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AN ADAPTIVE CONTROL SCHEME FOR OUTPUT FEEDBACK NONLINEAR SYSTEMS WITH ACTUATOR FAILURES[1]

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Abstract: An adaptive control scheme to achieve stability and output tracking is presented for the *output-feedback* nonlinear plant with unknown actuator failures. A state observer is designed for estimating the unavailable plant states, based on a chosen control strategy, in the presence of actuator failures with unknown failure values, time instants and pattern. An adaptive controller is developed by employing the backstepping technique, for which parameter update laws are derived to ensure asymptotic output tracking and signal boundedness of the closed-loop system, as shown by detailed stability analysis.

Keywords: Actuators, adaptive control, backstepping, failure compensation, nonlinear systems, tracking, stability.

1. INTRODUCTION

Actuator failures may lead to performance deterioration or dysfunction of control systems. For some critical systems such as flight control systems, actuator failures, if not compensated, may result in disasters. Some accidents caused by actuator failures could have been avoided, if it was successful to make use of the remaining working actuators which were actually enough for ensuring some desired system performance. To this end, effective controllers are needed to take action automatically, once actuator failures occur. However actuator failures are often unknown in terms of failure patterns, failure times and failure values, which not only introduce uncertainties into systems but also change the system structural properties. Hence, compensation schemes are expected to guarantee a specified control objective for any possible actuator failures. In order to handle failure uncertainties, an adaptive approach is a desirable one for designing control schemes with actuator failure compensation capacity.

Progresses have been made in the adaptive control of systems with actuator failures, including indirect adaptive LQ design (Ahmed-Zaid *et al.*, 1991), indirect adaptive multiple model switching design (Boskovic and Mehra, 1999), adaptive observer-based design (Wang and Daley, 1996), direct adaptive actuator failure compensation control of systems with known plant dynamics (Boskovic *et al.*, 1998).

Our research for the actuator failure compensation problem with direct adaptive designs was conducted in (Tao *et al.*, 2001*c*) and (Tao *et al.*, 2000) for linear systems, and some new results have also been presented in (Tao *et al.*, 2001*d*), (Tao *et al.*, 2001*b*) and (Tao *et al.*, 2001*a*) recently. In (Tang *et al.*, 2001) an adaptive state feedback control scheme for nonlinear systems in the *parametric strict-feedback* form is proposed. Now a more complex case is investigated in this paper, that is, designing an adaptive control scheme with output feedback for nonlinear systems in the presence of unknown actuator failures. The nonlinear systems considered in the paper are in the *output-feedback* form, and the actuator failures are characterized by the pattern that some of the system inputs are stuck at some fixed values not influenced by control action. In Section 2, we formulate the control problem we are investigating. In Section 3, an state observer for output feedback control design is proposed. With that state observer, an adaptive scheme based on the backstepping method is developed for asymptotic output tracking in spite of the actuator failures. The stability analysis for the closed-loop system is completed in Section 4.

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2. PROBLEM STATEMENT

Consider a *output-feedback* form nonlinear plant

$$\begin{aligned}
\dot{x}_i &= x_{i+1} + \varphi_i(y),\ i = 1, 2, \ldots, \rho - 1\\
\dot{x}_\rho &= x_{\rho+1} + \varphi_\rho(y) + \sum_{j=1}^{m} b_{\gamma,j}\beta_j(y)u_j\\
&\cdots\\
\dot{x}_{n-1} &= x_n + \varphi_{n-1}(y) + \sum_{j=1}^{m} b_{1,j}\beta_j(y)u_j\\
\dot{x}_n &= \varphi_n(y) + \sum_{j=1}^{m} b_{0,j}\beta_j(y)u_j\\
y &= x_1,
\end{aligned} \tag{2.1}$$

where $u_j \in R$, $j = 1, 2, \ldots, m$, are the inputs whose actuators may fail during operation, $x = [x_1, x_2, \ldots, x_n]^T \in R^n$ is the unmeasured state vector, $y \in R$ is the output, $\varphi_i(y)$, $i = 1, 2, \ldots, n$, and $\beta_j(y)$, $j = 1, 2, \ldots, m$, are known smooth nonlinear functions, and $b_{r,j}$, $r = 0, 1, \ldots, \gamma = n - \rho$, $j = 1, 2, \ldots, m$, are unknown constant parameters.

The actuator failures of interest are modeled as

$$u_j(t) = \bar{u}_j,\ t \geq t_j,\ j \in \{1, 2, \ldots, m\} \tag{2.2}$$

where the failure value $\bar{u}_j$ and failure time instant t_j are unknown, and so is the failure index j.

The basic assumption for the actuator failure compensation problem is that

(A1) the plant (2.1) is so designed that for any up to $m - 1$ actuator failures, the remaining actuators can still achieve a desired control objective, implemented *with* the knowledge of the plant parameters and failure parameters.

