26)$$

From (25) and (26), it is easy to see that if

$$-G+(A+\Delta A(s))^TX_1^{-1}(A+\Delta A(s))<0, \quad (27)$$

$$-W+(A_1+\Delta A_1(s))^T X_2^{-1}(A_1+\Delta A_1(s))x(s-\tau(s))<0, \quad (28)$$

$$-R+K^T(B+\Delta B(s))^T X_3^{-1}(B+\Delta B(s))K<0 \quad (29)$$

$$T=\varepsilon_3^{-1}(1-d)^{-1}E_2^T E_2 \quad (30)$$

then

$$\dot V(x_t)\le \frac{2}{\bar\tau}\int_{t-\bar\tau/2}^{t}\begin{bmatrix} z^T(t) & x^T(s)\end{bmatrix} H \begin{bmatrix} z(t) \\ x(s)\end{bmatrix} ds, \quad (31)$$

where

$$H=\begin{bmatrix} \Sigma_1+T_1+\frac{\bar\tau}{2}R & -\frac{\bar\tau}{2}P(A+A_1)A_1-\frac{\bar\tau}{2}T_1A_1 \\ -\frac{\bar\tau}{2}A_1^T(A+A_1)^T P-\frac{\bar\tau}{2}A_1^T T_1 & -\frac{\bar\tau}{2}Q+\frac{\bar\tau}{2}\varepsilon_2^{-1}A_1^T E_1^T E_1 A_1+\frac{\bar\tau^2}{4}A_1^T T_1 A_1 \end{bmatrix}.$$

Obviously, if $H<0$ holds, then there exists a constant $\lambda>0$ such that

$$\dot V(t,x_t)<-\lambda z^T(t)z(t)\ . \quad (32)$$

Next, we will prove that (4), (5), (6) and (7) implies $H<0$, (27), (28) and (29).

Using Schur complements, it can be shown that $H<0$ if and only if

$$\begin{bmatrix} \Sigma_1+\frac{\bar\tau}{2}R & -\frac{\bar\tau}{2}P(A+A_1)A_1 & H_1^T \\ -\frac{\bar\tau}{2}A_1^T(A+A_1)^T P & -\frac{\bar\tau}{2}Q+\frac{\bar\tau}{2}\varepsilon_2^{-1}A_1^T E_1^T E_1 A_1 & H_2^T \\ H_1 & H_2 & -M_2 \end{bmatrix}<0 \quad (33)$$

where

$$H_1^t=\begin{bmatrix} E_2 & \frac{\bar\tau}{2}I & \frac{\bar\tau}{2}I & \frac{\bar\tau}{2}I \end{bmatrix}$$

$$H_2^T=\begin{bmatrix} -\frac{\bar\tau}{2}A_1^T E_2^T & -\frac{\bar\tau^2}{4}A_1^T & -\frac{\bar\tau^2}{4}A_1^T & -\frac{\bar\tau^2}{4}A_1^T \end{bmatrix}$$

$$M_1=diag\begin{pmatrix} (1-d)\varepsilon_3 I & \frac{\bar\tau}{2}Q^{-1} & \frac{\bar\tau}{2}G^{-1} & \frac{\bar\tau}{2}(1-d)W^{-1}\end{pmatrix}.$$

Pre, post-multiplying the both sides of (33) with matrix $diag(P^{-1},Q^{-1},I,I,I,I)$ and denoting $X=P^{-1}$, $Y=KX$, $Y_1=Q^{-1}$, $Y_2=G^{-1}$, $Y_3=W^{-1}$ and $Y_4=P^{-1}RP^{-1}$, we have

$$\begin{bmatrix} \Sigma_2 & -\frac{\bar\tau}{2}(A+A_1)A_1Y_1 & XH_1^T \\ -\frac{\bar\tau}{2}Y_1A_1^T(A+A_1)^T & -\frac{\bar\tau}{2}Y_1+\frac{\bar\tau}{2}\varepsilon_2^{-1}Y_1A_1^T E_1^T E_1 A_1 Y_1 & Y_1H_2^T \\ H_1X & H_2Y_1 & -M_2 \end{bmatrix}<0 \quad (36)$$

