15th Triennial World Congress, Barcelona, Spain

THE PROBLEM OF H^∞ STATE ESTIMATION VIA LIMITED CAPACITY COMMUNICATION CHANNELS

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Abstract: The paper presents a new approach to the problem of state estimation via limited capacity communication channels. We consider the case when the state estimates are required at a distant location, and are to be transmitted via a limited capacity communication channel. A constructive method to design a linear state estimator with a limited capacity communication channel is proposed.

Keywords: state estimation, communication networks, H^∞ filtering

1. INTRODUCTION

In classical filtering theory (e.g., see Anderson and Moore, 1979) the standard assumption is that all data transmission required by the algorithm can be performed with infinite precision. However, in some new models, it is common to encounter situations where observation and control signals are sent via a communication channel with a limited capacity. This problem may arise when a large number of mobile units need to be controlled remotely by a single decision maker. Since the radio spectrum is limited, communication constraints are a real concern. In (Stilwell and Bishop, 2000), the problem of design of large-scale control systems for platoons of underwater vehicles highlights the need for control strategies that address reduced communications, since communication bandwidth is severely limited underwater. Another class of examples is offered by complex networked sensor systems containing a very large number of low power sensors. Furthermore, nowadays, it is becoming more common to use networks in systems, especially in those that are large scale and physically distributed; e.g. see (Brockett, 1995). All these new engineering applications motivate development of a new chapter of control theory in which control and communication issues are combined together, and all the limitations of the communication channels are taken into account. Communications requirements, especially regarding bandwidth limits, are often challenging obstacles to control system design. A natural question to ask is how much capacity is needed to achieve a specified estimation accuracy. One approach to this state estimation problem was proposed in (Wong and Brockett, 1997) and developed in (Nair and Evans, 1998). In this framework, the observation must be coded into a sequence of finite-valued symbols and transmitted via a digital communication channel. Another approach was proposed in (Savkin et al. , 2000) where the bandwidth limitation constraint was modeled in a manner that the state estimator can communicate with only one of several sensors at any time instant. The main disadvantage of both these approaches is that the state estimation systems proposed in (Wong and Brockett, 1997; Nair and Evans, 1998; Dokuchaev and Savkin, 1999; Savkin et al., 2000) are highly nonlinear and contain symbolic variables. In this paper, we propose a new problem statement. In our problem statement, the communication channel transmits a continuous time vector signal. The limited capacity of the channel means that the dimension of the signal transmitted by the channel is smaller than the dimension of the measured output of the system. Our goal is to design a linear time-invariant coder that transforms the measured output into the signal to be transmitted and a linear time-invariant decoder that transforms the transmitted signal into the state estimate. This approach simplifies the problem and allows to obtain more constructive and understandable results. The main advantage is that estimation system obtained in this paper is linear and time-invariant. This allows to apply conventional linear control theory to the problem of state estimation with bandwidth limitation constraints. Unlike (Wong and Brockett, 1997; Nair and Evans, 1998; Dokuchaev and Savkin, 1999) where the Kalman filtering problem was considered and (Savkin and et al., 2000) where the set-valued approach to state estimation was employed, in this paper we consider the case of H^∞ state estimation. The main result of the paper shows that the linear H^∞ control theory when suitably modified provides a good framework for the problem of limited communication state estimation.

It should be pointed out, that the proposed state estimation method is computationally non-expansive and easy to implement in real time. The obtained results can be extended to the case of uncertain linear systems; e.g., see (Savkin and Petersen, 1995, 1996; Petersen and Savkin, 1999).

2. PROBLEM STATEMENT

We consider the linear system defined on the infinite time interval $[0, \infty)$:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bw(t); \quad x(0) = 0; \\ z(t) &= Kx(t); \\ y(t) &= Cx(t) + v(t) \end{aligned} \tag{1}$$

where $x(t) \in \mathbf{R}^n$ is the *state*, $w(t) \in \mathbf{R}^p$ and $v(t) \in \mathbf{R}^l$ are the *disturbance inputs*, $z(t) \in \mathbf{R}^q$ is the *estimated output* and $y(t) \in \mathbf{R}^l$ is the *measured output*, A, B, K and C are given matrices.

Notation In this paper, $\|\cdot\|$ denotes the standard Eucledian norm, $\mathbf{L}_2[0, \infty)$ denotes the Hilbert space of square integrable vector-valued functions defined on $[0, \infty)$, $\|\cdot\|_2$ denotes the norm in $\mathbf{L}_2[0, \infty)$.