Suppose that there are p_k actuators failing at a time instant t_k, $k = 1, 2, \ldots, q$, and $t_0 < t_1 < t_2 < \cdots < t_q < \infty$. Obviously, it follows from Assumption (A1) that $\sum_{k=1}^{q} p_k \leq m-1$. In another word, at time $t \in (t_k, t_{k+1})$, $k = 0, 1, \ldots, q$, where $t_{q+1} = \infty$, there are $p = \sum_{i=1}^{k} p_i$ failed actuators, i.e., $u_j(t) = \bar{u}_j$, $j = j_1, \ldots, j_p$, $0 \leq p \leq m-1$, and $u_j(t) = v_j$, $j \neq j_1, \ldots, j_p$, where $v_j(t)$, $j = 1, 2, \ldots, m$, are applied control inputs from some feedback control design. The output-feedback plant (2.1) can be rewritten as

$$\begin{aligned}
\dot{x}_i &= x_{i+1} + \varphi_i(y),\ i = 1, 2, \ldots, \rho - 1\\
\dot{x}_\rho &= x_{\rho+1} + \varphi_\rho(y) + \sum_{j=j_1,\ldots,j_p} b_{\gamma,j}\beta_j(y)\bar{u}_j + \sum_{j\neq j_1,\ldots,j_p} b_{\gamma,j}\beta_j(y)v_j\\
&\cdots\\
\dot{x}_{n-1} &= x_n + \varphi_{n-1}(y) + \sum_{j=j_1,\ldots,j_p} b_{1,j}\beta_j(y)\bar{u}_j + \sum_{j\neq j_1,\ldots,j_p} b_{1,j}\beta_j(y)v_j\\
\dot{x}_n &= \varphi_n(y) + \sum_{j=j_1,\ldots,j_p} b_{0,j}\beta_j(y)\bar{u}_j + \sum_{j\neq j_1,\ldots,j_p} b_{0,j}\beta_j(y)v_j\\
y &= x_1.
\end{aligned} \tag{2.3}$$

The control objective is to design an output feedback control scheme for the plant (2.4) with p failed actuators, while p is changing at time instants t_k, $k = 1, 2, \ldots, q$, such that the plant output $y(t)$ asymptotically tracks a prescribed reference signal $y_r(t)$ (the ρth derivative of $y_r(t)$, $y_r^{(\rho)}(t)$, is piecewise continuous) and the closed-loop signals are all bounded, despite the presence of unknown actuator failures and unknown plant parameters. It is clear that the key task is to design adaptive feedback control laws for $v_j(t)$, $j = 1, 2, \ldots, m$, without knowing which of these $v_j(t)$ will have action on the plant dynamics, as there are arbitrary p failed actuators for $\forall\{j_1, \ldots, j_p\} \subset \{1, 2, \ldots, m\}$.

3. ADAPTIVE COMPENSATION CONTROL

For the plant (2.4) in the output-feedback form with p failed actuators at each time interval (t_k, t_{k+1}), $k = 0, 1, \ldots, q$, an adaptive control scheme will be developed in this section, which will be proved to achieve the control objective proposed above for $\forall t > t_0$ in the next section.

Since the failure pattern $u_j(t) = \bar{u}_j$, $j = j_1, \ldots, j_p$, $0 \leq p \leq m - 1$, is assumed to be unknown in this problem, a desirable adaptive control design is expected to achieve the control objective for any possible failure pattern. For a fixed failure pattern, there is a set of failed actuators, while there is another set of working actuators. For this set of working actuators, there is a resulting zero dynamics pattern for the plant (2.4). For closed-loop stabilization and tracking, the zero dynamic system of the plant (2.4) needs to be stable. As such zero dynamics depend on the pattern of working actuators whose number may range from 1 to m, which leads to many possible characterizations of required stable zero dynamics conditions related to different possible control designs, we choose to specify the following one with which a stable adaptive control scheme can be developed to achieve the control objective.

For the plant (2.1), we assume that

(A2) the polynomials $\sum_{j\neq j_1,\ldots,j_p} \mathrm{sign}[b_{\gamma,j}]B_j(s)$ are all stable ones, $\forall\{j_1, \ldots, j_p\} \subset \{1, 2, \ldots, m\}$, $\forall p \in \{0, 1, \ldots, m - 1\}$, where

$$\begin{aligned}
B_j(s) = {}& b_{\gamma,j}s^\gamma + b_{\gamma-1,j}s^{\gamma-1} + \cdots\\
&+ b_{1,j}s + b_{0,j},\ j = 1, 2, \ldots, m.
\end{aligned} \tag{3.1}$$

Remark 3.1. Assumption (A2) implies that the plant (2.1) is minimum phase for all possible actuator failure cases under a special control strategy (see (3.2) below). The plant (2.4) has relative degree ρ, and under the control strategy (3.2), it has a linear zero dynamics system $\dot{\eta} = A_z\eta + \phi(y)$, where the eigenvalues of $A_z \in R^{\gamma\times\gamma}$ are the roots of the polynomial $\sum_{j\neq j_1,\ldots,j_p} \mathrm{sign}[b_{\gamma,j}]B_j(s)$, which is stable from Assumption (A2). Note that while the zero dynamics depend on the actuator failures, that is, A_z

is determined by the failure pattern, $\phi(y)$ depends on both the failure pattern and failure values. □

For an adaptive control design, another assumption is required:

(A3) the sign of $b_{\gamma,j}$ is known, $j = 1, 2, \ldots, m$.

To develop a solution to the stated control problem, we choose the control strategy:

$$v_j = \mathrm{sign}[b_{\gamma,j}]\frac{1}{\beta_j(y)}v_0,\ j = 1, 2, \ldots, m, \quad (3.2)$$

where v_0 is a control signal derived from a backstepping design to be given in Section 3.2.

