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DECOMPOSITION FEEDBACK STABILIZATION OF SYSTEMS WITH DELAY

Alexandr G. Loukianov*, B. Castillo-Toledo*, J. Escoto Hernández*, and Vadim I. Utkin**

***CINVESTAV del IPN, Apdo. Postal 31-438, C.P. 44550, Guadalajara, Jal, México, e-mail: louk[toledo]@gdl.cinvestav.mx**
**** Department of Electrical Engineering, The Ohio-State University, Columbus, Ohio 43210-1272, e-mail: utkin@ee.eng.ohio-state.edu)*

Abstract: This paper applies the block control method to design a decomposed finite spectrum assignment control law suitable for multivariable linear time-delay systems. A block controllable form is introduced and a non-singular transformation that reduces the system to this form is proposed. Conditions of stability of the closed-loop system are derived. *Copyright © 2002 IFAC*
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1. INTRODUCTION

It is well known that delays often presented in various engineering systems may dramatically limit the performance and sometimes destabilize the closed-loop system dynamics. Therefore, the feedback stabilization problem has been extensively studied, and several controllers based on optimal control method (Zavarei, and Jamshidi, 1987; Feron, *et al*., 1992) including H_∞ and LMI approaches (see, Li, *et al*., 1997; Leyva-Ramos, and Pearson, 2000), have been proposed. The common feature of the mentioned papers is that their derivations are based on analysis of complete order system.

In this paper, in order to assign finite spectrum in linear systems with delay, the block control principle is applied. In order to achieve this, a special state representation must be used which will be referred as the Block Controllable Form with Delay (or BCD-form), consisting of a set of blocks that can be of different dimension. This approach has successfully been employed for decomposition control of linear (Dodds and Loukianov, 1997), and possibility of applying this approach to design controller for delayed systems, is investigated.

2. BLOCK DECOMPOSITION

Consider a linear time-delay system described by the following state equation

$$\dot{\mathbf{x}}(t)=\mathbf{A}\mathbf{x}(t)+\mathbf{C}\mathbf{x}(t-\tau)+\mathbf{B}\mathbf{u}(t)+\mathbf{D}\mathbf{u}(t-\tau) \qquad (1)$$

where

$\mathbf{x}\in R^n$, $\mathbf{u}\in R^m$, $\mathbf{x}(t)=\varphi(t)$, $\forall t\in[t_0-\tau,\ t_0]$, and $\varphi(t)$ is continuous vector-valued initial function.

The essential feature of the proposed method is the conversation of the system (1) into following BCD-form consisting of r blocks:

$$\begin{aligned}\dot{\mathbf{x}}_r(t)&=\mathbf{A}_r\mathbf{x}_r(t)+\mathbf{C}_r\mathbf{x}_r(t-\tau)\\&\quad+\bar{\mathbf{B}}_{r,1}[x_{r-1}(t)+\mathbf{\Pi}_r\mathbf{x}_{r-1}(t-\tau)]\end{aligned} \qquad (2a)$$

$$\dot{\mathbf{x}}_i(t)=\mathbf{A}_i\bar{\mathbf{x}}_i(t)+\mathbf{C}_i\bar{\mathbf{x}}_i(t-\tau)+\bar{\mathbf{B}}_{i,1}[\mathbf{x}_{i-1}(t)+\mathbf{\Pi}_i\mathbf{x}_{i-1}(t-\tau)]$$
$$i=2,\dots,r-1 \qquad (2b)$$

$$\dot{\mathbf{x}}_1(t)=\mathbf{A}_1\bar{\mathbf{x}}_1(t)+\mathbf{C}_1\bar{\mathbf{x}}_1(t-\tau)+\bar{\mathbf{B}}_{11}[\mathbf{u}(t)+\mathbf{\Pi}_1\mathbf{u}(t-\tau)] \qquad (2c)$$

where $\mathbf{x}=(\mathbf{x}_r,\dots,\mathbf{x}_1)^T$, $\bar{\mathbf{x}}_i=(\mathbf{x}_r,\dots,\mathbf{x}_i)^T$, $\mathbf{x}_i\in R^{n_i}$,

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$$rank\bar{\mathbf{B}}_{i,1} = n_i, \; i = 1, \cdots, r, \; \sum_{i=1}^{r} n_i = n. \qquad (3)$$

The integers $n_1, n_2, \cdots, n_r$ define the structure of the system and satisfy the following condition:

$$n_r \le n_{r-1} \le \cdots \le n_1 \le m.$$

The case, when ($n_i = n_{i-1}$), was considered by Loukianov and Escoto, (2000). Here, the case when

$$n_r < n_{r-1} < \cdots < n_1 < m \qquad (4)$$

will be investigated.

The initial system (1) is brought to the form (2a)-(2c) through the following iterative transformation procedure consisting of $(r-1)$ steps:

Step 1. The following assumptions will be carried out for each step of the procedure.

A11. $rank\mathbf{B} = n_1 < m$.

This means that some control components are linearly dependent. Then we can find a matrix $\mathbf{\Gamma}_1 \in R^{m \times n_1}$ such that

$$\bar{\mathbf{B}} = \mathbf{B\Gamma}_1 \qquad (5)$$

with $\bar{\mathbf{B}} \in R^{n \times n_1}$, and $rank\bar{\mathbf{B}} = n_1$. Using (5), a new control vector $\mathbf{v}$, $\mathbf{v} \in R^{n_1}$ of reduced dimension is defined as

$$\mathbf{u}(t) = \mathbf{\Gamma}_1 \mathbf{v}(t). \qquad (6)$$

So, the components of the new vector v are linearly independent. Using (5) and (6), the system (1) can be represented of the form

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{C}\mathbf{x}(t-\tau) + \bar{\mathbf{B}}\mathbf{v}(t) + \bar{\mathbf{D}}\mathbf{v}(t-\tau) \qquad (7)$$

with $\bar{\mathbf{D}} = \mathbf{D\Gamma}_1$, and $rank\bar{\mathbf{B}} = n_1$.

A12. There exist a matrix $\mathbf{\Pi}_1 \in R^{n_1 \times n_1}$ such that

$$\bar{\mathbf{D}} = \bar{\mathbf{B}}\mathbf{\Pi}_1.$$

This condition is commonly called *the matching condition* (Drajenovic, 1969). Using these assumptions, vector $\mathbf{x}$, and matrices $\mathbf{B}$, $\bar{\mathbf{B}}$, $\mathbf{D}$ and $\bar{\mathbf{D}}$ can be partitioned as

$$\mathbf{x} = \begin{bmatrix} \mathbf{x}_{12} \\ \mathbf{x}_1 \end{bmatrix}, \; \mathbf{B} = \begin{bmatrix} \mathbf{B}_{12} \\ \mathbf{B}_{11} \end{bmatrix}, \; \bar{\mathbf{B}} = \begin{bmatrix} \bar{\mathbf{B}}_{12} \\ \bar{\mathbf{B}}_{11} \end{bmatrix},$$

$$\mathbf{D} = \begin{bmatrix} \mathbf{D}_{12} \\ \mathbf{D}_{11} \end{bmatrix} \text{ and } \bar{\mathbf{D}} = \begin{bmatrix} \bar{\mathbf{D}}_{12} \\ \bar{\mathbf{D}}_{11} \end{bmatrix}$$

with

$\mathbf{x}_{12} \in R^{n-n_1}, \mathbf{x}_1 \in R^{n_1}, rank\mathbf{B}_{11} = rank\bar{\mathbf{B}}_{11} = n_1$.

