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THRESHOLD SETTING FOR TIME-DOMAIN DAMAGE DETECTION METHODS FOR UNCERTAIN TIME-VARYING DISCRETE-TIME SYSTEMS

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Abstract: The paper develops a new method of threshold setting for use in automated monitoring of a system to detect abnormal or degraded system behaviour. The method are designed for use with damage detection in time-domain methods when the model of the system is known, but it is uncertain and time-varying. The mathematical formulation of the dynamical system is based on a state-space model. Uncertainty in the system description is modelled by an unknown, norm bounded, additive perturbation of the system matrix. The method for upper bound estimates of the differences is presented. The suitability of the new method is demonstrated in damage detection example. *Copyright © 2002 IFAC*

Keywords: Fault detection, threshold selection, error estimation, uncertain dynamic systems, discrete-time systems, time-varying systems, non-stationary systems, state-space models.

1. INTRODUCTION

Systems are damaged as result of overloading, fatigue, ageing or environmental influences. In order to guarantee a safe working, it is necessary to determine and localise the early stages of damage. Uncertainty in the model description makes the problem more complicated. The main tasks for system monitoring are: receiving and interpretation data from process, to alarm when it is necessary and to provide further information.

Real control systems are described by differential or difference system equations, non-linear and non-stationary. Rarely we know exactly all coefficients. For analysis and simulation we exploit very often simplified linear models. Existence of uncertainty, non-stationarity or non-linearity in the system structure effects uncertainty at the state and output.

There is an extensive literature related to system monitoring. A set of techniques, called as *analytical redundancy* (e.g. Frank 1990, Gustafsson 1996, Isermann 1984, 1993, Srinivasan 1994, Willsky 1976) use mathematical models in conjunction with system measurements to detect system failures and sometimes to isolate the cause. An object of concern in analytical redundancy techniques is the robustness of the procedure. Frank (1990) discusses the difficulties inherent in setting thresholds, which are used in to distinguish a fault; He also presents some robustness techniques proposed to properly set thresholds in view of the fact that the models are imperfect. The use of adaptative thresholds, was stated in (Horak 1988, Puig 1999).

Our approach to threshold setting is designed for systems, for which the model is known and given as the space state model, but the its coefficients are uncertain with additive bounded uncertainty. The method is developed for discrete, time-varying systems, but it can be used for LTI models and it is possible to expand it for continuous-time systems. Having given the model in the state-space with time dependent coefficients the approach is for use rather with damage detection methods based on predictor-observer methodology e.g. Model Based Fault Detection (Orchard 2001). Basic structure for such methods are drawn on Fig. 1. The Observer could be state observer or output observer (in some cases identity).

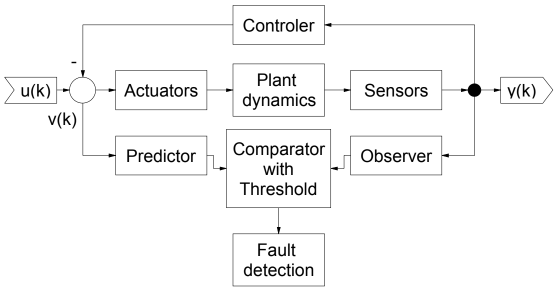


Fig. 1. Basic block diagram for damage detection using predictor-observer methods.

One of the most important parts in the structure is comparator with threshold. Very often the main method of choosing threshold has been based on experience and experiments. The paper introduces new method for calculating threshold, using system model for uncertain systems.

In practical applications input and output signals are filtered very often. Particularly when the level of noises is medium or high. It is possible to include the filter into system model, which should give better performance for obtained threshold.

2. NOTATION

Space of vector's sequence are given by Hilbert space (l^2) $(\mathbf{R}^q)^N = \mathbf{R}^q \times \mathbf{R}^q \times ... \times \mathbf{R}^q$. Elements of the space are sequences of vectors $\mathbf{z} = [\mathbf{z}(0)...\mathbf{z}(N-1)]^{\mathrm{T}}$ where $\mathbf{z}(i) \in \mathbf{R}^q$.