The state estimation problem is to find an estimate $\hat{z}$ of z in some sense using the measurement of y. One of the most common frameworks for the state estimation problem is based on the H^∞ theory. More precisely, the problem of H^∞ state estimation can be stated as follows; e.g., see (Nagpal and Khargonekar, 1991):

Given a constant $\gamma > 0$, find a causal unbiased filter $\hat{z}(t) = \mathcal{F}[t, y(\cdot)\mid_0^t]$ if it exists such that

$$J := \sup_{[w(\cdot), v(\cdot)] \in \mathbf{L}_2[0,\infty)} \frac{\|z(\cdot) - \hat{z}(\cdot)\|_2^2}{\|w(\cdot)\|_2^2 + \|v(\cdot)\|_2^2} < \gamma^2. \tag{2}$$

Let $m < l$. We will consider the problem of H^∞ state estimation via a communication channel of dimension m.

Suppose estimates of the output $z(t)$ are required at a distant location, and are to be transmitted via a limited capacity communication channel such that only m real numbers may be sent at each time t. We consider a system which consists of the coder, the transmission channel, and the decoder. Using the measurement $y(\cdot)\mid_0^t$, the coder produces a vector $\hat{y}(t)$ of dimension m which is transmitted via the channel and then received by the decoder. In its turn, the decoder produces an estimate $\hat{z}(t)$ which depends only on $\hat{y}(\cdot)\mid_0^t$.

We consider the class of linear time-invariant coders of the form

$$\begin{aligned} \dot{x}_c(t) &= A_c x_c(t) + B_c y(t); \quad x_c(0) = 0; \\ \hat{y}(t) &= C_c x_c(t) + D_c y(t) \end{aligned} \tag{3}$$

where $\hat{y}(t) \in \mathbf{R}^m$ is the signal transmitted via the communication channel. Note that the dimension of the coder state vector $x_c(t)$ may be arbitrary.

Also, we consider linear time-invariant decoders of the form

$$\begin{aligned} \dot{x}_d(t) &= Ax_d(t) + B_d \hat{y}(t); \quad x_d(0) = 0; \\ \hat{z}(t) &= Cx_d(t). \end{aligned} \tag{4}$$

The system (4) is an estimator for the system (1) and we wish to make the dynamics of (4) as close as possible to the dynamics of (1).

Consider the system (1). Let $\gamma > 0$ and $m < l$ be given. The H^∞ state estimation problem via a communication channel of dimension m with disturbance attenuation γ is said to have a solution if there exist a coder of the form (3) and a decoder of the form (4) such that the condition (2) holds.

The problem of H^∞ state estimation via limited capacity communication channel is to find matrix coefficients A_c, B_c, C_c and B_d (if they exist) to satisfy the H^∞ requirement (2). This problem is quit difficult. In this paper, we consider a simplified problem statement:

Suppose that B_d is given. Find coefficients of the coder (3) such that condition (2) holds.

Consider the system (1). Let $\gamma > 0$, $m < l$ and B_d be given. The H^∞ state estimation problem via a communication channel of dimension m with disturbance attenuation γ and the is said to have a solution with the decoder (4) if there exists a coder of the form (3) such that the requirement (2) holds.

3. THE MAIN RESULT

Our results require the following assumptions.

Assumption 1 The pair (A, B) is controllable.

Assumption 2 The pair (A, K) is observable.

Assumption 3 The pair (A, C) is observable.

Our solution to the above problem involves the following Riccati algebraic equations

$$AY + YA' + Y[\frac{1}{\gamma^2}K'K - C'C]Y + BB' = 0, \quad (5)$$

$$A'X_\epsilon + X_\epsilon A + X_\epsilon(\frac{1}{\epsilon}B_d B_d' - \frac{1}{\gamma^2}BB')X_\epsilon + K'K = 0. \quad (6)$$

Now we are in a position to present the main result of this paper.

Theorem 1. *Consider the system (1), suppose that Assumption 1-4 holds and the coefficient B_d of the decoder (4) is given. Let $\gamma > 0$ be a given constant and m be a given integer. Then, the following statements are equivalent:*

(i) The H^∞ state estimation problem via a communication channel of dimension m with disturbance attenuation γ has a solution with the decoder (4).

(ii) There exists a constant $\epsilon > 0$ such that the algebraic Riccati equations (5) and (6) have stabilizing solutions $Y \geq 0$ and $X_\epsilon \geq 0$ such that $\rho(YX_\epsilon) < \gamma^2$ where $\rho(\cdot)$ denotes the spectral radius of a matrix.

