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THE MULTISCALE STATE FUSION ESTIMATION FOR NONLINEAR SYSTEMS WITH MULTIRATE SENSORS

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Abstract: By combining strong tracking filter theory with state fusion estimation algorithm, we put forward a new algorithm of state fusion estimation for a class of nonlinear dynamic systems with all sensors having different sampling rates on the basis of distributed information. The algorithm is also extended to the joint state and parameter estimation of a class of nonlinear systems having time-varying parameters with unknown changing law. The effectiveness of the proposed algorithm is illustrated by computer simulations, which show that the new algorithm has strong robustness against model/plant mismatches. *Copyright ©2002 IFAC*

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1. INTRODUCTION

Information fusion techniques are of very importance for processing multi-source information, these techniques can be used to obtain more precise and complete estimates as well as judgments than those algorithms using only single source information, therefore, many scholars have paid much attention to this problem in the last decade. There are various information fusion techniques; the multi-sensor data fusion based on Kalman filter is one of the most significant methods. Kalman-filter-based data fusion algorithms, including state fusion and measurement fusion, have been widely studied over the last decade (Cheng, *et al.*, 1997; Smith, *et al.*, 1998). State fusion methods usually have lower computation and communication cost and have the advantages of parallel implementation and fault-tolerance than measurement fusion. It has also been pointed out that state fusion methods are only effective when Kalman filters are consistent, which restricts the practical application of state fusion methods (Mazor, *et al.*, 1998). In many realistic applications the real processes are often nonlinear, and the consequent Kalman filters based on linearized process models are usually inconsistent due to the modelling errors (Gan and Harris, 2001).

It is well known that the state estimates can be obtained by use of Kalman filter (KF) or extended Kalman filter (EKF) when the dynamic model matches accurately with the actual system. However, KF or EKF has the disadvantage of poor robustness against model uncertainties, therefore, one can obtain only inaccurate state estimates in practice by use of the KF or EKF (Bonivento and Tonielli, 1984). In order to overcome this limitation, we have proposed a strong tracking filter (STF) theory, which can be effectively used for the state and parameter estimation of a class of nonlinear systems (Bai, *et al*., 1998; zhou and Frank, 1996).

On the other hand, in nature and engineering practice, there are many processes having multiscale phenomena. Simultaneously, people often observe and measure these processes at different scales (or resolutions), thus it's natural for us to use multiscale idea to describe and analyse these processes. Therefore, combining the fusion algorithm with wavelet transform based multiscale analysis has been paid much attention recently.

In this paper, by combining the STF theory with multiscale estimation theory and state fusion approaches, we put forward a new state fusion estimation algorithm for a class of nonlinear systems with multi-sensors having different sampling rates. The proposed algorithm is optimal on the basis of global information by use of strong tracking filter. The proposed algorithm is then extended to the joint state and parameter estimation of a class of nonlinear systems. Computer simulation results show that the proposed fusion algorithm is more effective than those of EKF-based fusion algorithms (Mazor, et al, 1998; Bai, *et al*., 1998; zhou and Frank, 1996). In that

1) Having strong robustness against model uncertainties, i.e., having higher estimation accuracy when there are model/plant mismatches;

2) Having the ability to estimate time-varying parameters, in contrary, EKF-based fusion algorithms can be used to estimate constant parameters only. In the algorithm the wavelet transform is the bridge between different sampling information.

The paper is organized as follows: In the next section we present a class of nonlinear dynamic systems with multirate sensors and single model. Section 3 is the outline of discrete wavelet transform. In section 4 we propose a new multiscale state fusion estimation algorithm on the basis of STF for nonlinear dynamic systems described in section 2. In section 5 we extend our algorithm to joint state and parameter estimation of nonlinear systems. In section 6 we illustrate the effectiveness of the proposed algorithm by two numerical examples. Section 7 concludes this paper.