Scalar products in $(\mathbf{R}^q)^N$ is defined as follow

$$\langle \mathbf{z}, \mathbf{v} \rangle_{(\mathbf{R}^q)^N} = \sum_{i=0}^{N-1} \langle \mathbf{z}(i), \mathbf{v}(i) \rangle_{\mathbf{R}^q} = \sum_{i=0}^{N-1} \mathbf{z}^{\mathrm{T}}(i) \cdot \mathbf{v}(i) \qquad \mathbf{(1)}$$

where $\mathbf{z}, \mathbf{v} \in (\mathbf{R}^q)^N$. The induced H_2 norm has the form

$$\|\mathbf{z}\|_2 = \sqrt{\sum_{i=0}^{N-1} \langle \mathbf{z}(i), \mathbf{z}(i) \rangle_{\mathbf{R}^q}} = \sqrt{\sum_{i=0}^{N-1} \mathbf{z}^{\mathrm{T}}(i) \cdot \mathbf{z}(i)} \qquad (2)$$

and the H_∞ norm has the form $\|\mathbf{z}\|_\infty = \max_{0 \le i \le (N \cdot q - 1)} |z(i)|$

Operator's norm is defined by $\|\mathbf{M}^{\mathrm{F}}\| = \sup_{\mathbf{h} \in (\mathbf{R}^q)^N} \frac{\|\mathbf{M}^{\mathrm{F}}\mathbf{h}\|}{\|\mathbf{h}\|}$

3. NOMINAL AND PERTURBED SYSTEM

The *nominal*, non-stationary, linear control system Σ has the form

$$\mathbf{x}_p(k+1) = \mathbf{A}(k) \cdot \mathbf{x}_p(k) + \mathbf{B}(k) \cdot \mathbf{v}_p(k), \qquad (3)$$

$$\mathbf{y}_p(k) = \mathbf{C}(k) \cdot \mathbf{x}_p(k), \quad \mathbf{x}_p(0)=\mathbf{x}_0, \ k=0,1,...,N-1, \qquad (4)$$

where $\mathbf{x}_p(\cdot) \in (\mathbf{R}^n)^N$ is nominal state, $\mathbf{v}_p(\cdot) \in (\mathbf{R}^m)^N$ is nominal input, $\mathbf{y}_p(\cdot) \in (\mathbf{R}^p)^N$ is nominal output, and $\mathbf{A}(k) \in \mathcal{L}(\mathbf{R}^n)$, $\mathbf{B}(k) \in \mathcal{L}(\mathbf{R}^m, \mathbf{R}^n)$, $\mathbf{C}(k) \in \mathcal{L}(\mathbf{R}^n, \mathbf{R}^p)$ are system's matrices.

It is assumed that given are: *nominal system,* state space parameters $\mathbf{A}(k)$, $\mathbf{B}(k)$, $\mathbf{C}(k)$ for uncertain, linear, non-stationary discrete-time systems with the bounds for every matrix δ_A, δ_B, δ_C and a time horizon denoted by a positive integer N.

Real control system is different from (3-4) and may be described by *perturbed* model Σ_Δ as follows

$$\mathbf{x}_\Delta(k+1) = \mathbf{A}_\Delta(k) \cdot \mathbf{x}_\Delta(k) + \mathbf{B}_\Delta(k) \cdot \mathbf{v}_p(k), \qquad (5)$$

$$\mathbf{y}_\Delta(k) = \mathbf{C}_\Delta(k) \cdot \mathbf{x}_\Delta(k), \quad \mathbf{x}_\Delta(0)=\mathbf{x}_0, \ k=0,1,...,N-1 \qquad (6)$$

Uncertainty's structure depends on the system, nevertheless for solving it has been assumed specific (additive perturbation) model. Analyse has been carried out also for another model e.g. multiplicative uncertainty.