where

$$\Sigma_2=X(A+A_1)^T+(A+A_1)X+BY+Y^TB^T$$
$$+(\varepsilon_1+\frac{\bar\tau}{2}\varepsilon_2+\varepsilon_3+\varepsilon_4)DD^T+\varepsilon_1^{-1}XE_1^TE_1X$$
$$+\frac{\bar\tau}{2}A_1(X_1+X_2+X_3)A_1^T+\varepsilon_4^{-1}Y^TE_3^TE_3Y+\frac{\bar\tau}{2}Y_4\ ,$$

$$M_2=diag\begin{pmatrix} (1-d)\varepsilon_3 I & \frac{\bar\tau}{2}Y_1 & \frac{\bar\tau}{2}Y_2 & \frac{\bar\tau}{2}(1-d)Y_3\end{pmatrix}.$$

Using Lemma 1, it can be shown that (27), (28) and (29) hold if the following matrix inequalities hold

$$-G+A^T(X_1-\varepsilon_5DD^T)^{-1}A+\varepsilon_5^{-1}E_1^TE_1<0\ , \quad (35)$$

$$-W+A_1^T(X_2-\varepsilon_6DD^T)^{-1}A_1+\varepsilon_6^{-1}E_2^TE_2<0\ , \quad (36)$$

$$-R+K^TB^T(X_3-\varepsilon_7DD^T)^{-1}BK+\varepsilon_7^{-1}K^TE_3^TE_3K<0, \quad (37)$$

where ε_i $(i=5,6,7)$ are positive scalars, such that $X_1-\varepsilon_5DD^T>0$, $X_2-\varepsilon_6DD^T>0$ and $X_3-\varepsilon_7DD^T>0$.

Using transformations $Y=KX$, $Y_2=G^{-1}$, $Y_3=W^{-1}$ and $Y_4=P^{-1}RP^{-1}$, (35), (36) and (37) can be written as

$$-Y_2+Y_2A^T(X_1-\varepsilon_5DD^T)^{-1}AY_2+\varepsilon_5^{-1}Y_2E_1^TE_1Y_2<0\ , \quad (38)$$

$$-Y_3+Y_3A_1^T(X_2-\varepsilon_6DD^T)^{-1}A_1Y_3+\varepsilon_6^{-1}Y_3E_2^TE_2Y_3<0\ , \quad (39)$$

$$-Y_4+YB^T(X_3-\varepsilon_7DD^T)^{-1}BY+\varepsilon_7^{-1}YE_3^TE_3Y<0\ , \quad (40)$$

By Schur complements, we can show that (36), (38), (39) and (40) are equivalent to (4), (5), (6) and (7), respectively. In other words, (4), (5), (6) and (7) are sufficient conditions that guarantee $H<0$, (27), (28) and (29).

On the other hand, by Schur complements, $H<0$ also means

$$-Q+\frac{\bar\tau^2}{4}A_1^TQA_1<0\ . \quad (41)$$

By Schur complements and matrix theory, we can further prove that a positive scalar $0<\alpha<1$ exists such that

$$\begin{bmatrix} -\alpha Q & \frac{\bar\tau}{2}A_1^TQ \\ \frac{\bar\tau}{2}QA_1 & -Q \end{bmatrix}\le 0\ . \quad (42)$$

Using Lemma 2, we know that under the condition (41), $z(t)$ is a stable operator. Then, combining (32) and using Theorem 9.8.1 of (Hale,J., *et al.*, 1993), we can complete our proof.