Now, the following orthogonal transformation:

$$\mathbf{x}''(t) = \mathbf{M}_1 \mathbf{x}(t), \quad \mathbf{M}_1 = \begin{bmatrix} \mathbf{I}_{n-n_1} & -\bar{\mathbf{B}}_{12}\bar{\mathbf{B}}^{-1}{}_{11} \\ 0 & \mathbf{I}_{n_1} \end{bmatrix} \qquad (8)$$

$$\mathbf{M}_1\bar{\mathbf{B}} = \mathbf{M}_1 \begin{bmatrix} \bar{\mathbf{B}}_{12} \\ \bar{\mathbf{B}}_{11} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \bar{\mathbf{B}}_{11} \end{bmatrix}$$

is introduced. Then, from the condition **A12,** it follows

$$\mathbf{M}_1\bar{\mathbf{D}} = \mathbf{M}_1 \begin{bmatrix} \bar{\mathbf{D}}_{12} \\ \bar{\mathbf{D}}_{11} \end{bmatrix} = \mathbf{M}_1 \begin{bmatrix} \bar{\mathbf{B}}_{12}\mathbf{\Pi}_1 \\ \bar{\mathbf{B}}_{11}\mathbf{\Pi}_1 \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \bar{\mathbf{B}}_{11}\mathbf{\Pi}_1 \end{bmatrix}.$$

With the above transformation the system (7) is represented of the form

$$\dot{\mathbf{x}}'_2(t) = \mathbf{A}'_{22}\mathbf{x}'_2(t) + \mathbf{C}'_{22}\mathbf{x}'_2(t-\tau) + \mathbf{B}_2\mathbf{x}_1(t) + \mathbf{D}_2\mathbf{x}_1(t-\tau) \qquad (9a)$$

$$\dot{\mathbf{x}}_1(t) = \mathbf{A}_{12}\mathbf{x}'_2(t) + \mathbf{A}_{11}\mathbf{x}_1(t) + \mathbf{C}_{12}\mathbf{x}'_2(t-\tau) + \mathbf{C}_{11}\mathbf{x}_1(t-\tau) + \bar{\mathbf{B}}_{11}\left[\mathbf{v}(t) + \mathbf{\Pi}_1\mathbf{v}(t-\tau)\right] \qquad (9b)$$

where $\mathbf{x}'' = (\mathbf{x}'_2, \mathbf{x}_1)^T$, $\mathbf{x}'_2 \in R^{n-n_1}$, $\mathbf{x}_1 \in R^{n_1}$, and

$$\mathbf{M}_1\mathbf{A}\mathbf{M}_1^{-1} = \begin{bmatrix} \mathbf{A}'_{22} & \mathbf{B}_2 \\ \mathbf{A}_{12} & \mathbf{A}_{11} \end{bmatrix}, \; \mathbf{M}_1\mathbf{C}\mathbf{M}_1^{-1} = \begin{bmatrix} \mathbf{C}'_{22} & \mathbf{D}_2 \\ \mathbf{C}_{12} & \mathbf{C}_{11} \end{bmatrix}.$$

Note that from (5) it follows

$$\bar{\mathbf{B}} = \begin{bmatrix} \bar{\mathbf{B}}_{12} \\ \bar{\mathbf{B}}_{11} \end{bmatrix} = \mathbf{B\Gamma}_1 = \begin{bmatrix} \mathbf{B}_{12}\mathbf{\Gamma}_1 \\ \mathbf{B}_{11}\mathbf{\Gamma}_1 \end{bmatrix}, \text{ and } \bar{\mathbf{B}}_{11} = \mathbf{B}_{11}\mathbf{\Gamma}_1.$$

This relation will be used in the development of the control design in the following section.

Step 2. Now, the assumptions **A11** and **A12** for subsystem (9a) can be repeated as

A21. $rank\mathbf{B}_2 = n_2 < n_1$.

Similar to step 1, there exists a matrix $\mathbf{\Gamma}_2$, $\mathbf{\Gamma}_2 \in R^{n_1 \times n_2}$ such that

$$\mathbf{x}_1(t) = \mathbf{\Gamma}_2\mathbf{w}_1(t) \qquad (10)$$

and

$$rank\bar{\mathbf{B}}_2 = n_2, \; \bar{\mathbf{B}}_2 = \mathbf{B}_2\mathbf{\Gamma}_2$$

where $w_1 \in R^{n_2}$ is a new input vector for (9a). Then the subsystem (9a) with the transformation (10) can be presented of the form

$$\dot{\mathbf{x}}'_2(t) = \mathbf{A}'_{22}\mathbf{x}'_2(t) + \mathbf{C}'_{22}\mathbf{x}'_2(t-\tau) + \bar{\mathbf{B}}_2\mathbf{w}_1(t) + \bar{\mathbf{D}}_2\mathbf{w}_1(t-\tau) \qquad (11)$$

where $\bar{\mathbf{D}}_2 = \mathbf{B}_2\mathbf{\Gamma}_2$.

A22. There exists matrix $\mathbf{\Pi}_2 \in R^{n_2 \times n_2}$ such that

$$\bar{\mathbf{D}}_2 = \bar{\mathbf{B}}_2\mathbf{\Pi}_2. \qquad (12)$$

Then three different cases are possible depending of the value of n_2:

(i) $n_2 = 0$. This means that system (9a) is uncontrollable, and hence the initial system (1) is uncontrollable as well.

(ii) $n_2 = n - n_1$. In this case, defining

$$\mathbf{x}_2(t) = \mathbf{x}'_2(t), \; \mathbf{A}_{22} = \mathbf{A}'_{22}, \; \mathbf{C}_{22} = \mathbf{C}'_{22}, \; \bar{\mathbf{B}}_{21} = \bar{\mathbf{B}}_2$$

the transformed system (11), (9b) with (12):

$$\dot{\mathbf{x}}_2(t) = \mathbf{A}_{22}\mathbf{x}_2(t) + \mathbf{C}_{22}\mathbf{x}_2(t-\tau) + \bar{\mathbf{B}}_{21}[\mathbf{w}_1(t) + \mathbf{\Pi}_2\mathbf{w}_1(t-\tau)]$$

$$\dot{\mathbf{x}}_1(t) = \mathbf{A}_{12}\mathbf{x}_2(t) + \mathbf{A}_{11}\mathbf{x}_1(t) + \mathbf{C}_{12}\mathbf{x}_2(t-\tau) + \mathbf{C}_{11}\mathbf{x}_1(t-\tau) + \bar{\mathbf{B}}_{11}\left[\mathbf{v}(t) + \mathbf{\Pi}_1\mathbf{v}(t-\tau)\right]$$

with $rank\bar{\mathbf{B}}_{i,1} = n_i$, $i = 1,2$, $n_1 + n_2 = n$, is of the BCD-form.

(iii) $n_2 < n - n_1$. In this case, subsequent step is necessary, and the system (11) with state $\mathbf{x}'_2(t)$ and input $\mathbf{x}_1(t)$ is further decomposed and transformed.