For linear system with additive uncertainty we assume, following description

For matrix **A**

$$\mathbf{A}_\Delta(k)=\mathbf{A}(k)+\Delta_\mathbf{A}(k) \qquad (7)$$

where $\Delta_\mathbf{A}(k) \in \mathcal{L}(\mathbf{R}^n, \mathbf{R}^n)$, $k=0,1,...,N-1$, and

$$\|\Delta_\mathbf{A}(k)\| \le \delta_A < \infty, \qquad (8)$$

And similar for matrices **B** and **C**:

$\mathbf{B}_\Delta(k)=\mathbf{B}(k)+\Delta_\mathbf{B}(k)$, $\mathbf{C}_\Delta(k)=\mathbf{C}(k)+\Delta_\mathbf{C}(k)$, $k=0,1,...,N-1$

where $\Delta_\mathbf{B}(k) \in \mathcal{L}(\mathbf{R}^n, \mathbf{R}^m)$, $\Delta_\mathbf{C}(k) \in \mathcal{L}(\mathbf{R}^p, \mathbf{R}^n)$ and $\|\Delta_\mathbf{B}(k)\| \le \delta_B < \infty$, $\|\Delta_\mathbf{C}(k)\| \le \delta_C < \infty$

To obtain the norm of maximal output deviation, we needn't know the uncertainty matrices $\Delta_\mathbf{A}$, $\Delta_\mathbf{B}$, $\Delta_\mathbf{C}$, we have to know only their estimates δ_A, δ_B, δ_C.

The multiplicative model is easy for using with relative uncertainty. It has following description.

$$\mathbf{A}_{\Delta}(k) = (\mathbf{I} + \Delta_{MA}(k)) \cdot \mathbf{A}(k) \tag{9}$$

$$\mathbf{B}_{\Delta}(k) = (\mathbf{I} + \Delta_{MB}(k)) \cdot \mathbf{B}(k) \tag{10}$$

$$\mathbf{C}_{\Delta}(k) = (\mathbf{I} + \Delta_{MC}(k)) \cdot \mathbf{C}(k) \tag{11}$$

We can solve the multiplicative system by converting it to the additive model by following transformations

$$\Delta_{\mathbf{A}}(k) = \Delta_{MA} \cdot \mathbf{A}(k) \tag{12}$$

$$\Delta_{\mathbf{B}}(k) = \Delta_{MB} \cdot \mathbf{B}(k) \tag{13}$$

$$\Delta_{\mathbf{C}}(k) = \Delta_{MC} \cdot \mathbf{C}(k) \tag{14}$$

It should be clear that, by the transformation it is possible to obtain more conservative results.

4. OPERATOR'S DEFINITIONS

For the sake of simplicity we introduce three operators $\mathbf{L}^{\mathbf{F}} \in \mathcal{L}((\mathbf{R}^n)^N, (\mathbf{R}^n)^N)$, $\mathbf{K}^{\mathbf{F}} \in \mathcal{L}((\mathbf{R}^n)^N, \mathbf{R}^n)$ and $\mathbf{N}^{\mathbf{F}} \in \mathcal{L}(\mathbf{R}^n, (\mathbf{R}^n)^N)$, defined as follows

$$(\mathbf{L}^{\mathbf{F}}\mathbf{h})(k) = \sum_{i=0}^{k-2}\left[\prod_{j=i+1}^{k-1}\mathbf{A}(j)\right]\cdot\mathbf{h}(i) + \mathbf{h}(k-1) \tag{15}$$

$$(\mathbf{N}^{\mathbf{F}}\mathbf{z})(k) = \prod_{j=0}^{k-1}\mathbf{A}(j)\cdot\mathbf{z} \tag{16}$$

where $\mathbf{h}(i) \in \mathcal{L}(\mathbf{R}^n)$ i k=2,3,...,N.