To express the plant (2.4) with the control strategy (3.2), we define

$$\begin{aligned} x &= [x_1, x_2, \ldots, x_n]^T, \\ \varphi(y) &= [\varphi_1(y), \varphi_2(y), \ldots, \varphi_n(y)]^T \quad (3.3) \\ k_{1,r} &= \sum_{j\neq j_1,\ldots,j_p} \mathrm{sign}[b_{\gamma,j}]b_{r,j},\ r=0,1,\ldots,\gamma, \\ k_{2,rj} &= b_{r,j}\bar{u}_j,\ r=0,1,\ldots,\gamma,\ j=j_1,\ldots,j_p, \\ k_{2,rj} &= 0,\ r=0,1,\ldots,\gamma,\ j\neq j_1,\ldots,j_p \quad (3.4) \end{aligned}$$

and rewrite the plant (2.4) in a compact form as

$$\begin{aligned} \dot{x} &= Ax+\varphi(y)+\sum_{r=0}^{\gamma} e_{n-r}\sum_{j=1}^{m} k_{2,rj}\beta_j(y)+\sum_{r=0}^{\gamma} e_{n-r}k_{1,r}v_0 \\ y &= c^T x, \quad (3.5) \end{aligned}$$

where e_i is the ith coordinate vector in R^n, and

$$A=\begin{bmatrix} 0 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & \cdots & 0 & 0 \\ & & \cdots & \cdots & & \\ 0 & 0 & \cdots & 0 & 0 & 1 \\ 0 & 0 & \cdots & 0 & 0 & 0 \end{bmatrix} \in R^{n\times n},\ c=\begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix} \in R^{n\times 1}. \quad (3.6)$$

It follows from Assumption (A2) that the polynomial $k_{1,\gamma}s^{\gamma} + k_{1,\gamma-1}s^{\gamma-1} + \cdots + k_{1,1}s + k_{1,0}$ is stable, and, in addition, from (3.2), that $k_{1,\gamma} > 0$.

3.1 Observer Design

Since the states of the plant (3.5) are not available for feedback control, an observer is needed to estimate the unavailable state variables.

Choose a vector $l \in R^n$ such that $A_o = A - lc^T$ is stable, and define the filters

$$\begin{aligned} \dot{\xi} &= A_o\xi+ly+\varphi(y), \\ \dot{\zeta}_{rj} &= A_o\zeta_{rj}+e_{n-r}\beta_j(y),\ 0\le r\le\gamma,\ 1\le j\le m, \\ \dot{\mu}_r &= A_o\mu_r+e_{n-r}v_0,\ 0\le r\le\gamma. \quad (3.7) \end{aligned}$$

With the knowledge of $k_{1,r}$ and $k_{2,rj}$, $j=1, 2, \ldots, m$, $r = 0, 1, \ldots, \gamma$, the nominal state estimate vector is

$$\bar{x} = \xi + \sum_{r=0}^{\gamma}\sum_{j=1}^{m} k_{2,rj}\zeta_{rj} + \sum_{r=0}^{\gamma} k_{1,r}\mu_{rj}. \quad (3.8)$$

With $\epsilon = x - \bar{x}$, it follows from (3.5) and (3.7) that

$$\dot{\epsilon} = A_o\epsilon,\ \lim_{t\to\infty}\epsilon(t) = 0 \text{ exponentially}. \quad (3.9)$$

To construct an adaptive observer, we denote $\hat{k}_{1,r}$ and $\hat{k}_{2,rj}$ as the adaptive estimates of $k_{1,r}$ and $k_{2,rj}$, $j = 1, 2, \ldots, m$, $r = 0, 1, \ldots, \gamma$. Then, an adaptive estimate of x is

$$\hat{x} = \xi + \sum_{r=0}^{\gamma}\sum_{j=1}^{m} \hat{k}_{2,rj}\zeta_{rj} + \sum_{r=0}^{\gamma} \hat{k}_{1,r}\mu_{rj}, \quad (3.10)$$

for which the update laws for $\hat{k}_{1,r}$ and $\hat{k}_{2,rj}$ are developed next together with a failure compensation control law for $v_0(t)$ in (3.2) to ensure closed-loop stability and output tracking.

3.2 Backstepping Design

The backstepping technique (Krstić *et al.*, 1995) is now applied to derive a stable adaptive control scheme for the system (3.5), with a design procedure of ρ steps.

Define vectors of unknown constants

$$\begin{aligned} k_1 &= [k_{1,0}, k_{1,1}, \ldots, k_{1,\gamma-1}]^T, \\ k_2 &= [k_{2,01}, \ldots, k_{2,0m}, k_{2,11}, \ldots, k_{2,1m}, \ldots, k_{2,\gamma m}]^T, \quad (3.11) \end{aligned}$$

and vectors of signals

$$\begin{aligned} \omega_i &= [\mu_{0,i}, \mu_{1,i}, \ldots, \mu_{\gamma-1,i}]^T,\ i = 1, 2, \ldots, n, \\ \varepsilon_i &= [\zeta_{01,i}, \ldots, \zeta_{0m,i}, \zeta_{11,i}, \ldots, \\ &\quad \zeta_{1m,i}, \ldots, \zeta_{\gamma m,i}]^T,\ i = 1, 2, \ldots, n, \quad (3.12) \end{aligned}$$

where $\mu_{r,i}$ and $\zeta_{rj,i}$, $i = 1, 2, \ldots, n$, are the ith variable of μ_r, ζ_{rj}, $r = 0, 1, \ldots, \gamma-1$, $j = 1, 2, \ldots, m$.