2. SYSTEM DESCRIPTION

A class of single model, nonlinear dynamic systems with multi-rate sensors can be described by

$$x(N,k+1) = f(N;k,u(k),x(N,k)) + \Gamma(N,k)w(N,k) \quad (1)$$

$$z(i,k) = h(i;k,x(i,k)) + v(i,k),\ i = 1,2,\cdots,N \quad (2)$$

Where integer k is the discrete time variable, $x \in R^{n\times 1}$ is the state, i ($=1,2,\cdots,N$) denotes scale, nonlinear function f has one order continuous partial derivative with respect to the state in a region $R_0 \subset R^n$, system noise $w \in R^{n\times 1}$ is a Gaussian white sequence and has $w(N,k) \sim N(0,Q(N,k))$, $Q(N,k)$ is a semi-positive definite matrix. Γ is a known matrix.

The state variables are observed by N different sensors at scale i , the measurements are $z(i,k) \in R^{p_i\times 1}$, the nonlinear function $h(i;k,x(i,k))$ has one order continuous partial derivative also. Measurement noise $v(i,k) \in R^{p_i\times 1}$ is a Gaussian white sequence with $v(i,k) \sim N(0,R(i,k))$, and $R(i,k)$ is positive definite matrices.

Initial state $x(N,0)$ is a random vector with the mean x_0 and covariance P_0 . It is assumed that $x(N,0)$, $w(N,k), v(i,k)$ are independent of each other.

3. OUTLINE OF WAVELET TRANSFORM

Now consider a finite sequence at scale i with a length of a data-block $X_m(i)$ to be $M_i = 2^{i-1}$

$$X_m(i) = [x^T(i,(mM_i+1),\cdots,x^T(i,mM_i+M_i)]^T \quad (3)$$

The vector forms of wavelet transformation can be derived in terms of wavelet operators (Smith, *et al*., 1998)

$$X_{Vm}(i-1) = L_{i-1}^T \mathrm{diag}\{H_{i-1},\cdots,H_{i-1}\}L_i X_m(i) \quad (4)$$

$$X_{Dm}(i-1) = L_{i-1}^T \mathrm{diag}\{G_{i-1},\cdots,G_{i-1}\}L_i X_m(i) \quad (5)$$

For details, see Gan and Harris (2001).

4. OPTIMAL MULTISCALE FUSION ESTIMATION ALGORITHM

First, assume that we have obtained the optimal estimate $\hat{X}_{m|m}(N)$ for $X_m(N)$ based on global information and corresponding estimation error covariance matrix $P_{m|m}(N)$ at the finest scale N .

Theorem 1: When we get the actual measurement for $X_{m+1}(i)$ at every scale, we can obtain the optimal fusion estimate $\hat{X}_{m+1|m+1}(N)$ for $X_{m+1}(N)$ on the basis of global information and corresponding estimation error covariance matrix $P_{m+1|m+1}(N)$ as follows:

$$\hat{X}_{m+1|m+1}(N) = P_{m+1|m+1}(N)\Big[\sum_{i=1}^{N}(P^i_{m+1|m+1}(N))^{-1}\hat{X}^i_{m+1|m+1}(N) - (N-1)P^{-1}_{m+1|m}(N)\hat{X}_{m+1|m}(N)\Big] \quad (6)$$

$$P^{-1}_{m+1|m+1}(N) = \sum_{i=1}^{N}(P^i_{m+1|m+1}(N))^{-1} - (N-1)P^{-1}_{m+1|m}(N)\hat{X}_{m+1|m}(N) \quad (7)$$