Theorem 1. For every system Σ_p described by equations (3-4) the state and output trajectory can be written as follows

$$\mathbf{x}_p(k) = (\mathbf{N}^{\mathbf{F}}\mathbf{x}_0)(k) + (\mathbf{L}^{\mathbf{F}}\mathbf{B}\cdot\mathbf{v}_p)(k) \tag{17}$$

$$\mathbf{y}_p(k) = \mathbf{C}(k)\cdot\mathbf{x}_p(k), \tag{18}$$

Theorem 2. For every perturbed system Σ_Δ described by equations (5-6) the state and output trajectory can be written as follows

$$\mathbf{x}_\Delta(k) = \mathbf{x}_p(k) + (\mathbf{L}^{\mathbf{F}}\Delta_{\mathbf{A}}\cdot\mathbf{x}_\Delta)(k) + (\mathbf{L}^{\mathbf{F}}\Delta_{\mathbf{B}}\cdot\mathbf{v}_p)(k) \tag{19}$$

$$\mathbf{y}_\Delta(k) = \mathbf{C}(k)\cdot\mathbf{x}_\Delta(k) + \Delta_{\mathbf{C}}(k)\cdot\mathbf{x}_\Delta(k), \tag{20}$$

Theorems 1 and 2 were proofed using mathematical induction method (in manuscript). The proofs follow from linear system response results.

5. NORMS OF OPERATORS

It follows from the above formulas that effectivness of estimate (25) will highly depend on how good the estimates of the operator norms $\|\mathbf{C}\cdot\mathbf{L}^{\mathbf{F}}\|$ and $\|\mathbf{L}^{\mathbf{F}}\|$ are. In this section there are two methods presented which allows to obtain a very tight estimates for these norms.

The first method take advantage of matrix notations for discrete evolution operators (Orłowski 2001) and singular value decomposition.

In H^2 space the norm of operator is equal to the maximal singular value, e.g.

$$\left\|\mathbf{L}^{\mathbf{F}}\right\|_2 = \sigma_{\max}(\mathbf{L}^{\mathbf{F}}) \tag{21}$$

Interval operator's norm is given by

$$\sigma_{\min}(\mathbf{L}^{\mathbf{F}}) \le \frac{\left\|\mathbf{L}^{\mathbf{F}}\cdot\mathbf{h}\right\|_2}{\left\|\mathbf{h}\right\|_2} \le \sigma_{\max}(\mathbf{L}^{\mathbf{F}}) \tag{22}$$

where $\mathbf{L}^{\mathbf{F}} \in \mathbf{R}^{(n\cdot N)\times(n\cdot N)}$.

The best way to estimate the norm of $\|\mathbf{C}\cdot\mathbf{L}^{\mathbf{F}}\|$ is to obtain maximal singular value of operator $\mathbf{C}\cdot\mathbf{L}^{\mathbf{F}}$ e.g. $\left\|\mathbf{C}\cdot\mathbf{L}^{\mathbf{F}}\right\|_2 = \sigma_{\max}(\mathbf{C}\cdot\mathbf{L}^{\mathbf{F}})$ where $\mathbf{C}\cdot\mathbf{L}^{\mathbf{F}} \in \mathbf{R}^{(n\cdot N)\times(p\cdot N)}$.

When the time horizon N is large, then operators' size grow with power of two.

It is possible to reduce amounts of computations, using method based on solution of parametric optimisation problem using difference Riccati equation. The main idea of the second method have been presented for time invariant systems (Emirsajłow, Orłowski 1999) and for time-varying systems in (Orłowski 2000). Below we present only general result of its:

Theorem 3. $\left\|\mathbf{C}\cdot\mathbf{L}^{\mathbf{F}}\right\| < \gamma$ *if and only if the following difference Riccati equation for k=0,1,...,N-1*

$$\mathbf{R}(k) = -\mathbf{A}^{\mathrm{T}}\cdot\mathbf{R}(k+1)\cdot[\mathbf{R}(k+1)-\gamma^2\cdot\mathbf{I}]^{-1}\cdot\mathbf{R}(k+1)\cdot\mathbf{A} + \mathbf{A}^{\mathrm{T}}\cdot\mathbf{R}(k+1)\cdot\mathbf{A} + \mathbf{C}^{\mathrm{T}}\cdot\mathbf{C} \qquad \mathbf{R}(N)=\mathbf{0} \tag{23}$$

has a symmetric solution $\mathbf{R}(k) \in \mathcal{L}(\mathbf{R}^n)$.

Discrete difference Riccati equation stated in theorem 3 has a symmetric solution for all γ larger then the norm of operator. The infimum of the set of γ, for which equation (23) has a symmetric solution approximate the norm of operator.