Step 1: With the output tracking error

$$z_1 = y - y_r, \quad (3.13)$$

where y_r is a reference signal with the piecewise continuous ρth derivative $y_r^{(\rho)}$, it follows from (3.5)–(3.8) that

$$\begin{aligned} \dot{z}_1 &= \epsilon_2 + \bar{x}_2 + \varphi_1(y) - \dot{y}_r \\ &= \epsilon_2 + k_{1,\gamma}\mu_{\gamma,2} + \omega_2^T k_1 + \varepsilon_2^T k_2 + \xi_2 + \varphi_1(y) - \dot{y}_r. \quad (3.14) \end{aligned}$$

Choose the auxiliary error signal

$$z_2 = \mu_{\gamma,2} - \hat{\kappa}\dot{y}_r - \alpha_1, \quad (3.15)$$

where $\hat{\kappa}$ is an estimate of $\kappa = \frac{1}{k_{1,\gamma}}$, and

$$\begin{aligned} \alpha_1 &= \hat{\kappa}(-c_1z_1 - d_1z_1 - \hat{x}_2 - \varphi_1(y)) \\ &= \hat{\kappa}(-c_1z_1 - d_1z_1 - \omega_2^T\hat{k}_1 - \varepsilon_2^T\hat{k}_2 - \xi_2 - \varphi_1(y)). \quad (3.16) \end{aligned}$$

Substituting (3.15) into (3.14) results in

$$\begin{aligned} \dot{z}_1 &= -c_1z_1 - d_1z_1 + k_{1,\gamma}z_2 + \bar{x}_2 - \hat{x}_2 - k_{1,\gamma}\tilde{\kappa}(\dot{y}_r + \alpha_1) \\ &= -c_1z_1 - d_1z_1 + \epsilon_2 + \hat{k}_{1,\gamma}z_2 + \tilde{k}_{1,\gamma}(\mu_{\gamma,2} - \hat{\kappa}\dot{y}_r - \alpha_1) \\ &\quad + \omega_2^T\tilde{k}_1 + \varepsilon_2^T\tilde{k}_2 - k_{1,\gamma}\tilde{\kappa}(\dot{y}_r + \alpha_1), \quad (3.17) \end{aligned}$$

where $\tilde{k}_{1,\gamma} = k_{1,\gamma} - \hat{k}_{1,\gamma}$, $\tilde{k}_1 = k_1 - \hat{k}_1$, $\tilde{k}_2 = k_2 - \hat{k}_2$, $\tilde{\kappa} = \kappa - \hat{\kappa}$, and c_1, d_1 are some positive constants.

The time-derivativ e of the partial Lyapunov candidate function $V_1 = \frac{1}{2}z_1^2 + \frac{k_{1,\gamma}}{2\lambda_1}\tilde{\kappa}^2$, where $\lambda_1 > 0$ is a chosen constant gain, and $k_{1,\gamma}$ is positive due to Assumption (A2), is derived as

$$\begin{aligned}\dot{V}_1 =& -c_1 z_1^2 - d_1 z_1^2 + z_1\epsilon_2 + \hat{k}_{1,\gamma} z_1 z_2 + \upsilon_1 \tilde{k}_{1,\gamma} + \\ & \tau_1^T \tilde{k}_1 + \nu_1^T \tilde{k}_2 - z_1 k_{1,\gamma}(\dot{y}_r + \alpha_1)\tilde{\kappa} - \frac{k_{1,\gamma}}{\lambda_1}\tilde{\kappa}\dot{\hat{\kappa}} \\ =& -c_1 z_1^2 - d_1 z_1^2 + z_1\epsilon_2 + \hat{k}_{1,\gamma} z_1 z_2 + \upsilon_1 \tilde{k}_{1,\gamma} + \tau_1^T \tilde{k}_1 + \nu_1^T \tilde{k}_2\end{aligned}$$

with the choice of the adaptive law for $\hat{\kappa}$:

$$\dot{\hat{\kappa}} = -\lambda_1 z_1(\dot{y}_r + \alpha_1), \qquad (3.18)$$

where υ_1, τ_1 and ν_1, which are the first tuning functions for $\hat{k}_{1,\gamma}$, $\hat{k}_1$ and $\hat{k}_2$, are given as

$$\upsilon_1 = z_1(\mu_{\gamma,2} - \hat{\kappa}\dot{y}_r - \alpha_1),\ \tau_1 = z_1\omega_2,\ \nu_1 = z_1\varepsilon_2. \quad (3.19)$$

Step $i = 2, 3, \ldots, \rho - 1$**:** According to (3.15), define z_i, $i = 2, 3, \ldots, \rho - 1$, in a similar way:

$$z_i = \mu_{\gamma,i} - \hat{\kappa} y_r^{(i-1)} - \alpha_{i-1}. \qquad (3.20)$$

Differentiating (3.20), we obtain

$$\begin{aligned}\dot{z}_i =& \dot{\mu}_{\gamma,i} - \hat{\kappa} y_r^{(i)} - \dot{\hat{\kappa}} y_r^{(i-1)} - \dot{\alpha}_{i-1} \\ =& \mu_{\gamma,i+1} - l_i\mu_{\gamma,1} - \hat{\kappa} y_r^{(i)} - \dot{\hat{\kappa}} y_r^{(i-1)} - \frac{\partial\alpha_{i-1}}{\partial\hat{\kappa}}\dot{\hat{\kappa}} \\ & -\frac{\partial\alpha_{i-1}}{\partial y}(\epsilon_2 + k_{1,\gamma}\mu_{\gamma,2} + \omega_2^T k_1 + \varepsilon_2^T k_2 + \xi_2 + \varphi_1(y)) \\ & -\sum_{q=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial y_r^{(q-1)}} y_r^{(q)} - \sum_{q=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\mu_{\gamma,q}}(\mu_{\gamma,q+1} - l_q\mu_{\gamma,1}) \\ & -\sum_{q=1}^{i}\frac{\partial\alpha_{i-1}}{\partial\xi_q}(\xi_{q+1} - l_q\xi_1 + l_q y + \varphi_q(y)) \\ & -\sum_{q=1}^{i}\frac{\partial\alpha_{i-1}}{\partial\varepsilon_q}(\varepsilon_{q+1} - l_q\varepsilon_1) - \sum_{q=1}^{i}\frac{\partial\alpha_{i-1}}{\partial\omega_q}(\omega_{q+1} - l_q\omega_1) \\ & -\frac{\partial\alpha_{i-1}}{\partial\hat{k}_{1,\gamma}}\dot{\hat{k}}_{1,\gamma} - \frac{\partial\alpha_{i-1}}{\partial\hat{k}_2}\dot{\hat{k}}_1 - \frac{\partial\alpha_{i-1}}{\partial\hat{k}_3}\dot{\hat{k}}_2. \qquad (3.21)\end{aligned}$$