Proof: At scale N, by use of the STF with initial condition $\hat{x}(N, mM \mid mM)$ and $P(N, mM \mid mM)$, we can obtain prediction $\hat{X}_{m+1|m}(N)$, predicted error covariance matrix $P_{m+1|m}(N)$, estimation $\hat{X}^N_{m+1|m+1}(N)$ and estimation error covariance matrix $P^N_{m+1|m+1}(N)$ on the basis of sensors at the finest scale N. For integer k, $k = mM+1, \cdots, mM+M$, we have

$$\hat{x}^N(N, k+1 \mid k+1) = \hat{x}(N, k+1 \mid k) + K(N, k+1) g(N, k+1) \tag{8}$$

$$\hat{x}(N, k+1 \mid k) = f(N; k, u(k), \hat{x}(N, k \mid k)) \tag{9}$$

$$\begin{aligned} P(N,k+1 \mid k) &= l(k+1) F(N;k,u(k),\hat{x}(N,k \mid k)) P(N,k \mid k) \\ &\cdot F^T(N;k,u(k),\hat{x}(N,k \mid k)) + \Gamma(N,k) Q(N,k) \Gamma^T(N,k) \end{aligned} \tag{10}$$

$$\begin{aligned} K(N,k+1) &= P(N,k+1 \mid k) H^T(N,k+1,\hat{x}(N,k+1 \mid k)) \\ &\cdot [H(N,k+1,\hat{x}(N,k+1 \mid k)) P(N,k+1 \mid k) \\ &\cdot H^T(N,k+1,\hat{x}(N,k+1 \mid k)) + R(N,k+1)]^{-1} \end{aligned} \tag{11}$$

$$g(N,k+1) = z(N,k+1) - h(N,k+1,\hat{x}(N,k+1 \mid k)) \tag{12}$$

$$\begin{aligned} &P^N(N,k+1 \mid k+1) \\ &= [I - K(N,k+1) H(N;k+1,\hat{x}(N,k+1 \mid k))] P(N,k+1 \mid k) \end{aligned} \tag{13}$$

In Eq. (10)

$$F(N;k,u(k),\hat{x}(N,k \mid k)) = \left. \frac{\partial f(N;k,u(k),x(N,k))}{\partial x} \right|_{x(N,k)=\hat{x}(N,k|k)} \tag{14}$$

In Eqs. (11) and (13)

$$H(N;k+1,\hat{x}(N,k+1 \mid k)) = \left. \frac{\partial h(N,k+1,x(N,k+1))}{\partial x} \right|_{x(N,k+1)=\hat{x}(N,k+1|k)} \tag{15}$$

The adaptive fading factor $l(k+1)$ in Eq.(10) is determined by(Bai, *et al*., 1998; zhou and Frank, 1996)

$$l(k+1) = \begin{cases} l_0, & l_0 \ge 1 \\ 1, & l_0 < 1 \end{cases} \tag{16}$$

where

$$l_0 = \frac{\mathrm{tr}[N(k+1)]}{\mathrm{tr}[M(k+1)]} \tag{17}$$

In Eq.(17)

$$\begin{aligned} N(k+1) &= V_0(k+1) - H(N;k+1,\hat{x}(N,k+1 \mid k)) \\ &\cdot Q(N,k) H^T(N;k+1,\hat{x}(N,k+1 \mid k)) - R(N,k+1) \end{aligned} \tag{18}$$

$$\begin{aligned} M(k+1) &= H(N;k+1,\hat{x}(N,k+1 \mid k)) F(N;k,\hat{x}(k \mid k)) \\ &\cdot P(N,k \mid k) F^T(N,k,\hat{x}(N,k \mid k)) H^T(N,k+1,\hat{x}(N,k+1 \mid k)) \end{aligned} \tag{19}$$

$$V_0(k+1) = \begin{cases} g(1) g^T(1), & k = 0 \\ \dfrac{r V_0(k) + g(k+1) g^T(k+1)}{1+r}, & k \ge 1 \end{cases} \tag{20}$$

Where in Eq.(20), $r = 0.98$ is a forgetting factor .

Thereby, at scale N we have obtained the state predicted estimates $\hat{X}_{m+1|m}(N)$ as well as the state predicted error covariance matrix $P_{m+1|m}(N)$, and also got the state estimates $\hat{X}^N_{m+1|m+1}(N)$ as well as the state estimation error matrix $P^N_{m+1|m+1}(N)$ on the basis of sensor at scale N.