6. TRAJECTORY DEVIATION NORM

The main purpose of this paragraph is to develop techniques for estimating the upper bounds for the difference $\left\| \mathbf{y}_\Delta(\cdot) - \mathbf{y}_p(\cdot) \right\|_{(\mathbf{R}^p)^N}$ and their applications in damage detection problems for uncertain systems.

Theorem 4. *For every* $\Delta_\mathbf{A} \in \mathcal{L}(\mathbf{R}^n, \mathbf{R}^n)^N$, $\Delta_\mathbf{B} \in \mathcal{L}(\mathbf{R}^n, \mathbf{R}^m)^N$, $\Delta_\mathbf{C} \in \mathcal{L}(\mathbf{R}^p, \mathbf{R}^n)^N$, defined in paragraph 3 and

$$\delta_\mathrm{A} < \left\| \mathbf{L}^\mathrm{F} \right\|^{-1} \tag{24}$$

are satisfied, the distance $\left\| \mathbf{y}_\Delta(\cdot) - \mathbf{y}_p(\cdot) \right\|_{(\mathbf{R}^p)^N}$ can be estimated as follows

$$\left\| \mathbf{y}_\Delta(\cdot) - \mathbf{y}_p(\cdot) \right\|_{(\mathbf{R}^p)^N} \le \frac{\left[\left\| \mathbf{C} \cdot \mathbf{L}^\mathrm{F} \right\| \cdot \delta_\mathrm{A} + \delta_\mathrm{C} \right] \cdot \left[\left\| \mathbf{x}_p \right\| + \left\| \mathbf{L}^\mathrm{F} \right\| \cdot \delta_\mathrm{B} \cdot \left\| \mathbf{v}_p \right\| \right]}{1 - \left\| \mathbf{L}^\mathrm{F} \right\| \cdot \delta_\mathrm{A}} + \left\| \mathbf{C} \cdot \mathbf{L}^\mathrm{F} \right\| \cdot \delta_\mathrm{B} \cdot \left\| \mathbf{v}_p \right\| \tag{25}$$

Proof: It is a standard result of functional analysis, if we transform (19) with triangle inequality there is

$$\left\| \mathbf{x}_\Delta(k) \right\|_{\mathbf{R}^n} \le \left\| \mathbf{x}_p(k) \right\|_{\mathbf{R}^n} + \left\| (\mathbf{L}^\mathrm{F} \Delta_\mathbf{A} \cdot \mathbf{x}_\Delta)(k) \right\|_{\mathbf{R}^n} + \left\| (\mathbf{L}^\mathrm{F} \Delta_\mathbf{B} \cdot \mathbf{v}_p)(k) \right\|_{\mathbf{R}^n}$$

then

$$\left\| \mathbf{x}_\Delta(k) \right\|_{\mathbf{R}^n} - \left\| (\mathbf{L}^\mathrm{F} \Delta_\mathbf{A} \cdot \mathbf{x}_\Delta)(k) \right\|_{\mathbf{R}^n} \le \left\| \mathbf{x}_p(k) \right\|_{\mathbf{R}^n} + \left\| (\mathbf{L}^\mathrm{F} \Delta_\mathbf{B} \cdot \mathbf{v}_p)(k) \right\|_{\mathbf{R}^n}$$

$$\left(1 - \left\| \mathbf{L}^\mathrm{F} \Delta_\mathbf{A} \right\|_{(\mathbf{R}^n, \mathbf{R}^n)^N} \right) \cdot \left\| \mathbf{x}_\Delta \right\|_{(\mathbf{R}^n)^N} \le \left\| \mathbf{x}_p \right\|_{(\mathbf{R}^n)^N} + \left\| \mathbf{L}^\mathrm{F} \Delta_\mathbf{B} \cdot \mathbf{v}_p \right\|_{(\mathbf{R}^n)^N}$$