Step k. The system obtained at $(k-1)^{th}$ step has the following form:

$$\begin{aligned}\dot{\mathbf{x}}'_k(t) &= \mathbf{A}'_{kk}\mathbf{x}'_k(t) + \mathbf{C}'_{kk}\mathbf{x}_k(t-\tau) \\ &\quad + \mathbf{B}_k\mathbf{x}_{k-1}(t) + \mathbf{D}_k\mathbf{x}_{k-1}(t-\tau)\end{aligned} \tag{13a}$$

$$\begin{aligned}\dot{\mathbf{x}}_i(t) &= \mathbf{A}_i\bar{\mathbf{x}}_i(t) + \mathbf{C}_i\bar{\mathbf{x}}_i(t-\tau) \\ &\quad + \bar{\mathbf{B}}_{i,1}[\mathbf{w}_{i-1}(t) + \mathbf{\Pi}_i\mathbf{w}_{i-1}(t-\tau)], \quad i = 2,...,k-1\end{aligned} \tag{13b}$$

$$\dot{\mathbf{x}}_1(t) = \mathbf{A}_1\bar{\mathbf{x}}_1(t) + \mathbf{C}_1\bar{\mathbf{x}}_1(t-\tau) + \bar{\mathbf{B}}_{11}[\mathbf{v}(t) + \mathbf{\Pi}_1\mathbf{v}(t-\tau)] \tag{13c}$$

where $\mathbf{x}_i \in R^{n_i}$, and $\mathbf{x}'_k$ is a vector of dimension $(n - n_1 - \cdots - n_{k-1})$, $\mathbf{w}_i \in R^{n_{i+1}}$, $i = 1,...,k-1$, and

$$rank\bar{\mathbf{B}}_{i,1} = n_i, \quad i = 1,...,k-1.$$

For this step we generalize assumptions **A11**, **A12, A21** and **A22** as follows:

Ak1. $rank\mathbf{B}'_k = n_k < n_{k-1}$.

Then, it is necessary to find a transformation

$$\mathbf{x}_{k-1}(t) = \mathbf{\Gamma}_k\mathbf{w}_{k-1}(t), \quad \bar{\mathbf{B}}_k = \mathbf{B}_k\mathbf{\Gamma}_k \tag{14}$$

such that the matrix $\bar{\mathbf{B}}_k$ has full rank, i.e.

$$rank\bar{\mathbf{B}}_k = n_k$$

where $\mathbf{x}_{k-1} \in R^{n_{k-1}}$, $\mathbf{\Gamma}_k \in R^{n_{k-1} \times n_k}$, and $\mathbf{w}_{k-1} \in R^{n_k}$ is a new input vector for (13a). Using (14), the subsystem (13a) is represented as

$$\begin{aligned}\dot{\mathbf{x}}'_k(t) &= \mathbf{A}'_{kk}\mathbf{x}'_k(t) + \mathbf{C}'_{kk}\mathbf{x}'_k(t-\tau) \\ &\quad + \bar{\mathbf{B}}_k\mathbf{w}_{k-1}(t) + \bar{\mathbf{D}}_k\mathbf{w}_{k-1}(t-\tau)\end{aligned} \tag{15}$$

where $\bar{\mathbf{D}}_k = \mathbf{D}_k\mathbf{\Gamma}_k$.

Ak2. There exists a matrix $\mathbf{\Pi}_k \in R^{n_k \times n_k}$ such that

$$\bar{\mathbf{D}}_k = \bar{\mathbf{B}}_k\mathbf{\Pi}_k. \tag{16}$$

If $n_k = n - \sum_{j=1}^{k-1} n_j$, then after defining $\mathbf{x}_k(t) = \mathbf{x}'_k(t)$ $\mathbf{A}_{kk} = \mathbf{A}'_{kk}$, $\mathbf{C}_{kk} = \mathbf{C}'_{kk}$, $\bar{\mathbf{B}}_{k,1} = \bar{\mathbf{B}}_k$ the algorithm terminates giving the equations (15), (13b)-(13c) with (16) as the BCD-form. But if $n_k < n - \sum_{j=1}^{k-1} n_j$, then the subsystem (15) can be divided as

$$\mathbf{x}'_k = \begin{bmatrix}\mathbf{x}_{k,2} \\ \mathbf{x}_k\end{bmatrix}, \quad \bar{\mathbf{B}}_k = \begin{bmatrix}\bar{\mathbf{B}}_{k,2} \\ \bar{\mathbf{B}}_{k,1}\end{bmatrix} \text{ and } \bar{\mathbf{D}}_k = \begin{bmatrix}\bar{\mathbf{D}}_{k,2} \\ \bar{\mathbf{D}}_{k,1}\end{bmatrix}$$

where, x_k and $x_{k,2}$ are vectors $n_k \times 1$ and $\left(n - \sum_{j=1}^{k-1} n_j - n_k\right) \times 1$ respectively, $rank\bar{\mathbf{B}}_{k,1} = n_k$.

Note that from $\bar{\mathbf{B}}_k = \mathbf{B}_k\mathbf{\Gamma}_k$ (14), it follows

$$\bar{\mathbf{B}}_k = \begin{bmatrix}\bar{\mathbf{B}}_{k,2} \\ \bar{\mathbf{B}}_{k,1}\end{bmatrix} = \mathbf{B}_k\mathbf{\Gamma}_k = \begin{bmatrix}\mathbf{B}_{k,2}\mathbf{\Gamma}_k \\ \mathbf{B}_{k,1}\mathbf{\Gamma}_k\end{bmatrix}$$

or

$$\bar{\mathbf{B}}_{k,1} = \mathbf{B}_{k,1}\mathbf{\Gamma}_k.$$

Proceeding as in the first step, under the previous assumptions, the similar to (8) transformation:

$$\mathbf{x}''_k(t) = \mathbf{M}_k\mathbf{x}'_k(t), \quad \mathbf{M}_k = \begin{bmatrix}\mathbf{I}_{n-n_1-\cdots-n_k} & -\bar{\mathbf{B}}_{k,2}\bar{\mathbf{B}}_{k,1}^{-1} \\ 0 & \mathbf{I}_{n_k}\end{bmatrix}$$

$$\mathbf{M}_k\bar{\mathbf{B}}_k = \mathbf{M}_k\begin{bmatrix}\bar{\mathbf{B}}_{k,2} \\ \bar{\mathbf{B}}_{k,1}\end{bmatrix} = \begin{bmatrix}\mathbf{0} \\ \bar{\mathbf{B}}_{k,1}\end{bmatrix},$$

$$\mathbf{M}_k\bar{\mathbf{D}}_k = \mathbf{M}_k\begin{bmatrix}\bar{\mathbf{D}}_{k2} \\ \bar{\mathbf{D}}_{k,1}\end{bmatrix} = \mathbf{M}_k\begin{bmatrix}\bar{\mathbf{B}}_{k2}\mathbf{\Pi}_k \\ \bar{\mathbf{B}}_{k,1}\mathbf{\Pi}_k\end{bmatrix} = \begin{bmatrix}\mathbf{0} \\ \bar{\mathbf{B}}_{k,1}\mathbf{\Pi}_k\end{bmatrix}$$