Design the stabilizing function α_i as

$$\begin{aligned}\alpha_i =& -z_{i-1} - c_i z_i - d_i\left(\frac{\partial\alpha_{i-1}}{\partial y}\right)^2 z_i + l_i\mu_{\gamma,1} \\ & +\frac{\partial\alpha_{i-1}}{\partial y}(\hat{k}_{1,\gamma}\mu_{\gamma,2} + \omega_2^T\hat{k}_1 + \varepsilon_2^T\hat{k}_2 + \xi_2 + \varphi_1(y)) \\ & +\sum_{q=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial y_r^{(q-1)}} y_r^{(q)} + \sum_{q=1}^{i}\frac{\partial\alpha_{i-1}}{\partial\xi_q}(\xi_{q+1} - l_q\xi_1 + l_q y + \varphi_q(y)) \\ & +\sum_{q=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\mu_{\gamma,q}}(\mu_{\gamma,q+1} - l_q\mu_{\gamma,1}) + \sum_{q=1}^{i}\frac{\partial\alpha_{i-1}}{\partial\varepsilon_q}(\varepsilon_{q+1} - l_q\varepsilon_1) \\ & +\sum_{q=1}^{i}\frac{\partial\alpha_{i-1}}{\partial\omega_q}(\omega_{q+1} - l_q\omega_1) + (y_r^{(i-1)} + \frac{\partial\alpha_{i-1}}{\partial\hat{\kappa}})\dot{\hat{\kappa}} \\ & +\frac{\partial\alpha_{i-1}}{\partial\hat{k}_{1,\gamma}}\lambda_2\upsilon_i - \sum_{q=2}^{i-1}\frac{\partial\alpha_{q-1}}{\partial\hat{k}_{1,\gamma}}\lambda_2\frac{\partial\alpha_{i-1}}{\partial y}\mu_{\gamma,2}z_q \\ & +\frac{\partial\alpha_{i-1}}{\partial\hat{k}_1}\Gamma_1\tau_i - \sum_{q=2}^{i-1}\frac{\partial\alpha_{q-1}}{\partial\hat{k}_1}\Gamma_1\frac{\partial\alpha_{i-1}}{\partial y}\omega_2 z_q \\ & +\frac{\partial\alpha_{i-1}}{\partial\hat{k}_2}\Gamma_2\nu_i - \sum_{q=2}^{i-1}\frac{\partial\alpha_{q-1}}{\partial\hat{k}_2}\Gamma_2\frac{\partial\alpha_{i-1}}{\partial y}\varepsilon_2 z_q, \qquad (3.22)\end{aligned}$$

where c_i and d_i, $i = 2, 3, \ldots, \rho-1$, are some positive constants to be chosen, and υ_i, τ_i and ν_i, $i = 2, 3, \ldots, \rho - 1$, are the tuning functions given as

$$\begin{aligned}&\upsilon_i = \upsilon_{i-1} - \frac{\partial\alpha_{i-1}}{\partial y}\mu_{\gamma,2}z_i,\ \tau_i = \tau_{i-1} - \frac{\partial\alpha_{i-1}}{\partial y}\omega_2 z_i, \\ &\nu_i = \nu_{i-1} - \frac{\partial\alpha_{i-1}}{\partial y}\varepsilon_2 z_i. \qquad (3.23)\end{aligned}$$

In troducing the partial Lyapunov candidate function $V_i = V_{i-1} + \frac{1}{2}z_i^2$, with the definition $z_{i+1} = \mu_{\gamma,i+1} - \hat{\kappa}y_r^{(i)} - \alpha_i$, we deriv e the time-derivative of V_i from (3.21) and (3.22) as

$$\begin{aligned}\dot{V}_i =& -\sum_{q=1}^{i}c_q z_q^2 - d_1 z_1^2 - \sum_{q=2}^{i}d_q\left(\frac{\partial\alpha_{q-1}}{\partial y}\right)^2 z_q^2 + z_i z_{i+1} \\ & +z_1\epsilon_2 - \sum_{q=2}^{i}z_q\frac{\partial\alpha_{q-1}}{\partial y}\epsilon_2 + \upsilon_i\tilde{k}_{1,\gamma} + \tau_i^T\tilde{k}_1 + \nu_i^T\tilde{k}_2 \\ & +\sum_{q=2}^{i}z_q\frac{\partial\alpha_{q-1}}{\partial\hat{k}_{1,\gamma}}(\lambda_2\upsilon_i - \dot{\hat{k}}_{1,\gamma}) + \sum_{q=2}^{i}z_q\frac{\partial\alpha_{q-1}}{\partial\hat{k}_1}(\Gamma_1\tau_i - \dot{\hat{k}}_1) \\ & +\sum_{q=2}^{i}z_q\frac{\partial\alpha_{q-1}}{\partial\hat{k}_2}(\Gamma_2\nu_i - \dot{\hat{k}}_2). \qquad (3.24)\end{aligned}$$