Then we decompose $\hat{X}_{m+1|m}(N)$ into every coarser scale $i (1 \le i < N)$ by wavelet transform to generate the smoothness signal $\hat{X}_{Vm+1|m}(i)$ and the detail signal $\hat{X}_{Dm+1|m}(j)$, $(i \le j < N)$

$$\hat{X}_{Vm+1|m}(i) = L_i^T \mathrm{diag}\left\{ \prod_{r=i}^{N-1} H_r, \cdots, \prod_{r=i}^{N-1} H_r \right\} L_N \hat{X}_{m+1|m}(N) \tag{21}$$

$$\hat{X}_{Dm+1|m}(j) = L_i^T \mathrm{diag}\left\{ G_j \prod_{r=j+1}^{N-1} H_r, \cdots, G_j \prod_{r=j+1}^{N-1} H_r \right\} L_N \hat{X}_{m+1|m}(N) \,. \tag{22}$$

$$\begin{aligned} &\hat{X}_{Vm+1|m}(i) \\ &= [\hat{x}_V^T(i, mM_i + 1 \mid mM_i), \cdots, \hat{x}_V^T(i, mM_i + M_i \mid mM_i + M_i - 1)]^T \end{aligned} \tag{23}$$

$$\begin{aligned} &\hat{X}_{Dm+1|m}(j) \\ &= [\hat{x}_V^T(j, mM_j + 1 \mid mM_j), \cdots, \hat{x}_V^T(j, mM_j + M_j \mid mM_j + M_j - 1)]^T \end{aligned} \tag{24}$$

Note that

$$\hat{\hat{X}}_{m+1|m}(i) = \left[\hat{X}^T_{Vm+1|m}(i), \hat{X}^T_{Dm+1|m}(i+1), \cdots, \hat{X}^T_{Dm+1|m}(i+1) \right]^T \tag{25}$$

$$T(i) = \left[\prod_{r=N-1}^{i} H_r^T, \prod_{r=N-1}^{i+1} H_r^T G_i^T, \cdots, \prod_{r=N-1}^{N-1} H_{N-1}^T G_{N-2}^T, G_{N-1}^T \right]^T \tag{26}$$

$$\begin{aligned} \overline{P}_{m+1|m}(i) &= \begin{bmatrix} P_{VVm+1|m}(i) & P_{VDm+1|m}(i) \\ P_{DVm+1|m}(i) & P_{DDm+1|m}(i) \end{bmatrix} \\ &= L_i^T \mathrm{diag}\{T(i), \ \cdots, \ T(i)\} L_N P_{m+1|m}(N) \\ &\cdot L_N^T \mathrm{diag}\{T^T(i), \ \cdots, \ T^T(i)\} L_i \end{aligned} \tag{27}$$

Where

$$\begin{aligned} P_{VVm+1|m}(i) &= L_i \mathrm{diag}\left\{ \prod_{r=i}^{N-1} H_r \ \cdots \ \prod_{r=i}^{N-1} H_r \right\} L_N P_{VVm+1|m}(N) \\ &\cdot L_N^T \mathrm{diag}\left\{ \prod_{r=N-1}^{i} H_r^T \ \cdots \ \prod_{r=N-1}^{i} H_r^T \right\} L_i \,. \end{aligned} \tag{28}$$

$$P_{VDm+1|m}(i) = [P_{VDm+1|m}(i,i), \cdots, P_{VDm+1|m}(i, n-1)] \tag{29}$$

$$\begin{aligned} P_{VDm+1|m}(i,j) &= L_i \mathrm{diag}\left[\prod_{r=i}^{N-1} H_r, \cdots, \prod_{r=i}^{N-1} H_r \right] L_N P_{m+1|m}(N) \\ &\cdot L_N^T \mathrm{diag}\left[\prod_{r=N-1}^{j+1} H_r^T G_j^T, \cdots, \prod_{r=N-1}^{j+1} H_r^T G_j^T \right] L_j \end{aligned} \tag{30}$$