is used. Then, the system (13a)-(13c) is governed by

$$\begin{aligned}\dot{\mathbf{x}}'_{k+1}(t) &= \mathbf{A}'_{k+1}\mathbf{x}'_{k+1}(t) + \mathbf{C}'_{k+1}\mathbf{x}'_{k+1}(t-\tau) \\ &\quad + \mathbf{B}_{k+1}\mathbf{x}_k(t) + \mathbf{D}_{k+1}\mathbf{x}_k(t-\tau) \\ \dot{\mathbf{x}}_k(t) &= \mathbf{A}_k\bar{\mathbf{x}}_k(t) + \mathbf{C}_k\bar{\mathbf{x}}_k(t-\tau) \\ &\quad + \bar{\mathbf{B}}_{k,1}[\mathbf{w}_{k-1}(t) + \mathbf{\Pi}_k\mathbf{w}_{k-1}(t-\tau)] \\ \dot{\mathbf{x}}_i(t) &= \mathbf{A}_i\bar{\mathbf{x}}_i(t) + \mathbf{C}_i\bar{\mathbf{x}}_i(t-\tau) \\ &\quad + \bar{\mathbf{B}}_{i,1}[\mathbf{w}_{i-1}(t) + \mathbf{\Pi}_i\mathbf{w}_{i-1}(t-\tau)], \quad i = 2,...,k-1 \\ \dot{\mathbf{x}}_1(t) &= \mathbf{A}_1\bar{\mathbf{x}}_1(t) + \mathbf{C}_1\bar{\mathbf{x}}_1(t-\tau) + \bar{\mathbf{B}}_{11}[\mathbf{v}(t) + \mathbf{\Pi}_1\mathbf{v}(t-\tau)]\end{aligned}$$

where $\mathbf{x}''_k = (\mathbf{x}'_{k+1}, \quad \mathbf{x}_k)^T$ and

$$rank\bar{\mathbf{B}}_{i,i} = n_i, \quad i = 1,...,k.$$

From the previous algorithm, the following result is stated:

THEOREM 1. *Assume that*

A) *The system* (1) *is controllable.*

B) *At each step of the BCD-form algorithm, assumptions* **Ak1** *and* **Ak2** *hold.*

Then, there exists an integer $r \le n$ *such that the system* (1) *takes the form* (2a)-(2c).

<u>Remark</u>: The assumptions **Ak1**, k=*1,…,r-1* mean that the system (1) has the structure (4).

3. FEEDBACK CONTROLLER DESIGN

In this section a state feedback control law is developed for transformed system (2a)-(2c). It is more convenient to renumber the state variables of (2a)-(2c) as

$$\begin{aligned}\dot{\mathbf{x}}_1(t) &= \mathbf{A}_{11}\mathbf{x}_1(t) + \mathbf{C}_{11}x_1(t-\tau) + \bar{\mathbf{B}}_1\mathbf{v}_1(t) \\ \mathbf{v}_1(t) &= [\mathbf{w}_2(t) + \mathbf{\Pi}_1\mathbf{w}_2(t-\tau)]\end{aligned} \tag{17a}$$

$$\begin{aligned}\dot{\mathbf{x}}_i(t) &= \mathbf{A}_i\bar{\mathbf{x}}_i(t) + \mathbf{C}_i\bar{\mathbf{x}}_i(t-\tau) + \bar{\mathbf{B}}_i\mathbf{v}_i(t) \\ \mathbf{v}_i(t) &= [\mathbf{w}_{i+1}(t) + \mathbf{\Pi}_i\mathbf{w}_{i+1}(t-\tau)], \quad i = 2,...,r-1\end{aligned} \tag{17b}$$

$$\begin{aligned}\dot{\mathbf{x}}_r(t) &= \mathbf{A}_r\bar{\mathbf{x}}_r(t) + \mathbf{C}_r\bar{\mathbf{x}}_r(t-\tau) + \bar{\mathbf{B}}_r\mathbf{v}_r(t) \\ \mathbf{v}_r(t) &= [\mathbf{v}(t) + \mathbf{\Pi}_r\mathbf{v}(t-\tau)]\end{aligned} \tag{17c}$$

where $\mathbf{x} = (\mathbf{x}_1,...,\mathbf{x}_r)^T$, $\bar{\mathbf{x}}_i = (\mathbf{x}_1,...,\mathbf{x}_i)^T$, $\mathbf{x}_i \in R^{n_i}$, $\mathbf{w}_i \in R^{n_{i+1}}$, $\mathbf{\Pi}_i \in R^{n_i \times n_i}$, and

$$\mathbf{u} = \mathbf{\Gamma}_r \mathbf{v}, \quad \mathbf{\Gamma}_r \in R^{m \times n_r} \tag{17d}$$

$$\mathbf{x}_i = \mathbf{\Gamma}_{i-1} \mathbf{w}_i, \; \mathbf{\Gamma}_{i-1} \in R^{n_i \times n_{i-1}}, \quad i = 2, ..., r \tag{17e}$$

$$\overline{\mathbf{B}}_i = \mathbf{B}_i \mathbf{\Gamma}_i, \quad \mathbf{B}_i \in R^{n_i \times n_{i-1}}, \quad i = 1, \cdots, r \tag{17f}$$

$$rank\overline{\mathbf{B}}_i = rank\mathbf{B}_i = n_i, \quad i = 1, \cdots, r, \quad \sum_{i=1}^{r} n_i = n.$$

The control strategy for system (17a)-(17f) can be designed considering v_i as a fictitious control vector in the i^{th} block. This procedure is outlined in the following.

Step 1. Let the fictitious control v_1 in the first block (17a) be chosen of the form

$$\mathbf{v}_1(t) = \mathbf{v}_{1c}(t) + \overline{\mathbf{B}}_1^{-1}\left[\mathbf{\Lambda}_1 \mathbf{z}_1(t) + \mathbf{E}_{11}\mathbf{z}_2(t)\right] \tag{18a}$$

where $\mathbf{z}_1 = \mathbf{x}_1$ and $\mathbf{z}_2 \in R^{n2}$ are new variable vectors, $\mathbf{\Lambda}_1 \in R^{n_1 \times n_1}$ is a Hurtwitz matrix with desired eigenvalues, $\mathbf{E}_{11}$, $\mathbf{E}_{11} \in R^{n_1 \times n_2}$ will be defined during the procedure, and $\mathbf{v}_{1c}(t)$ is calculated from equation $\dot{\mathbf{z}}_1(t) = 0$ along the trajectories of the 1st block (17a) as

$$\mathbf{v}_{1c}(t) = -\overline{\mathbf{B}}_1^{-1}\left[\mathbf{A}_{11}\mathbf{x}_1(t) + \mathbf{C}_{11}\mathbf{x}_1(t-\tau)\right]. \tag{18b}$$

The transformed 1^{st} block in new coordinates $z_1(t)$ and $\mathbf{z}_2(t)$ with input (18a) y (18b) has the following desired form without delay:

$$\dot{\mathbf{z}}_1(t) = \mathbf{\Lambda}_1 \mathbf{z}_1(t) + \mathbf{E}_{11}\mathbf{z}_2(t). \tag{19}$$

Substituting (18b) in (18a) and multiplying by the matrix $\overline{\mathbf{B}}_1$ from the right, gives

$$\overline{\mathbf{B}}_1 \mathbf{v}_1(t) = -\left[\mathbf{A}_1 \mathbf{x}_1(t) + \mathbf{C}_1 \mathbf{x}_1(t-\tau)\right] + \left[\mathbf{\Lambda}_1 \mathbf{z}_1(t) + \mathbf{E}_{11}\mathbf{z}_2(t)\right]. \tag{20}$$