Step ρ**:** Considering $z_\rho = \mu_{\gamma,\rho} - \hat{\kappa}y_r^{(\rho-1)} - \alpha_{\rho-1}$, w e obtain its time-derivativ e

$$\begin{aligned}\dot{z}_\rho =& \dot{\mu}_{\gamma,\rho} - \hat{\kappa}y_r^{(\rho)} - \dot{\hat{\kappa}}y_r^{(\rho-1)} - \dot{\alpha}_{\rho-1} \\ =& \mu_{\gamma,\rho+1} - l_\rho\mu_{\gamma,1} + v_0 - \hat{\kappa}y_r^{(\rho)} - \dot{\hat{\kappa}}y_r^{(\rho-1)} - \frac{\partial\alpha_{\rho-1}}{\partial\hat{\kappa}}\dot{\hat{\kappa}} \\ & -\frac{\partial\alpha_{\rho-1}}{\partial y}(\epsilon_2 + k_{1,\gamma}\mu_{\gamma,2} + \omega_2^T k_1 + \varepsilon_2^T k_2 + \xi_2 + \varphi_1(y)) \\ & -\sum_{q=1}^{\rho-1}\frac{\partial\alpha_{\rho-1}}{\partial\mu_{\gamma,q}}(\mu_{\gamma,q+1} - l_q\mu_{\gamma,1}) - \sum_{q=1}^{\rho-1}\frac{\partial\alpha_{\rho-1}}{\partial y_r^{(q-1)}}y_r^{(q)} \\ & -\sum_{q=1}^{\rho}\frac{\partial\alpha_{\rho-1}}{\partial\xi_q}(\xi_{q+1} - l_q\xi_1 + l_q y + \varphi_q(y)) \\ & -\sum_{q=1}^{\rho}\frac{\partial\alpha_{\rho-1}}{\partial\omega_q}(\omega_{q+1} - l_q\omega_1) - \sum_{q=1}^{\rho}\frac{\partial\alpha_{\rho-1}}{\partial\varepsilon_q}(\varepsilon_{q+1} - l_q\varepsilon_1) \\ & -\sum_{j=1}^{m}\frac{\partial\alpha_{\rho-1}}{\partial\zeta_{\gamma j,\rho}}\beta_j(y) - \frac{\partial\alpha_{\rho-1}}{\partial\hat{k}_{1,\gamma}}\dot{\hat{k}}_{1,\gamma} - \frac{\partial\alpha_{\rho-1}}{\partial\hat{k}_1}\dot{\hat{k}}_1 - \frac{\partial\alpha_{\rho-1}}{\partial\hat{k}_2}\dot{\hat{k}}_2.\end{aligned}$$

Design the control signal $v_0(t)$ for (3.2) as

$$v_0 = \alpha_\rho + \hat{\kappa}y_r^{(\rho)}, \qquad (3.25)$$

where α_ρ is constructed as

$$\alpha_\rho = -z_{\rho-1} - c_\rho z_\rho - d_\rho\left(\frac{\partial\alpha_{\rho-1}}{\partial y}\right)^2 z_\rho + l_\rho\mu_{\gamma,1} - \mu_{\gamma,\rho+1}$$

$$+\frac{\partial\alpha_{\rho-1}}{\partial y}(\epsilon_2+\hat{k}_{1,\gamma}\mu_{\gamma,2}+\omega_2^T\hat{k}_1+\varepsilon_2^T\hat{k}_2+\xi_2+\varphi_1(y))$$
$$+\sum_{q=1}^{\rho-1}\frac{\partial\alpha_{\rho-1}}{\partial\mu_{\gamma,q}}(\mu_{\gamma,q+1}-l_q\mu_{\gamma,1})+\sum_{q=1}^{\rho-1}\frac{\partial\alpha_{\rho-1}}{\partial y_r^{(q-1)}}y_r^{(q)}$$
$$+\sum_{q=1}^{\rho}\frac{\partial\alpha_{\rho-1}}{\partial\xi_q}(\xi_{q+1}-l_q\xi_1+l_q y+\varphi_q(y))$$
$$+\sum_{q=1}^{\rho}\frac{\partial\alpha_{\rho-1}}{\partial\omega_q}(\omega_{q+1}-l_q\omega_1)+\sum_{q=1}^{\rho}\frac{\partial\alpha_{\rho-1}}{\partial\varepsilon_q}(\varepsilon_{q+1}-l_q\varepsilon_1)$$
$$+\sum_{j=1}^{m}\frac{\partial\alpha_{\rho-1}}{\partial\zeta_{\gamma j,\rho}}\beta_j(y)+(y_r^{(i-1)}+\frac{\partial\alpha_{\rho-1}}{\partial\hat{\kappa}})\dot{\hat{\kappa}}$$
$$+\frac{\partial\alpha_{\rho-1}}{\partial\hat{k}_{1,\gamma}}\lambda_2 v_\rho-\sum_{q=2}^{\rho-1}\frac{\partial\alpha_{q-1}}{\partial\hat{k}_{1,\gamma}}\lambda_2\frac{\partial\alpha_{\rho-1}}{\partial y}\mu_{\gamma,2}z_q$$
$$+\frac{\partial\alpha_{\rho-1}}{\partial\hat{k}_1}\Gamma_1\tau_\rho-\sum_{q=2}^{\rho-1}\frac{\partial\alpha_{q-1}}{\partial\hat{k}_1}\Gamma_1\frac{\partial\alpha_{\rho-1}}{\partial y}\omega_2 z_q$$
$$+\frac{\partial\alpha_{\rho-1}}{\partial\hat{k}_2}\Gamma_2\nu_\rho-\sum_{q=2}^{\rho-1}\frac{\partial\alpha_{q-1}}{\partial\hat{k}_2}\Gamma_2\frac{\partial\alpha_{\rho-1}}{\partial y}\varepsilon_2 z_q, \quad (3.26)$$