$$P_{DVm+1|m}(i) = (P_{VDm+1|m}(i))^T \tag{31}$$

$$P_{DDm+1|m}(i) = \left[P_{DDm+1|m}(l,j) \right], \ l, j = i, i+1, \cdots N-1 \tag{32}$$

$$\begin{aligned} P_{DDm+1|m}(l,j) &= L_l \mathrm{diag}\left[G_l \prod_{r=l+1}^{N-1} H_r, \cdots, G_l \prod_{r=l+1}^{N-1} H_r \right] L_N P_{m+1|m}(N) \\ &\cdot L_N^T \mathrm{diag}\left[\prod_{r=N-1}^{j+1} H_r^T G_j^T, \cdots, \prod_{r=N-1}^{j+1} H_r^T G_j^T \right] L_j \end{aligned} \tag{33}$$

Notice that $\hat{X}_{Dm+1|m}(i)$, $\hat{X}_{Dm+1|m}(i+1)$, $\cdots$, $\hat{X}_{Dm+1|m}(N-1)$ are detail signals at different scale spaces, the relationship between them and $\hat{X}_{Vm+1|m}(i)$ are $P_{VDm+1|m}(i)$ and $P_{DVm+1|m}(i)$.

At scale i, we can update $\hat{X}_{Vm+1|m}(i)$ using EKF on the basis of measurement of sensor i and obtain the local estimates $\hat{X}_{Vm+1|m+1}(i)$ and estimation error matrix $P_{VVm+1|m+1}(i)$

$$\begin{aligned}\hat{x}_V(i,k+1|k+1) &= \hat{x}_V(i,k+1|k) + K(i,k+1) \\ &\cdot [z(i,k+1) - h(i;k+1,\hat{x}_V(i,k+1|k))]\end{aligned} \tag{34}$$

$$\begin{aligned}K(i,k+1) &= P_{VV}(i,k+1|k)H^T(i,k+1,\hat{x}_V(i,k+1|k) \\ &\cdot (i,k+1,\hat{x}_V(i,k+1|k))P(i,k+1|k) \\ &\cdot H^T(i,k+1,\hat{x}_V(i,k+1|k)) + R(i,k+1)]^{-1}\end{aligned} \tag{35}$$

$$\begin{aligned}&P_{VV}(i,k+1|k+1) \\ &= [I - K(i,k+1)H(i;k+1,\hat{x}_V(i,k+1|k))]P_{VV}(i,k+1|k)\end{aligned} \tag{36}$$

$$\begin{aligned}&H(i,k+1,\hat{x}_V(i,k+1|k)) \\ &= \left.\frac{\partial h(i;k+1,x(i,k+1))}{\partial x}\right|_{x(i,k+1)=\hat{x}_V(i,k+1|k)}\end{aligned} \tag{37}$$

At the same time, we note that the detail signals $\hat{X}_{Dm+1|m}(r)$ ($r = N-1,\cdots,i$) are still not updated, therefore, we denote

$$\hat{X}_{Dm+1|m+1}(j) = \hat{X}_{Dm+1|m}(j) \ , j = N-1,\ldots,i \tag{38}$$

Because of the relativity between $\hat{X}_{Dm+1|m}(j)$ ($j = N-1,\cdots,i$) and $\hat{X}_{Vm+1|m}(i)$, so the error covariance matrices $P_{VDm+1|m}(i)$ and $P_{DVm+1|m}(i)$ can be updated as

$$\begin{aligned}&P_{VD}(i,j;k+1|k+1) \\ &= [I - K(i,k+1)H(i,k+1)]P_{VD}(i,j;k+1|k)\end{aligned} \tag{39}$$