Now from the relationship (17a)-(17f), namely

$$\overline{\mathbf{B}}_1 = \mathbf{B}_1 \mathbf{\Gamma}_1, \quad \mathbf{v}_1(t) = \mathbf{w}_2(t) + \mathbf{\Pi}_1 \mathbf{w}_2(t-\tau),$$

$$\mathbf{x}_2(t) = \mathbf{\Gamma}_1 \mathbf{w}_2(t) \text{ and } \mathbf{w}_2(t) = \mathbf{\Gamma}_1^+ \mathbf{x}_2(t)$$

with $\mathbf{\Gamma}_1^+ = (\mathbf{\Gamma}_1^T \mathbf{\Gamma}_1)^{-1} \mathbf{\Gamma}_1^T$, it follows that

$$\begin{aligned} \overline{\mathbf{B}}_1 v_1(t) &= \mathbf{B}_1 \mathbf{\Gamma}_1 \mathbf{v}_1(t) = \\ & \mathbf{B}_1[\mathbf{\Gamma}_1 \mathbf{w}_2(t) + \mathbf{\Gamma}_1 \mathbf{\Pi}_1 \mathbf{w}_2(t-\tau)] = \\ & \mathbf{B}_1[\mathbf{x}_2(t) + \mathbf{\Gamma}_1 \mathbf{\Pi}_1 \mathbf{\Gamma}_1^+ \mathbf{x}_2(t-\tau)]. \end{aligned} \tag{21}$$

Set

$$\mathbf{E}_{11} = \mathbf{B}_1$$

then from (20) and (21), it follows

$$\mathbf{B}_1\left[\mathbf{x}_2(t) + \mathbf{\Gamma}_1 \mathbf{\Pi}_1 \mathbf{\Gamma}_1^+ \mathbf{x}_2(t-\tau)\right] = -(\mathbf{A}_1 - \mathbf{\Lambda}_1)\mathbf{x}_1(t) - \mathbf{C}_1 \mathbf{x}_1(t-\tau) + \mathbf{B}_1 \mathbf{z}_2(t). \tag{22}$$

In order to define the transformation for $\mathbf{z}_2$, it is assumed that the elements of the matrix $\mathbf{B}_1$ can be rearranged such that the square matrix

$$\tilde{\mathbf{B}}_2 := \begin{bmatrix} \mathbf{B}_1 \\ \mathbf{E}_{12} \end{bmatrix}$$

with $\mathbf{E}_{12} = [\mathbf{0} \quad \mathbf{I}_{n_2 - n_1}]$, $\mathbf{E}_{12} \in R^{(n_2 - n_1) \times n_2}$ has rank n_2. Then set the following equality:

$$\mathbf{E}_{12}[\mathbf{x}_2(t) + \mathbf{\Gamma}_1 \mathbf{\Pi}_1 \mathbf{\Gamma}_1^+ \mathbf{x}_2(t-\tau)] = \mathbf{E}_{12} z_2. \tag{23}$$

From equations (22) and (23) the following nonsingular transformation for $\mathbf{z}_2$:

$$\mathbf{z}_2(t) = \tilde{\mathbf{B}}_2^{-1}\begin{bmatrix} (\mathbf{A}_1 - \mathbf{\Lambda}_1)\mathbf{x}_1(t) + \mathbf{C}_1 \mathbf{x}_1(t-\tau) \\ \mathbf{0} \end{bmatrix} + \left[\mathbf{x}_2(t) + \mathbf{\Gamma}_1 \mathbf{\Pi}_1 \mathbf{\Gamma}_1^+ x_2(t-\tau)\right] \tag{24}$$

is derived. Now using (17e), namely

$$\mathbf{x}_2(t) = \mathbf{\Gamma}_1 \mathbf{w}_2(t) \quad \text{and} \quad \mathbf{w}_2(t) = \mathbf{\Gamma}_1^+ \mathbf{x}_2(t)$$

the second term of (24) can be represented as

$$\begin{aligned} & \mathbf{x}_2(t) + \mathbf{\Gamma}_1 \mathbf{\Pi}_1 \mathbf{\Gamma}_1^+ \mathbf{x}_2(t-\tau) = \\ & \mathbf{\Gamma}_1 \mathbf{w}_2(t) + \mathbf{\Gamma}_1 \mathbf{\Pi}_1 \mathbf{w}_2(t-\tau) = \\ & \mathbf{\Gamma}_1[\mathbf{w}_2(t) + \mathbf{\Pi}_1 \mathbf{w}_2(t-\tau)] = \mathbf{\Gamma}_1 \mathbf{v}_1(t). \end{aligned}$$

Then, the transformation (24) is represented of the following form:

$$\mathbf{z}_2(t) = \tilde{\mathbf{B}}_2^{-1}\begin{bmatrix} (\mathbf{A}_1 - \mathbf{\Lambda}_1)\mathbf{x}_1(t) + \mathbf{C}_1 \mathbf{x}_1(t-\tau) \\ \mathbf{0} \end{bmatrix} + \mathbf{\Gamma}_1 \mathbf{v}_1(t) \tag{25a}$$

$$\mathbf{v}_1(t) = \mathbf{w}_2(t) + \mathbf{\Pi}_1 \mathbf{w}_2(t-\tau) \tag{25b}$$

$$\mathbf{x}_2(t) = \mathbf{\Gamma}_1 \mathbf{w}_2(t). \tag{25c}$$

It is clear that the transformation (25a)-(25c) is stable if all the eigenvalues of the matrix $\mathbf{\Pi}_1$ are located inside the unit circle. From this, the equation (19) is represented as

$$\dot{\mathbf{z}}_1(t) = \mathbf{\Lambda}_1 \mathbf{z}_1(t) + \mathbf{B}_1 \mathbf{z}_2(t).$$

Step 2. Taking the derivative of (24) along the trajectories of the 1^{st} and 2^{nd} blocks of (17a)-(17c), gives

$$\dot{\mathbf{z}}_2(t) = \sum_{j=1}^{2}\begin{bmatrix} \mathbf{A}_{2,j}\mathbf{x}_j(t) + \mathbf{C}_{2,j}^1 \mathbf{x}_j(t-\tau) \\ + \mathbf{C}_{2,j}^2 \mathbf{x}_j(t-2\tau) \end{bmatrix} + \overline{\mathbf{B}}_2 \mathbf{v}_2^1(t) \tag{26a}$$

with

$$\mathbf{v}_2^1(t) = \mathbf{v}_2(t) + \overline{\mathbf{B}}_2^{-1} \mathbf{\Gamma}_1 \mathbf{\Pi}_1 \mathbf{\Gamma}_1^+ \overline{\mathbf{B}}_2 \mathbf{v}_2(t-\tau) \tag{26b}$$

$$\mathbf{v}_2(t) = \mathbf{w}_3(t) + \mathbf{\Pi}_2 \mathbf{w}_3(t-\tau) \tag{26c}$$

$$\mathbf{x}_3(t) = \mathbf{\Gamma}_2 \mathbf{w}_3(t). \tag{26d}$$

As on the first step the fictitious control input $\mathbf{v}_2^1(t)$ in (26a) is chosen similar to (18a) of the form

$$\mathbf{v}_2^1(t) = \mathbf{v}_{2c}^1(t) + \overline{\mathbf{B}}_2^{-1}\left[\mathbf{\Lambda}_2 \mathbf{z}_2(t) + \mathbf{E}_{21}\mathbf{z}_3(t)\right] \tag{27a}$$

where $\mathbf{z}_3 \in R^{n3}$ is the new vector, $\mathbf{\Lambda}_2 \in R^{n_2 \times n_2}$ is a Hurtwitz matrix, $\mathbf{E}_{21} \in R^{n2 \times n3}$, and $\mathbf{v}_{2c}^1(t)$ is calculated from equation $\dot{\mathbf{z}}_2(t) = 0$ of the form

$$\mathbf{v}_{2c}^1(t) = -\overline{\mathbf{B}}_2^{-1}\sum_{j=1}^{2}\begin{bmatrix} \mathbf{A}_{2,j}\mathbf{x}_j(t) + \mathbf{C}_{2,j}^1 \mathbf{x}_j(t-\tau) \\ + \mathbf{C}_{2,j}^2 \mathbf{x}_j(t-2\tau) \end{bmatrix}. \tag{27b}$$