in which $c_\rho>0$ and $d_\rho>0$ are designing constants. Consider the Lyapunov candidate function

$$V=V_\rho=\frac{1}{2}z^Tz+\frac{k_{1,\gamma}}{2\lambda_1}\tilde{\kappa}^2+\frac{1}{2\lambda_2}\tilde{k}_{1,\gamma}^2+\frac{1}{2}\tilde{k}_1^T\Gamma_1^{-1}\tilde{k}_1$$
$$+\frac{1}{2}\tilde{k}_2^T\Gamma_2^{-1}\tilde{k}_2+\sum_{i=1}^{\rho}\frac{1}{2d_i}\epsilon^TP\epsilon, \quad (3.27)$$

where P, which is positive definite and symmetric, satisfies the Lyapunov equation $PA_o+A_o^TP=-I$, and $\lambda_2>0, \Gamma_1=\Gamma_1^T>0, \Gamma_2=\Gamma_2^T>0$. The time-derivative of V is

$$\dot{V}=-\sum_{i=1}^{\rho}c_iz_i^2-d_1z_1^2-\sum_{i=2}^{\rho}d_i(\frac{\partial\alpha_{i-1}}{\partial y})^2z_i^2+z_1\epsilon_2$$
$$-\sum_{i=2}^{\rho}z_i\frac{\partial\alpha_{i-1}}{\partial y}\epsilon_2-\sum_{i=1}^{\rho}\frac{1}{2d_i}\epsilon^T\epsilon+(v_\rho-\frac{1}{\lambda_2}\dot{\hat{k}}_{1,\gamma})\tilde{k}_{1,\gamma}$$
$$+(\tau_\rho^T-\dot{\hat{k}}_1^T\Gamma_1^{-1})\tilde{k}_1+(\nu_\rho^T-\dot{\hat{k}}_2^T\Gamma_2^{-1})\tilde{k}_2$$
$$+\sum_{i=2}^{\rho}z_i\frac{\partial\alpha_{i-1}}{\partial\hat{k}_{1,\gamma}}(\lambda_2v_\rho-\dot{\hat{k}}_{1,\gamma})+\sum_{i=2}^{\rho}z_i\frac{\partial\alpha_{i-1}}{\partial\hat{k}_1}(\Gamma_1\tau_\rho-\dot{\hat{k}}_1)$$
$$+\sum_{i=2}^{\rho}z_i\frac{\partial\alpha_{i-1}}{\partial\hat{k}_2}(\Gamma_2\nu_\rho-\dot{\hat{k}}_2). \quad (3.28)$$

With the choice of the update laws

$$\dot{\hat{k}}_{1,\gamma}=\lambda_2v_\rho,\ \dot{\hat{k}}_1=\Gamma_1\tau_\rho,\ \dot{\hat{k}}_2=\Gamma_2\nu_\rho, \quad (3.29)$$

we have

$$\dot{V}=-\sum_{i=1}^{\rho}c_iz_i^2-d_1z_1^2-\sum_{i=2}^{\rho}d_i(\frac{\partial\alpha_{i-1}}{\partial y})^2z_i^2$$
$$+z_1\epsilon_2-\sum_{i=2}^{\rho}z_i\frac{\partial\alpha_{i-1}}{\partial y}\epsilon_2-\sum_{i=1}^{\rho}\frac{1}{2d_i}\epsilon^T\epsilon$$
$$=-\sum_{i=1}^{\rho}c_iz_i^2-d_1(z_1-\frac{1}{2d_1}\epsilon_2)^2-\sum_{i=2}^{\rho}d_i(\frac{\partial\alpha_{i-1}}{\partial y}z_i+\frac{1}{2d_i}\epsilon_2)^2$$
$$-\sum_{i=1}^{\rho}\frac{1}{4d_i}\epsilon_2^2-\sum_{i=1}^{\rho}\frac{1}{2d_i}(\epsilon_1^2+\epsilon_3^2+\cdots+\epsilon_n^2)$$
$$\le\sum_{i=1}^{\rho}c_iz_i^2-\sum_{i=1}^{\rho}\frac{1}{4d_i}\epsilon_2^2-\sum_{i=1}^{\rho}\frac{1}{2d_i}(\epsilon_1^2+\epsilon_3^2+\cdots+\epsilon_n^2). \quad (3.30)$$

In summary, we have developed an adaptive actuator failure compensation scheme for the system (3.5), which consists of the controller law

$$v_j=\mathrm{sign}[b_{\gamma,j}]\frac{1}{\beta_j(y)}v_0,\ j=1,2,\ldots,m,$$
$$v_0=\alpha_\rho+\hat{\kappa}y_r^{(\rho)} \quad (3.31)$$

and the adaptive laws for updating the controller parameters,

$$\dot{\hat{\kappa}}=-\lambda_1z_1(\dot{y}_r+\alpha_1)$$
$$\dot{\hat{k}}_{1,\gamma}=\lambda_2v_\rho,\ \dot{\hat{k}}_1=\Gamma_1\tau_\rho,\ \dot{\hat{k}}_2=\Gamma_2\nu_\rho, \quad (3.32)$$

where α_ρ, v_ρ, τ_ρ and ν_ρ are derived from the recursive backstepping procedure.

4. STABILITY ANALYSIS

Now we prove that with the adaptive control design proposed in Section 3, the closed-loop signal boundedness and asymptotic output tracking are guaranteed for $\forall t>t_0$, so that the stated actuator failure compensation objective is achieved.