$$P_{DV}(i,j;k+1|k+1) = (P_{VD}(i,j;k+1|k))^T \tag{40}$$

Now we use Eq.(6) to synthesize the updated $\hat{X}_{Vm+1|m+1}(i)$ with $\hat{X}_{Dm+1|m+1}(j)$ ($j = N-1,\cdots,i$) and obtain the estimate $\hat{X}^i_{m+1}(N)$ for $X_{m+1}(N)$ at scale N based on the sensor at scale i and the corresponding estimation error covariance matrix $P^i_{m+1|m+1}(N)$ as

$$\hat{X}^i_{m+1|m+1}(N) = L_N^T \mathrm{diag}\left[T^T(i), \ \cdots, \ T^T(i)\right]L_i \hat{\bar{X}} \tag{41}$$

$$\begin{aligned}P^i_{m+1|m+1}(N) &= L_N^T \mathrm{diag}\left[T^T(i),\cdots,T^T(i)\right]L_i \bar{P}_{m+1|m+1}(i) \\ &\cdot L_i^T \mathrm{diag}\left[T(i),\cdots,T(i)\right]L_N\end{aligned} \tag{42}$$

with

$$\bar{P}_{m+1|m+1}(i) = \begin{bmatrix} P_{VVm+1|m+1}(i) & P_{VDm+1|m+1}(i) \\ P_{DVm+1|m+1}(i) & P_{DDm+1|m+1}(i) \end{bmatrix} \tag{43}$$

At scale N, we have obtained N estimates for $X_{m+1}(N)$ based on different sensors at different scale $N, N-1,\cdots,1$, that is,

$$\hat{X}^N_{m+1|m+1}(N), \hat{X}^{N-1}_{m+1|m+1}(N), \cdots, \hat{X}^1_{m+1|m+1}(N)$$

And also get N corresponding estimation error covariance matrix $P^i_{VVm+1|m+1}(N)$ ($i = N\cdots,1$). By fusing them (Hong, 1991), one finally generates the full-scale optimal estimate $\hat{X}_{m+1|m+1}(N)$ for $X_{m+1}(N)$ based on global information and obtain the corresponding estimation error covariance matrix $P_{m+1|m+1}(N)$, which are just the (6) and (7).

5. EXTENSION TO JOINT STATE AND PARAMETER ESTIMATION OF NONLINERAR SYSTEMS

Now we consider the following nonlinear systems

$$\begin{aligned}x(N,k+1) = \ & f(N;k,u(k),x(N,k),\theta(N,k)) \\ &+ \Gamma(N,k)w(N,k)\end{aligned} \tag{44}$$

$$z(i,k) = h(i,k,x(i,k),\theta(i,k)) + v(i,k) \ , i = 1,\cdots,N \tag{45}$$

Where $q(k) \in R^{l\times 1}$ are unknown parameters with unknown changing law, we assume that $q(k)$ are locally identifiable, other variables are the same as in Eqs. (1) and (2). The problem now is to estimate $q(N,k)$ and $x(N,k)$ simultaneously. To do this, we add the following equation

$$q(i,k+1) = q(i,k) \ , i = 1,2,\cdots,N \tag{46}$$

and let
$$x_e(i,k) = \begin{bmatrix} x(i,k) \\ \theta(i,k) \end{bmatrix} \tag{47}$$

We obtain the following equivalent form of systems (44) and (45)

$$\begin{aligned}x_e(N,k+1) = \ & f_e(N;k,u(k),x_e(N,k)) \\ &+ \Gamma_e(N,k)w(N,k)\end{aligned} \tag{48}$$

$$z(i,k) = h_e(i;k,x_e(i,k)) + v(i,k) \ , i = 1,2,\cdots,N \tag{49}$$

$$f_e(N,u(k),x_e(k)) = \begin{bmatrix} f(N;k,u(k),x(N,k),\theta(N,k)) \\ \theta(N,k) \end{bmatrix} \tag{50}$$

$$h_e(i;k,x_e(i,k)) = h(i;k,x(i,k),\theta(i,k)) \tag{51}$$

$$\Gamma_e(N,k) = \begin{bmatrix} \Gamma(N,k) \\ 0_{l\times q} \end{bmatrix} \tag{52}$$

Obviously, the state fusion algorithms obtained in section 4 can be directly applied to systems (48) and

(49) to get the fusion estimate $\hat{x}_e(k|k)$, i.e., $\hat{x}(k|k)$ and $\hat{q}(k|k)$.