Thus, equation (26a) with (27a) and (27b) takes a form similar to equation (19)

$$\dot{\mathbf{z}}_2(t) = \mathbf{\Lambda}_2 \mathbf{z}_2(t) + \mathbf{E}_{21}\mathbf{z}_3(t). \tag{28}$$

Combining the equations (27a) and (27b), gives

$$\bar{\mathbf{B}}_2\mathbf{v}_2^1(t) = \tag{29}$$
$$-\sum_{j=1}^{2}\left[\mathbf{A}_{2,j}\mathbf{x}_j(t)+\mathbf{C}_{2,j}^1\mathbf{x}_j(t-\tau)+\mathbf{C}_{2,j}^2\mathbf{x}_j(t-2\tau)\right]$$
$$+\left[\mathbf{\Lambda}_2\mathbf{z}_2(t)+\mathbf{E}_{21}\mathbf{z}_3(t)\right].$$

From the other hand, taking into account the relationships (17f) and (26b)-(26c), namely

$$\bar{\mathbf{B}}_2 = \mathbf{B}_2\mathbf{\Gamma}_2,$$
$$\mathbf{v}_2^1(t) = \mathbf{v}_2(t) + \bar{\mathbf{B}}_2^{-1}\mathbf{\Gamma}_1\mathbf{\Pi}_1\mathbf{\Gamma}_1^{+}\bar{\mathbf{B}}_2\mathbf{v}_2(t-\tau),$$
$$\mathbf{v}_2(t) = \mathbf{w}_3(t) + \mathbf{\Pi}_2\mathbf{w}_3(t-\tau)$$

it can be obtained first

$$\bar{\mathbf{B}}_2 v_2^1 = \mathbf{B}_2\mathbf{\Gamma}_2\mathbf{v}_2^1 =$$
$$\mathbf{B}_2\mathbf{\Gamma}_2\left[\mathbf{v}_2(t) + \bar{\mathbf{B}}_2^{-1}\mathbf{\Gamma}_1\mathbf{\Pi}_1\mathbf{\Gamma}_1^{+}\bar{\mathbf{B}}_2\mathbf{v}_3(t-\tau\right] =$$
$$\mathbf{B}_2\mathbf{\Gamma}_2\left\{\begin{array}{l}\left[\mathbf{w}_3(t)+\mathbf{\Pi}_2\mathbf{w}_3(t-\tau)\right]\\ +\bar{\mathbf{B}}_2^{-1}\mathbf{\Gamma}_1\mathbf{\Pi}_1\mathbf{\Gamma}_1^{+}\bar{\mathbf{B}}_2\left[\mathbf{w}_3(t-\tau)+\mathbf{\Pi}_2\mathbf{w}_3(t-2\tau)\right]\end{array}\right\}$$

and then, using $\mathbf{x}_3(t) = \mathbf{\Gamma}_2\mathbf{w}_3(t)$, $\mathbf{w}_3(t) = \mathbf{\Gamma}_2^{+}\mathbf{x}_3(t)$, and $\mathbf{\Gamma}_2^{+} = (\mathbf{\Gamma}_2^{T}\mathbf{\Gamma}_2)^{-1}\mathbf{\Gamma}_2^{T}$, gives

$$\bar{\mathbf{B}}_2\mathbf{v}_2^1 = \tag{30}$$
$$\mathbf{B}_2\left\{\begin{array}{l}\left[\mathbf{x}_3(t)+\mathbf{\Gamma}_2\mathbf{\Pi}_2\mathbf{\Gamma}_2^{+}\mathbf{x}_3(t-\tau)\right]\\ +\mathbf{\Gamma}_2\bar{\mathbf{B}}_2^{-1}\mathbf{\Gamma}_1\mathbf{\Pi}_1\mathbf{\Gamma}_1^{+}\bar{\mathbf{B}}_2\mathbf{\Gamma}_2^{+}\\ \times\left[\mathbf{x}_3(t-\tau)+\mathbf{\Gamma}_2\mathbf{\Pi}_2\mathbf{\Gamma}_2^{+}\mathbf{x}_3(t-2\tau)\right]\end{array}\right\}$$

Now, set $\mathbf{E}_{21}$ in (28) and (29)

$$\mathbf{E}_{21} = \mathbf{B}_2$$

then, from (29) and (30), it follows

$$\mathbf{B}_2\left\{\begin{array}{l}\left[\mathbf{x}_3(t)+\mathbf{\Gamma}_2\mathbf{\Pi}_2\mathbf{\Gamma}_2^{+}\mathbf{x}_3(t-\tau)\right]+\mathbf{\Gamma}_2\bar{\mathbf{B}}_2^{-1}\mathbf{\Gamma}_1\mathbf{\Pi}_1\mathbf{\Gamma}_1^{+}\bar{\mathbf{B}}_2\mathbf{\Gamma}_2^{+}\\ \times\left[\mathbf{x}_3(t-\tau)+\mathbf{\Gamma}_2\mathbf{\Pi}_2\mathbf{\Gamma}_2^{+}\mathbf{x}_3(t-2\tau)\right]\end{array}\right\} =$$
$$-\sum_{j=1}^{2}\left[\mathbf{A}_{2,j}\mathbf{x}_j(t)+\mathbf{C}_{2,j}^1\mathbf{x}_j(t-\tau)+\mathbf{C}_{2,j}^2\mathbf{x}_j(t-2\tau)\right]$$
$$+\mathbf{\Lambda}_2\mathbf{z}_2(t)+\mathbf{B}_2\mathbf{z}_3(t). \tag{31a}$$

It is clear that this transformation between $\mathbf{x}_3$ and $\mathbf{z}_3$ is singular since the matrix $\mathbf{B}_2$ has dimension $n_2\times n_3$, and $n_2 < n_3$. Therefore, assuming that, the elements of the matrix $\mathbf{B}_2$ can be rearranged such that the square matrix

$$\tilde{\mathbf{B}}_3 := \begin{bmatrix}\mathbf{B}_2\\ \mathbf{E}_{22}\end{bmatrix}$$

has rank n_3, the following transformation is added

$$\mathbf{E}_{22}\left\{\begin{array}{l}\left[\mathbf{x}_3(t)+\mathbf{\Gamma}_2\mathbf{\Pi}_2\mathbf{\Gamma}_2^{+}x_3(t-\tau)\right]\\ +\mathbf{\Gamma}_2\bar{\mathbf{B}}_2^{-1}\mathbf{\Gamma}_1\mathbf{\Pi}_1\mathbf{\Gamma}_1^{+}\bar{\mathbf{B}}_2\mathbf{\Gamma}_2^{+}\times\\ \left[\mathbf{x}_3(t-\tau)+\mathbf{\Gamma}_2\mathbf{\Pi}_2\mathbf{\Gamma}_2^{+}\mathbf{x}_3(t-2\tau)\right]\end{array}\right\} = \mathbf{E}_{22}\mathbf{z}_3 \tag{31b}$$

where $\mathbf{E}_{22} = [\mathbf{0} \quad \mathbf{I}_{n_3-n_2}]$ and $\mathbf{E}_{22}\in R^{(n_3-n_2)\times n_3}$.