Theorem 4.1. The adaptive output feedback control scheme consisting of the controller (3.31) and the filters (3.7) along with the parameter update laws (3.32) applied to the system (3.5), based on Assumptions (A2)–(A3), ensures global boundedness of all closed-loop signals and global asymptotic output tracking: $\lim_{t\to\infty}(y(t)-y_r(t))=0$.

Proof: For each time interval (t_k,t_{k+1}), $k=0,1,\ldots,q$, a Lyapunov function V such as (3.27) can be constructed. Starting from the first time interval, it can be seen from (3.30) that $V(t)\le V(t_0)$ and $\dot{V}\le 0$ for $\forall t\in[t_0,t_1)$. Hence we conclude that z, $\hat{\kappa}$, $\hat{k}_{1,\gamma}$, $\hat{k}_1$, $\hat{k}_2$ and ϵ are bounded for $t\in[t_0,t_1)$. From the boundedness of y_r and z_1, y is bounded. It follows from (3.7) that ξ and ζ_{rj}, $r=0,1,\ldots,\gamma$, $j=1,2,\ldots,m$, are bounded. Also from (3.7), we obtain that

$$\mu_{r,i}=e_i^T(sI-A_o)^{-1}e_{n-r}v_0 \quad (4.1)$$

for $0\le r\le\gamma$, $1\le i\le n$, where e_i is the ith coordinate vector in R^n. Express the plant (3.5) in the differential equation form

$$y^{(n)}=\sum_{i=1}^{n}\varphi_i^{(n-i)}(y)+\sum_{r=0}^{\gamma}\sum_{j=1}^{m}k_{2,rj}\beta_j^{(r)}(y)+\sum_{r=0}^{\gamma}k_{1,r}v_0^{(r)}. \quad (4.2)$$

As in (Krstić *et al.*, 1995), rewrite (4.2) in the input-output form with transfer function $G(s) = \frac{1}{k_{1,\gamma}s^{\gamma}+\cdots+k_{1,1}s+k_{1,0}}$, "input" signal

$$\varpi=y^{(n)}-\sum_{i=1}^{n}\varphi_i^{(n-i)}(y)-\sum_{r=0}^{\gamma}\sum_{j=1}^{m}k_{2,rj}\beta_j^{(r)}(y), \quad (4.3)$$

and "output" signal v_0:

$$v_0=G(s)[\varpi]. \quad (4.4)$$

Substituting (4.4) into (4.1), we have

$$\mu_{r,i}=e_i^T(sI-A_o)^{-1}e_{n-r}G(s)[\varpi], \quad (4.5)$$

which results in the boundedness of $\mu_{r,i}$, $r = 0,1,\ldots,\gamma$, $i = 1,2,\ldots,n$, because y is bounded, $\varphi_i(\cdot)$, $i=1,2,\ldots,n$, and $\beta_j(\cdot)$, $j=1,2,\ldots,m$, are smooth, and the matrix A_o and the polynomial $k_{1,\gamma}s^{\gamma}+\cdots+k_{1,1}s+k_{1,0}$ are stable. It is in turn implied from (3.8) and the boundedness of ϵ that x is bounded. According to (4.4), it can also be seen that v_0 is a bounded signal. Since $\beta_j(y)\neq 0$ for $\forall y\in R$, the boundedness of v_j is guaranteed too, $j = 1,2,\ldots,m$. Therefore, all closed-loop signals are bounded for $t\in[t_0,t_1)$.

At time $t = t_1$, there occur p_1 actuator failures, which result in the abrupt change of κ, $k_{1,\gamma}$, k_1 and k_2. Since the change of values of these parameters are finite and z is continuous, it can be concluded from $\dot V\leq 0$ that $V(t)\leq V(t_1^+)=V(t_1^-)+\bar V_1 \leq V(t_0)+\bar V_1$ for $t\in(t_1,t_2)$ with a positive constant $\bar V_1$. By repeating the argument above, the boundedness of all the signals are proved for the time interval (t_1,t_2). Continuing in the same way, finally we have that $V(t)\leq V(t_q^+)=V(t_q^-)+\bar V_q \leq V(t_0)+\sum_{k=1}^{q}\bar V_k$ for $t\in(t_q,\infty)$ with some positive constants $\bar V_k$, $k=1,2,\ldots,q$. Due to the finite times of actuator failures, it can be obtained that $V(t)$ is bounded for $\forall t>t_0$, and so are all the closed-loop signals.

To prove output tracking, considering the last time interval (t_q,∞) with a positive initial $V(t_q^+)$, we see that it follows from (3.30) that $z\in L^2$ and $\epsilon\in L^2$. On the other hand, we can conclude that $\dot z\in L^\infty$ and $\dot\epsilon\in L^\infty$ from the boundedness of the closed-loop signals. In turn it follows that over the time interval (t_q,∞), $\lim_{t\to\infty}z_1(t)=0$, which means that the output tracking error $y(t)-y_r(t)$ is such that $\lim_{t\to\infty}(y(t)-y_r(t)) = 0$. ∇

5. CONCLUDING REMARKS

An adaptive output feedback actuator failure compensation control scheme for a canonical class of nonlinear systems: those in the *output-feedback* form, is presented as a further development of the backstepping technique. In order to estimate the unmeasured states of the controlled plant, an adaptive observer for the uncertain plant with an equivalent unknown disturbance due to actuator failures is constructed and used for an adaptive control design. Based on the state observer, a backstepping method is applied to develop an adaptive compensation scheme to ensure that the plant output tracks a reference output asymptotically and that all closed-loop signals are bounded, in spite of the unknown actuator failures in addition to unknown plant parameters.

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