Q.E.D

Remark : For a given *d*, the largest $\bar\tau$, which ensures the robust stabilization, can be determined by solving a convex optimization problem (Boyd.S., *et al.*, 1994). Moreover, it can be found from (3) that only upper bound of the time delay is needed to implement the controller (3) although the controller has feedback of the current state and the past state information.

3. EXAMPLE

Example 1: Consider the following system (Carlos E. De Souza., *et al.*, 1999)

$$\dot x(t)=[A+\Delta A(t)]x(t)+[A_1+\Delta A_1(t)]x(t-\tau(t))+Bu(t), \quad (43)$$

where $A=\begin{bmatrix}0 & 0\\ 0 & 1\end{bmatrix}$, $A_1=\begin{bmatrix}-1 & -1\\ 0 & -0.9\end{bmatrix}$, $B=\begin{bmatrix}0\\ 1\end{bmatrix}$, $\|\Delta A(t)\|\le 0.2$ and $\|\Delta A_1(t)\|\le 0.2$. $\tau(t)$ satisfies $0\le\tau(t)\le\bar\tau$ and $\dot\tau(t)\le d<1$.

Choose $D_1=D_2=\begin{bmatrix}0.2 & 0\\ 0 & 0.2\end{bmatrix}$, $E_1=E_2=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$. In (Carlos E. De Souza., *et al.*, 1999), it was shown that the maximum allowable value of τ that guarantees the system (43) with $d = 0$ is stable via a memoryless controller is 0.3346. However, by applying Theorem 1, we found that the upper bound of τ which guarantees (43) with $d = 0$ is stable via the controller (3) is 0.5492. In other words, we can design a controller as

$$u(t) = -[0.2132 \quad 311.0461]\left[x(t) + \int_{t-0.2746}^{t} \begin{bmatrix} -1 & -1 \\ 0 & -0.9 \end{bmatrix} x(s)ds \right],$$

which guarantees that the closed-loop system is asymptotically stable for any $\tau \in [0 \quad 0.5492]$. If $d = 0.5$, the upper bound of τ which guarantees (43) is stable via the controller (3) is 0.3927. If $d = 0.9$, the upper bound of τ is 0.1298.

Example 2**:** Consider the following system (Li Xi., *et al*., 1997)

$$\dot{x}(t) = [A + \Delta A(t)]x(t) + [A_1 + \Delta A_1(t)]x(t-\tau) + Bu(t), \quad (44)$$

where $A = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$, $A_1 = \begin{bmatrix} -2 & -0.5 \\ 0 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $\|\Delta A(t)\| \le 0.2$ and $\|\Delta A_1(t)\| \le 0.2$. $\tau(t)$ satisfies $0 \le \tau(t) \le \bar{\tau}$ and $\dot{\tau}(t) \le d < 1$.

Choose $D_1 = D_2 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}$, $E_1 = E_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

In , it was shown that the maximum allowable value of τ that guarantees the system (44) with $d = 0$ is stable via a memoryless controller is 0.3015. However, by applying Theorem 1, we found that the upper bound of τ which guarantees (44) is stable via the controller (3) is 0.5084. In other words, we can design a controller as

$$u(t) = -[0.2521 \quad 101.7509]\left[x(t) + \int_{t-0.2542}^{t} \begin{bmatrix} -2 & -0.5 \\ 0 & -1 \end{bmatrix} x(s)ds \right],$$

which guarantees the closed-loop system is asymptotically stable for any $\tau \in [0 \quad 0.5084]$.

If $d = 0.5$, the upper bound of τ which guarantees (44) is stable via the controller (3) is 0.3858. If $d = 0.9$, the upper bound of τ is 0.1538.

4. CONCLUSIONS

In this paper, a robust controller with delay compensation was proposed for uncertain systems with time varying delay based on the Lyapunov function method. Like the memoryless controller case, it is not required to know the exact value of the time delay although the designed controller in this paper depends on the current state and past state information. Since more information on the state is used, the given controller can achieve a better performance than memoryless controllers in the existing references.

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