6. SIMULATION STUDY

Example 1: The first example comes from a pulsive system model of a ship (Zhou and Frank, 1996)

$$x(2,k+1) = 0.1a\,x^2(2,k) + x(2,k) + 0.1b\,u(2,k) + w(2,k)$$
$$z(i,k+1) = x(i,k+1) + v(i,k+1),\ i = 2,1$$

Where parameter a is the resistance of the hull, b is the efficiency of the ship engine, x is the velocity of the ship. The nominal values of a and b are $a^0 = -0.58$ and $b^0 = 0.2$, respectively, $Q = [0.00001]$. There are two sensors observing the system at scale 2 and 1 with $R(2) = [0.002]$ and $R(1) = [0.001]$, respectively. We take $x(0) = 0$, $P(0|0) = 100$. The following model/plant mismatch cases are tested: case 1: $a = 0.75a^0, b = b^0$; case 2: $a = a^0, b = 1.2b^0$; case 3: $a = 0.6a^0,\ b = 0.8b^0$

$u(k)$ is a squared wave with magnitude 0.9 and 1.1, respectively, the period is 200. One of the simulation results is shown in Fig.1, which presents the results in case 3. To test the tracking ability, in case 1 to case 3, we have added a state sudden change at $k = 300$.

Table 1 The accumulated absolute errors by use of the multiscale estimation

Case	Parameter	Filter	Sensor 1	Multiscale fusion estimation with sensor 1 and 2
Normal case	$a = a^0$ $b = b^0$	EKF	34.3529	30.0139
		STF	11.5339	6.1997
Case 1	$a = 0.75a^0$ $b = b^0$	EKF	28.1175	23.5369
		STF	21.3445	5.9453
Case 2	$a = a^0$ $b = 1.2b^0$	EKF	34.1480	29.9969
		STF	22.1862	6.0255
Case 3	$a = 0.6a^0$ $b = 0.8b^0$	EKF	24.3217	19.7929
		STF	20.6452	5.9901

Table 1 gives the corresponding accumulated absolute errors in every case. All data in Table 1 are the average value of 100 Monte Carlo runs.

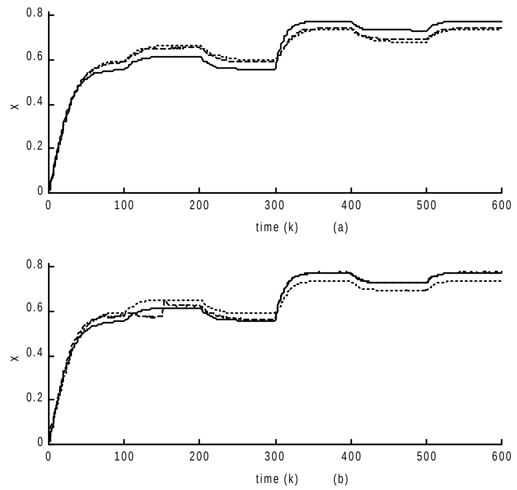

Fig.1. Simulation results of example 1 in case 3. In (a) "----" is the estimates by use of EKF based on sensor 1, "_ _" *is the fusion estimates by use of multiscale EKF; In (b) "----" is the estimates by use of STF based on sensor 1,* "_ _" is the fusion estimates by use of multiscale STF.

From Table 1 and Fig.1 we find that: 1) In the normal case, the state estimation precision and tracking abilities by use of EKF and STF are equivalent, but the state fusion estimation precision of STF is higher. 2) When there are parameter mismatches the STF has stronger tracking ability and higher estimation precision than the EKF. The fusion estimation by use of STF has the highest estimation precision.