Combining (31a) with (31b) gives the non-singular transformation along $\mathbf{z}_3$ and $\mathbf{x}_3$ as

$$\mathbf{z}_3 = \tilde{\mathbf{B}}_3^{-1}\left\{\begin{array}{c}\sum_{j=1}^{2}\left[\begin{array}{l}\mathbf{A}_{2,j}\mathbf{x}_j(t)+\mathbf{C}_{2,j}^1\mathbf{x}_j(t-\tau)\\ +\mathbf{C}_{2,j}^2\mathbf{x}_j(t-2\tau)\end{array}\right]+\mathbf{\Lambda}_2\mathbf{z}_2\\ \mathbf{0}\end{array}\right\}$$
$$+\left[\mathbf{x}_3(t)+\mathbf{\Gamma}_2\mathbf{\Pi}_2\mathbf{\Gamma}_2^{+}\mathbf{x}_3(t-\tau)\right] \tag{32}$$
$$+\mathbf{\Gamma}_2\bar{\mathbf{B}}_2^{-1}\mathbf{\Gamma}_1\mathbf{\Pi}_1\mathbf{\Gamma}_1^{+}\bar{\mathbf{B}}_2\mathbf{\Gamma}_2^{+}\left[\mathbf{x}_3(t-\tau)+\mathbf{\Gamma}_2\mathbf{\Pi}_2\mathbf{\Gamma}_2^{+}\mathbf{x}_3(t-2\tau)\right]$$

Using (17e), namely

$$\mathbf{x}_3(t) = \mathbf{\Gamma}_2\mathbf{w}_3(t) \quad \text{and} \quad \mathbf{w}_3(t) = \mathbf{\Gamma}_2^{+}\mathbf{x}_3(t),$$
$$\mathbf{\Gamma}_2^{+} = (\mathbf{\Gamma}_2^{T}\mathbf{\Gamma}_2)^{-1}\mathbf{\Gamma}_2^{T}, \quad \mathbf{\Gamma}_2^{+}\mathbf{\Gamma}_2 = \mathbf{I}_{n2}$$

the second term of the transformation (32), can be represented as

$$\left[\mathbf{x}_3(t)+\mathbf{\Gamma}_2\mathbf{\Pi}_2\mathbf{\Gamma}_2^{+}\mathbf{x}_3(t-\tau)\right]+\mathbf{\Gamma}_2\bar{\mathbf{B}}_2^{-1}\mathbf{\Gamma}_1\mathbf{\Pi}_1\mathbf{\Gamma}_1^{+}\bar{\mathbf{B}}_2\mathbf{\Gamma}_2^{+}$$
$$\times\left[\mathbf{x}_3(t-\tau)+\mathbf{\Gamma}_2\mathbf{\Pi}_2\mathbf{\Gamma}_2^{+}\mathbf{x}_3(t-2\tau)\right]$$
$$= \left[\mathbf{\Gamma}_2\mathbf{w}_3(t)+\mathbf{\Gamma}_2\mathbf{\Pi}_2\mathbf{w}_3(t-\tau)\right]+$$
$$\mathbf{\Gamma}_2\bar{\mathbf{B}}_2^{-1}\mathbf{\Gamma}_1\mathbf{\Pi}_1\mathbf{\Gamma}_1^{+}\bar{\mathbf{B}}_2\mathbf{\Gamma}_2^{+}\left[\mathbf{\Gamma}_2\mathbf{w}_3(t-\tau)+\mathbf{\Gamma}_2\mathbf{\Pi}_2\mathbf{w}_3(t-2\tau)\right]$$
$$= \mathbf{\Gamma}_2\left\{\begin{array}{l}\left[\mathbf{w}_3(t)+\mathbf{\Pi}_2\mathbf{w}_3(t-\tau)\right]+\bar{\mathbf{B}}_2^{-1}\mathbf{\Gamma}_1\mathbf{\Pi}_1\mathbf{\Gamma}_1^{+}\bar{\mathbf{B}}_2\times\\ \left[\mathbf{w}_3(t-\tau)+\mathbf{\Pi}_2\mathbf{w}_3(t-2\tau)\right]\end{array}\right\}$$
$$= \mathbf{\Gamma}_2\left[\mathbf{v}_2(t)+\bar{\mathbf{B}}_2^{-1}\mathbf{\Gamma}_1\mathbf{\Pi}_1\mathbf{\Gamma}_1^{+}\bar{\mathbf{B}}_2\mathbf{v}_2(t-\tau))\right]$$
$$= \mathbf{\Gamma}_2\mathbf{v}_2^1(t)$$

Then the transformation (32) can be presented in the compact form

$$\mathbf{z}_3 = \tilde{\mathbf{B}}_2^{-1}\left\{\begin{array}{c}\sum_{j=1}^{2}\left[\begin{array}{l}\mathbf{A}_{2,j}\mathbf{x}_j(t)+\mathbf{C}_{2,j}^1\mathbf{x}_j(t-\tau)\\ +\mathbf{C}_{2,j}^2\mathbf{x}_j(t-2\tau)\end{array}\right]+\mathbf{\Lambda}_2\mathbf{z}_2\\ \mathbf{0}\end{array}\right\}$$
$$+\mathbf{\Gamma}_2\mathbf{v}_2^1(t). \tag{33a}$$

with

$$\mathbf{v}_2^1(t) = \mathbf{v}_2(t)+\bar{\mathbf{B}}_2^{-1}\mathbf{\Gamma}_1\mathbf{\Pi}_1\mathbf{\Gamma}_1^{+}\bar{\mathbf{B}}_2\mathbf{v}_2(t-\tau) \tag{33b}$$
$$\mathbf{v}_2(t) = \mathbf{w}_3(t)+\mathbf{\Pi}_2\mathbf{w}_3(t-\tau) \tag{33c}$$
$$\mathbf{x}_3(t) = \mathbf{\Gamma}_2\mathbf{w}_3(t). \tag{33d}$$

Again this transformation is stable if all the eigenvalues of matrix $\mathbf{\Pi}_1$ and $\mathbf{\Pi}_2$ are located inside the unit circle. From this, equation (28) is represented of the form

$$\dot{\mathbf{z}}_2(t) = \mathbf{\Lambda}_2\mathbf{z}_2(t)+\mathbf{B}_2\mathbf{z}_3(t).$$

This procedure can be performed iteratively, obtaining on the k^{st} step $(k = 3,\cdots r-1)$ the following recursive transformation:

$$\mathbf{z}_{k+1}(t) = \tag{34a}$$
$$= \tilde{\mathbf{B}}_k^{-1}\left\{\sum_{j=1}^{k}\left[\begin{array}{l}\mathbf{A}_{k,j}\mathbf{x}_j(t)+\mathbf{C}_{k,j}^1\mathbf{x}_j(t-\tau)+\\ \cdots+C_{k,j}^k x_j(t-k\tau)\end{array}\right]+\mathbf{\Lambda}_k\mathbf{z}_k\right\}$$
$$+\mathbf{\Gamma}_k\mathbf{v}_k^1(t)$$