If assumption (24) is satisfying, then *uncertain state* norm we can write as follow

$$\left\| \mathbf{x}_\Delta \right\|_{(\mathbf{R}^n)^N} \le \frac{\left\| \mathbf{x}_p \right\|_{(\mathbf{R}^n)^N} + \left\| \mathbf{L}^\mathrm{F} \right\| \cdot \delta_\mathbf{B} \cdot \left\| \mathbf{v}_p \right\|_{(\mathbf{R}^n)^N}}{1 - \left\| \mathbf{L}^\mathrm{F} \right\| \cdot \delta_\mathbf{A}} \tag{26}$$

Output difference $\mathbf{y}_\Delta$-$\mathbf{y}_\mathbf{p}$ is given

$$\mathbf{y}_\Delta(k) - \mathbf{y}_p(k) = \mathbf{C}(k) \cdot (\mathbf{x}_\Delta(k) - \mathbf{x}_p(k)) + \Delta_\mathbf{C}(k) \cdot \mathbf{x}_\Delta(k)$$
$$= \mathbf{C}(k) \cdot \left[(\mathbf{L}^\mathrm{F} \Delta_\mathbf{A} \cdot \mathbf{x}_\Delta)(k) + (\mathbf{L}^\mathrm{F} \Delta_\mathbf{B} \cdot \mathbf{v}_p)(k) \right] + \Delta_\mathbf{C}(k) \cdot \mathbf{x}_\Delta(k)$$

After normalization we obtain

$$\left\| \mathbf{y}_\Delta(\cdot) - \mathbf{y}_p(\cdot) \right\|_{(\mathbf{R}^p)^N} \le$$
$$\le \left[\left\| \mathbf{C} \cdot \mathbf{L}^\mathrm{F} \right\| \cdot \delta_\mathrm{A} + \delta_\mathrm{C} \right] \cdot \left\| \mathbf{x}_\Delta \right\|_{(\mathbf{R}^n)^N} + \left\| \mathbf{C} \cdot \mathbf{L}^\mathrm{F} \right\| \cdot \delta_\mathrm{B} \cdot \left\| \mathbf{v}_p \right\|_{(\mathbf{R}^m)^N}$$

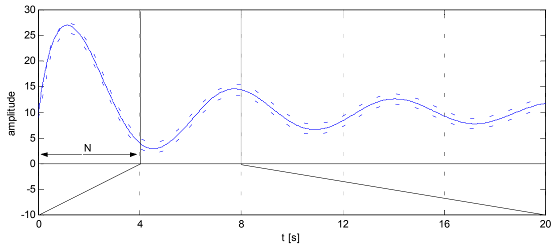
When we substitute (26), we have

$$\left\| \mathbf{y}_\Delta(\cdot) - \mathbf{y}_p(\cdot) \right\|_{(\mathbf{R}^p)^N}$$
$$\le \left[\left\| \mathbf{C} \cdot \mathbf{L}^\mathrm{F} \right\| \cdot \delta_\mathrm{A} + \delta_\mathrm{C} \right] \cdot \frac{\left\| \mathbf{x}_p \right\|_{(\mathbf{R}^n)^N} + \left\| \mathbf{L}^\mathrm{F} \right\| \cdot \delta_\mathbf{B} \cdot \left\| \mathbf{v}_p \right\|_{(\mathbf{R}^n)^N}}{1 - \left\| \mathbf{L}^\mathrm{F} \right\| \cdot \delta_\mathrm{A}}$$
$$+ \left\| \mathbf{C} \cdot \mathbf{L}^\mathrm{F} \right\| \cdot \delta_\mathrm{B} \cdot \left\| \mathbf{v}_p \right\|_{(\mathbf{R}^m)^N}$$

It is equivalent to equation (25). □

7. DAMAGE DETECTION

Method for damage detection for uncertain systems using output uncertainty estimation is presented. The algorithm for this method is drawn on block diagram below.