Example 2: Considering the following nonlinear systems

$$x(2,k+1) = 0.1\,a(2,k)\,x^2(1,k) + x(1,k)$$
$$+ 0.02\,u(1,k) + w(2,k)$$

Where the parameter $a(k)$ is time varying with unknown changing law, which is to be estimated. $Q = [0.001]$, $R(2) = [0.001]$, $R(1) = [0.002]$.

$$z(2,k+1) = a^3(2,k+1)x(2,k+1) + v(2,k),$$
$$z(1,k+1) = a(1,k+1)x(1,k+1) + v(1,k).$$

In the simulations we select: $x(0) = 0$, $a(0) = -0.58$, $\hat{x}(0|0) = 0$; $\hat{a}(0|0) = -0.38$, $P(0|0) = 0.3I_2$, and we choose $a(k)$ arbitrarily as follows:

$$a(k+1) = \begin{cases} a(k) + w_2(k), & 0 \le k \le 198; \\ a(199) + a^0/10 + w_2(199), & k = 199; \\ a(k) + w_2(k), & 200 \le k \le 399; \\ a(k) - a^0/500 + w_2(k), & 400 \le k \le 499; \\ a(k) + w_2(k), & 500 \le k \le 698 \\ a(k) + a^0/5 + w_2(k), & k = 699; \\ a(k) + w_2(k), & 700 \le k \le 899. \end{cases}$$

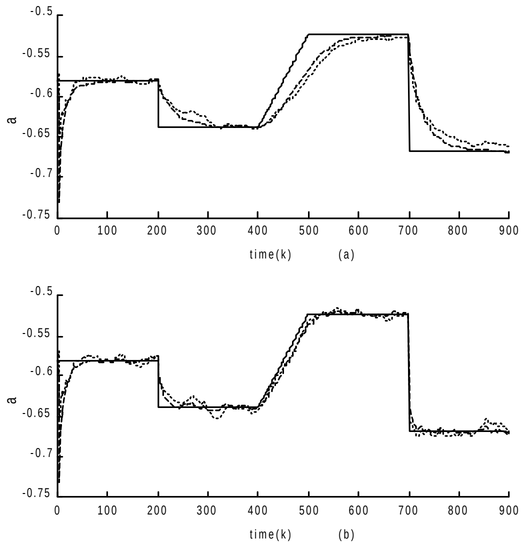

Fig.2. Simulation results of example 2. In (a) "----" is estimates by use of EKF based on sensor 1, "_ _" is fusion estimates by use of MEKF; In (b) "----" is estimates by use of STF based on sensor 1, "_ _" is fusion estimates by use of MSTF.

With $w(2,k) \sim N[0,0.00001]$. The simulation results are shown in Fig.2 and Table 2.

Table 2 The accumulated absolute errors

	Sensor 1	Multiscale fusion Estimates with sensor 1 and 2
EKF	47.2140	27.6767
STF	20.1746	12.8410

Table 2 and Fig.2 show that, when the model parameters is time-varying with unknown changing law, no matter whether the system runs in steady-state or not, changing abruptly or slowly, the state and parameter estimation precision of STF is much higher than that of EKF. The state fusion estimation precision on the basis of STF is the highest; the STF has strong robustness against model uncertainties and has strong tracking ability to the state and time-varying parameters (see Figs.1-2). From Figs.1-2 and Tables 1-2, it is illustrated that STF can effectively solve the problem pointed out in Mazor, *et al*.(1998) and overcome the main limitation of KF or EKF.

7. CONCLUSIONS

In this paper, by combining the STF theory with multiscale state fusion approaches, we have proposed a new state fusion estimation algorithm for a class of nonlinear systems with multi-rate sensors having different sampling rates. Computer simulation results show that the proposed multiscale fusion algorithm is more effective than those of EKF-based fusion algorithms.

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