Furthermore, suppose that condition (ii) holds. Then, the coder (3) with

$$A_c = \begin{pmatrix} \tilde{A}_c & -\tilde{B}_c C \\ -B\tilde{C}_c & A \end{pmatrix};$$

$$B_c = \begin{pmatrix} \tilde{B}_c \\ 0 \end{pmatrix}; \qquad C_c = \begin{pmatrix} \tilde{C}_c & 0 \end{pmatrix};$$

$$\tilde{A}_c = A + B\tilde{C}_c - \tilde{B}_c C + \frac{1}{\gamma^2}BB'X_\epsilon,$$

$$\tilde{B}_c = (I - \frac{1}{\gamma^2}YX_\epsilon)^{-1}YC', \quad \tilde{C}_c = \frac{1}{\epsilon}B_d'X_\epsilon \quad (7)$$

solves the H^∞ state estimation problem via a communication channel of dimension m with disturbance attenuation γ with the decoder (4).

Proof $((i) \Rightarrow (ii))$ Assume that condition (i) holds and consider the decoder (4). Introduce a new vector variable $\tilde{x} := x(t) - x_d(t)$. Then $\tilde{x}(t)$ satisfies the equation

$$\dot{\tilde{x}}(t) = A\tilde{x}(t) + B\hat{y}(t) + Bw(t); \quad \tilde{x}(0) = 0. \quad (8)$$

Furthermore, introduce

$$\tilde{z}(t) := K\tilde{x}(t); \qquad \tilde{y}(t) := C\tilde{x}(t) + v(t). \quad (9)$$

Now consider the linear system (8), (9) and suppose that $\hat{y}(t)$ is the control input, $w(t)$ and $v(t)$ are the disturbance inputs, $\tilde{y}(t)$ is the measured output, $\tilde{z}(t)$ is the controlled output. The requirement (2) can now be re-written as

$$J := \sup_{[w(\cdot),v(\cdot)]\in \mathbf{L}_2[0,\infty)} \frac{\|\tilde{z}(\cdot)\|_2^2}{\|w(\cdot)\|_2^2 + \|v(\cdot)\|_2^2} < \gamma^2. \quad (10)$$

Now the coder (3) can be considered as an output feedback controller with input $\tilde{y}(\cdot)$ and output $\hat{y}(\cdot)$. Furthermore, this controller solves the output feedback H^∞ control problem (10). Note that the H^∞ control problem (10) is a singular problem. The approach taken to solve this singular problem is a perturbation approach such as contained in (Petersen, 1987; Khargonekar et al., 1990). As it was shown in (Khargonekar et al., 1990), since the controller (3) solves the H^∞ control problem (10), there exists a small $\epsilon > 0$ such that the same controller solves the non-singular H^∞ control problem

$$J := \sup_{[w(\cdot),v(\cdot)]\in \mathbf{L}_2[0,\infty)} \frac{\|\tilde{z}(\cdot)\|_2^2 + \epsilon\|\tilde{y}(\cdot)\|_2^2}{\|w(\cdot)\|_2^2 + \|v(\cdot)\|_2^2} < \gamma^2. \quad (11)$$

Now condition (ii) follows from the standard H^∞ control theory; e.g., see (Petersen *et al.*, 2000), p.73. This completes the prof of this part of the theorem.

$((ii) \Rightarrow (i))$

Suppose that condition (ii) holds. Then, it follows from the standard H^∞ control theory (e.g., see (Petersen et al., 2000, pp. 73 and 78) that the linear output feedback controller

$$\dot{\tilde{x}}_c(t) = \tilde{A}_c\tilde{x}_c(t) + \tilde{B}_c\tilde{y}(t); \quad \tilde{x}_c(0) = 0;$$
$$\hat{y}(t) = C_c\tilde{x}_c(t) \quad (12)$$

with the coefficients defined by (7) solves the H^∞ control problem (11) for the system (8), (9). Since $\tilde{z} = z - \hat{z}$, the H^∞ requirement (2) follows from (2). Furthermore, since $\tilde{y}(t) = y(t) - Cx_d(t)$ and $x_d(t)$ is defined by (4), (12) can be re-written in the form (3) with the coefficients defined by (7). This completes the proof of the theorem. □

4. CONCLUSIONS

A new approach to the state estimation problem via limited capacity communication channels was proposed. A constructive method to design a linear state estimation system that satisfies the standard H^∞ filtering requirement and a bandwidth limitation constraint was presented.

Acknowledgement. The work was supported by the Australian Research Council.

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