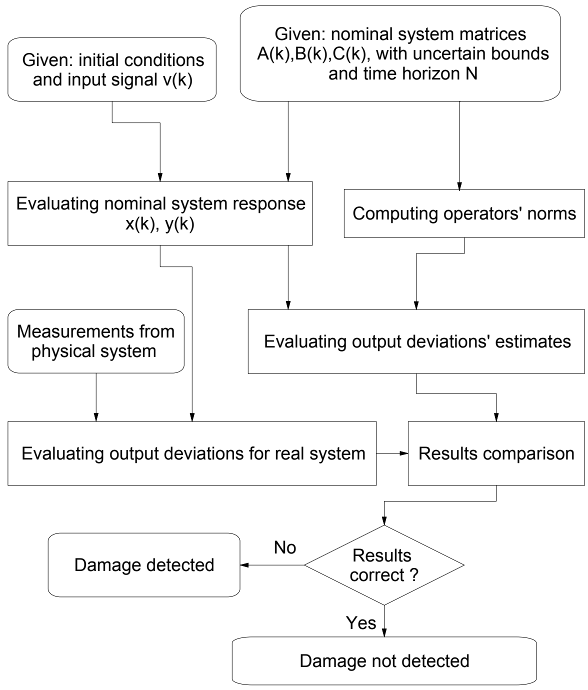


Fig. 2. Block diagram for damage detection process for one time segment.

Block diagram for process of damage detection is shown on fig. 2. For every section of N samples, where N is arbitrary positive integer, analysis goes as follow:

Firstly, system matrices $\mathbf{A}(k)$, $\mathbf{B}(k)$, $\mathbf{C}(k)$, bounds δ_A, δ_B, δ_C time horizon N and input function $v(k)$ have to be known at least at the beginning of every time section. On the next step norms of operators, state and output trajectory have to be evaluated. Then it is possible to calculate the estimates of output deviation. When the measurements from physical system are compared to evaluated output (or state) trajectory it produces an error. When the input, process and output noises are negligible, damage will be detected, it can happen if signal error is larger then the evaluated estimate. In the case of large noises, signal error should be filtered before comparison with estimate of deviation. It is possible to use Kalman or only low-pass filter, but special cases of noises will be considered in the future works.

The analysis are doing periodically, where the length of the time horizon is equal to N samples and symbolically is shown on upper part of fig. 2. N can be fixed or varied. In the case of time invariant system ($\mathbf{A}(k)$, $\mathbf{B}(k)$, $\mathbf{C}(k)$ are independent on k) and fixed N, norms of operators are constant, which allow to calculate it only one time. The deviations could be evaluated for trajectory or for final vector, but this paper develops only method for trajectory deviations estimates. Norm can be estimated in H_2 or H_∞ space. Larger time horizon is better when the noise covariance large is. The shorter time horizon we have, the shorter time of damage detection is.

The next section focus on numerical computations and consists an example of threshold evaluate with application in damage detection problem using block diagram on fig. 2.

8. NUMERICAL EXAMPLE

Let us consider example of damage detection in satellite positioning control.

The linearized and normalized equations of motion of the satellite around the translunar equilibrum point are (Jones, Bishop 1993)

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 7.3809 & 0 & 0 & 0 & 2 & 0 \\ 0 & -2.1904 & 0 & -2 & 0 & 0 \\ 0 & 0 & -3.1904 & 0 & 0 & 0 \end{bmatrix} \cdot \mathbf{x} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \mathbf{u} \tag{27}$$

$$\mathbf{y} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \cdot \mathbf{x} \tag{28}$$

where $\mathbf{x} = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \end{bmatrix}^T$

The state vector $\mathbf{x}$ consists of the satellite position x_{1-3} and velocity x_{4-6}. The inputs $\mathbf{u}$ are the engine thrust accelerations.

The equations have been discretized (sampling period T=0.02s). The aim of control is to minimize input and final error energy. The functional is given by

$$J(\mathbf{u}) = \sum_{i=0}^{N-1} \langle \mathbf{x}(i), \mathbf{x}(i) \rangle_{\mathbf{R}^n} + \sum_{i=0}^{N-1} \langle \mathbf{u}(i), \mathbf{u}(i) \rangle_{\mathbf{R}^m} + 1000 \cdot \langle \mathbf{x}(N), \mathbf{x}(N) \rangle_{\mathbf{R}^n} \tag{29}$$

Feedback matrix is obtained from standard minimisation problem. Input signal $\mathbf{u}(k)$ is delta Croneckera for the first input and zeros for inputs 2,3. Initial state is equal $\mathbf{x}_0 = [\text{-0.82} \;\; 1.38 \;\; 0.89 \;\; \text{-0.60} \;\; 1.17 \;\; \text{-0.27}]^T$. The perturbation norm is $\delta_A = 0.001$. Time horizon N=30.