with

$$\mathbf{v}_k^1(t) = \mathbf{v}_k^2(t) \tag{34b}$$
$$+\bar{\mathbf{B}}_k^{-1} \cdots \bar{\mathbf{B}}_2^{-1}\mathbf{\Gamma}_k \cdots \mathbf{\Gamma}_2\mathbf{\Pi}_1\mathbf{\Gamma}_2^+ \cdots \mathbf{\Gamma}_k^+\bar{\mathbf{B}}_2 \cdots \bar{\mathbf{B}}_k\mathbf{v}_k^2(t-\tau)$$
$$\mathbf{v}_k^2(t) = \mathbf{v}_k^3(t) \tag{34c}$$
$$+\bar{\mathbf{B}}_{k-1}^{-1} \cdots \bar{\mathbf{B}}_3^{-1}\mathbf{\Gamma}_{k-1} \cdots \mathbf{\Gamma}_3\mathbf{\Pi}_1\mathbf{\Gamma}_3^+ \cdots \mathbf{\Gamma}_{k-1}^+\bar{\mathbf{B}}_3 \cdots \bar{\mathbf{B}}_{k-1}\mathbf{v}_k^3(t-\tau)$$
$$\vdots$$
$$\mathbf{v}_k^{k-1}(t) = \mathbf{v}_k(t) + \bar{\mathbf{B}}_k^{-1}\mathbf{\Gamma}_k\mathbf{\Pi}_{k-1}\mathbf{\Gamma}_k^+\bar{\mathbf{B}}_k\mathbf{v}_k(t-\tau) \tag{34d}$$
$$\mathbf{v}_k(t) = \mathbf{w}_{k+1}(t) + \mathbf{\Pi}_k\mathbf{w}_{k+1}(t-\tau) \tag{34e}$$
$$\mathbf{x}_{k+1}(t) = \mathbf{\Gamma}_k\mathbf{w}_{k+1}(t) \tag{34f}$$

and $\mathbf{\Lambda}_k$ is a Hurtwitz matrix.

On the last step system (17a)-(17c) can be represented in the new coordinates defined by (24), (32) and (33,) of the following form:

$$\dot{\mathbf{z}}_1(t) = \mathbf{\Lambda}_1\mathbf{z}_1(t) + \mathbf{B}_1\mathbf{z}_2(t) \tag{35a}$$
$$\dot{\mathbf{z}}_i(t) = \mathbf{\Lambda}_i\mathbf{z}_i(t) + \mathbf{B}_i\mathbf{z}_{i+1}(t), \quad i = 2, \cdots r-1 \tag{35b}$$
$$\dot{\mathbf{z}}_r(t) = \sum_{j=1}^{r}\begin{bmatrix}\mathbf{A}_{k,j}\mathbf{x}_j(t) + \mathbf{C}_{k,j}^1\mathbf{x}_j(t-\tau) + \\ \cdots + \mathbf{C}_{k,j}^k\mathbf{x}_j(t-r\tau)\end{bmatrix} + \bar{\mathbf{B}}_r\mathbf{v}_r^1(t) \tag{35c}$$

where $\mathbf{z} = (\mathbf{z}_1, \cdots, \mathbf{z}_r)^T$, $\mathbf{z}_i \in R^{n_i}$, $i = 1, \cdots r$, and

$$\mathbf{v}_r^1(t) = \mathbf{v}_r^2(t) + \bar{\mathbf{B}}_r^{-1} \cdots \bar{\mathbf{B}}_2^{-1}\mathbf{\Gamma}_r \cdots \mathbf{\Gamma}_1\mathbf{\Pi}_1\mathbf{\Gamma}_1^+ \cdots \mathbf{\Gamma}_r^+\bar{\mathbf{B}}_2 \cdots \bar{\mathbf{B}}_r\mathbf{v}_r^2(t-\tau) \tag{36a}$$
$$\mathbf{v}_r^2(t) = \mathbf{v}_r^3(t) + \bar{\mathbf{B}}_{r-1}^{-1} \cdots \bar{\mathbf{B}}_3^{-1}\mathbf{\Gamma}_{r-1} \cdots \mathbf{\Gamma}_2\mathbf{\Pi}_2\mathbf{\Gamma}_2^+ \cdots \mathbf{\Gamma}_{r-1}^+\bar{\mathbf{B}}_3 \cdots \bar{\mathbf{B}}_{r-1}\mathbf{v}_r^3(t-\tau) \tag{36b}$$
$$\vdots$$
$$\mathbf{v}_r^{r-1}(t) = \mathbf{v}_r(t) + \bar{\mathbf{B}}_r^{-1}\mathbf{\Gamma}_{r-1}\mathbf{\Pi}_{r-1}\mathbf{\Gamma}_{r-1}^+\bar{\mathbf{B}}_r\mathbf{v}_r(t-\tau) \tag{36c}$$
$$\mathbf{v}_r(t) = \mathbf{v}(t) + \mathbf{\Pi}_r\mathbf{v}(t-\tau) \tag{36d}$$
$$\mathbf{u}(t) = \mathbf{\Gamma}_r\mathbf{v}(t). \tag{36e}$$

A choice of the control $\mathbf{v}_r^1(t)$ in the equation (35c) similar to (18a) of the form

$$\mathbf{v}_r^1(t) = \mathbf{v}_{rc}^1(t) + \bar{\mathbf{B}}_r^{-1}\mathbf{\Lambda}_r\mathbf{z}_r(t) \tag{37a}$$

with a Hurtwitz matrix $\mathbf{\Lambda}_r \in R^{n_r \times n_r}$, and

$$\mathbf{v}_{rc}^1(t) = -\bar{\mathbf{B}}_r^{-1}\sum_{j=1}^{r}\begin{bmatrix}\mathbf{A}_{k,j}\mathbf{x}_j(t) + \mathbf{C}_{k,j}^1\mathbf{x}_j(t-\tau) \\ + \cdots + \mathbf{C}_{k,j}^k x_j(t-r\tau)\end{bmatrix} \tag{37b}$$

provides the closed-loop dynamics of the states $\mathbf{z}_1(t),...,\mathbf{z}_n(t)$ determined by the following system:

$$\dot{\mathbf{z}}_1(t) = \mathbf{\Lambda}_1\mathbf{z}_1(t) + \mathbf{B}_1\mathbf{z}_2(t) \tag{38a}$$
$$\dot{\mathbf{z}}_i(t) = \mathbf{\Lambda}_i\mathbf{z}_i(t) + \mathbf{B}_i\mathbf{z}_{i+1}(t), \; i = 2, \cdots r-1 \tag{38b}$$
$$\dot{\mathbf{z}}_r(t) = \mathbf{\Lambda}_r\mathbf{z}_r(t) \tag{38c}$$

with Hurwitz matrices $\mathbf{\Lambda}_1, \ldots, \mathbf{\Lambda}_r$, and without delay.

The stability conditions of the closed loop system are presented in the following theorem.

TEOREMA 2. *Assume that*

1) *The system* (1) *is transformable into BCD-form* (17a)-(17(c).
2) *All the eigenvalues of the matrices* $\mathbf{\Pi}_i$, $i = 1, \cdots, r$ *are located inside the unit circle*.

Then the system (1) *or* (17a)-(17c) *with control strategy* (37a)-(37b) *is asymptotically stable.*

Proof: The stability of closed-loop system (17a)-(17c) and (37a)-(37b) is determined by the eigenvalues of systems (38a)-(38)c which may be chosen arbitrarily, and by the property of the internal dynamics presented by the state and control variables transformations (36a)-(36e). It is clear that the internal dynamics is asymptotically stable if the condition 2) of the Theorem 2 holds.

4. CONCLUSIONS

The decomposition block control method has been formulated for control of linear time-delay systems that can be transformed into BCD-form. The proposed transformation and control design procedures have recursive character that simplifies the solution of the problem. The stability of the closed loop system is tested. The proposed method enables to solve an important problem of the classical control theory: pole placement by state feedback for linear systems with delay.

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