Following operators' H_2 norms have been computed: $\|\mathbf{L}^F\|_2 = 19.0$, $\|\mathbf{C} \cdot \mathbf{L}^F\|_2 = 18.0$.

After 15 steps (0.3s) the damage effects on coefficient $\mathbf{A}_{6,6}$ and change it value (the new one is 25). Outputs y_1 and y_2 work correctly, only the damage is observable on the third output.

Figure 3 shows norm of output $\|\mathbf{y}(\cdot)\|_\infty$ for systems: damaged and uncertain not damaged. Also H_∞ bounds for system without noise have been marked.

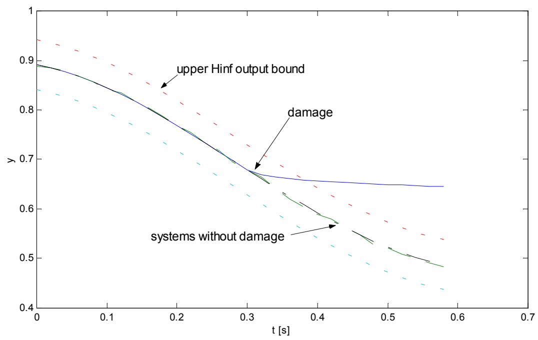


Fig. 3. Norm of output $\|\mathbf{y}(\cdot)\|_\infty$ for nominal system with damage (solid), without damage (dashdot), uncertain system (dashed) and upper and lower bounds (dotted).

Figure 4 shows norms of output error $\|\mathbf{y}(\cdot)\|_2$ for: nominal system with fault, uncertain one and uncertain with white noise (mean value m=0, standard deviation $\sigma = 0.002$). Upper error bounds has been marked.

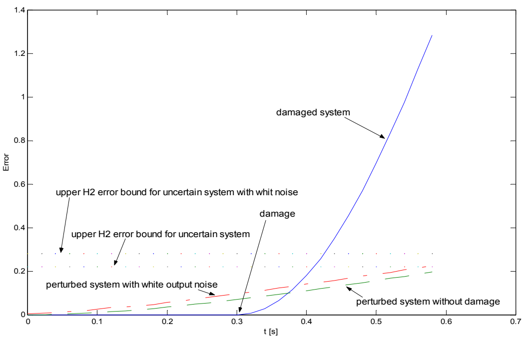


Fig. 4. H_2 norms of error $\|\mathbf{y}_\Delta(\cdot) - \mathbf{y}_p(\cdot)\|_2$ for nominal system with damage (solid), uncertain system without noise (dashed), with white noise (dash-dot), and upper bounds (dotted).

9. CONCLUSION

It should be rather clear that tools for setting threshold for damage detection problems and estimates presented in this work are only one of the possibilities.

The analysis of the uncertain system and the deviations' estimates are accomplished in time domain and in finite time horizon. For time invariant and periodically varying systems, the operators are invariant and could be evaluated only once. The estimates computed in H_∞ space are more conservative then their equivalents in H_2. Nevertheless, when the process and measurement noises are normal and not negligible, it is easier to estimate the energy or power of the noises than the peak noise value.

It seems, however to be true that tight estimates for operator norms $\|\mathbf{C}\cdot\mathbf{L}^\mathbf{F}\|$ and $\|\mathbf{L}^\mathbf{F}\|$ will always play a crucial role. For this reason, presented methods provide a very effective solution to this problem. The developed estimates can be used also in various control desgn tasks for perturbed non-stationar linear discrete time systems.

It is possible to connect this method with another techniques e.g. statistical process control, system identification. Future work will focus on sensibility the threshold when the model assumed for identification is different to real system, and identificated parameters of the system are different from real.

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