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FUZZY PREDICTIVE CONTROL: CONTINUOUS STIRRED–TANK REACTOR CASE STUDY

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Abstract: A nonlinear predictive controller is presented in the paper. The greatest advantage of predictive functional control (PFC) is very fast and easy calculation of control variable. Combined with the Takagi-Sugeno (TS) fuzzy model, the PFC becomes nonlinear and is called fuzzy PFC (FPFC). The controller was evaluated by simulation on the well-known process, the continuous stirred-tank reactor (CSTR), which exhibits strongly nonlinear characteristics. The TS model of the process was obtained by identification. At the end the resulting controller has been compared to a conventional PI controller. Copyright © 2002 IFAC

Keywords: dynamic system, nonlinear control systems, fuzzy control, process control, predictive control, fuzzy modelling, process identification

1. INTRODUCTION

Predictive control development started with the expansion of computer control. At the beginning, the predictive control algorithms were applied in the process technology. Today the predictive controllers are used in many areas, where high–quality control is required. The principle of predictive control is to calculate the control action in the way to minimize the difference between the predicted process output and the reference trajectory. Basic predictive methods are Generalized Predictive Control (GPC) (Clarke *et. al.*, 1987), Dynamics Matrix Control (DMC) (Cutler, Ramaker, 1980) and Predictive Functional Control (PFC) (Richalet *et. al.*, 1978), (Richalet, 1993).

Strongly nonlinear characteristics of many process requires nonlinear control. Nonlinearities can be divided into two groups. In the first group are processes with linear dynamics and static nonlinear transformations on the input or the output of the system. Such processes can be modelled with Wiener or Hamerstein models. The nonlinearities can be compensated using the static nonlinear mapping with the inverted nonlinear characteristics. The second group consists of processes with the nonlinear dynamics. To control such processes the controller should have the nonlinear dynamics too. In many cases suitable controllers are actually linear controllers with changeable parameters. Looking widely the parameters could not only be changed on the nonlinear base, but also according to the operating conditions. This idea, which was originally used only to accommodate changes in process gain, is called gain scheduling. Like the predictive algorithms the gain scheduling is difficult and expensive to implement in the analogue technique and very easy in the computer–controlled systems.

In the paper the PFC algorithm is combined with the global linear model of the process given in the form of Takagi–Sugeno fuzzy model. The fuzzy scheduling used in the proposed algorithm is not directly applied to the controller parameters but is a part of the process model, which is a part of the controller. The method called fuzzy PFC (FPFC) is capable of controlling highly nonlinear processes with long time delay and exhibits remarkable robustness with the respect to the model mismatch and unmodeled dynamics. It was evaluated on a strongly nonlinear process by simulation. The control problem of the continuous stirred–tank reactor (CSTR) was presented by Morningred *et. al.* (1992). The formulation of the model in the state space domain leads to a simple solution, when deriving the control law for the second order process with complex poles of the transfer function. Since the control law is derived in the state space domain, can be treated as a universal derivation for all kinds of stable processes.

The rest of the paper is organized as follows: Fuzzy identification concept is presented in Section 2. In Section 3 the principles of predictive functional control based on fuzzy model are described, the case study is the theme of Section 4 and the conclusion is given in Section 5.

2. FUZZY IDENTIFICATION

The fuzzy model consists of three operations. Fuzzification at the input of the model is transformation of input antecedent variables to the antecedent linguistic variables using membership functions. Defuzzification at the output is the opposite operation, which transforms consequent linguistic variables to output. Between fuzzification and defuzzification is inference. It consists of if–then rules which link antecedent variables to consequent variables. Generally there are two types of fuzzy models: The Mamdani and the Takagi–Sugeno type. In our case the Takagi-Sugeno model was more appropriate to describe the process. The model was obtained using fuzzy identification, which was already described in the literature (Takagi, Sugeno, 1985), so at this point let us just see the quick review. The rule of TS model can have the following form:

$$\begin{array}{l} \mathrm{R}^i : \text{if } \; av \; \text{ is } \; A^i \; \text{ then} \\ y_{pi}(k+1) = a_{1i} y_p(k) + a_{2i} y_p(k-1) + b_i u(k-D) + r_i \end{array}, \quad (1)$$

where a_{1i}, a_{2i} and b_i are linear model parameters, r_i is the offset and D is the delay. The TS model is actually linear combination of linear models

$$y_p(k+1) = \sum_{i=1}^{K} \beta_i(k) y_{pi}(k+1), \quad (2)$$

weighted with normalized degree of fulfilment $\beta_i(k)$, which can be obtained from degrees of membership $\mu_i(k)$:

$$\beta_i(k) = \frac{\mu_i(k)}{\sum_{j=1}^{K} \mu_j(k)}. \quad (3)$$

That solution offers very compact presentation of nonlinear model. It was used in the FPFC method presented in the paper.

Eq. (2) can be divided to the series of K linear equations as follows:

$$\begin{array}{c} \beta_1(k) y_p(k+1) = \beta_1(k) a_{11} y_p(k) + \beta_1(k) a_{21} y_p(k-1) + \\ + \beta_1(k) b_1 u(k-D) + \beta_1(k) r_1 \\ \vdots \\ \beta_i(k) y_p(k+1) = \beta_i(k) a_{1i} y_p(k) + \beta_i(k) a_{2i} y_p(k-1) + \\ + \beta_i(k) b_i u(k-D) + \beta_i(k) r_i \\ \vdots \\ \beta_K(k) y_p(k+1) = \beta_K(k) a_{1K} y_p(k) + \beta_K(k) a_{2K} y_p(k-1) + \\ + \beta_K(k) b_K u(k-D) + \beta_K(k) r_K \end{array}. \quad (4)$$

The use of least squares identification method requires the regression matrix $\mathbf{\Psi}_i$ and the output data vector $\mathbf{Y}_p^i$:

$$\boldsymbol{\psi}_i(k) = \left[\beta_i(k) y_p(k) \quad \beta_i(k) y_p(k-1) \quad \beta_i(k) u(k-D) \quad \beta_i(k) 1 \right], \quad (5)$$

$$\mathbf{\Psi}_i = \begin{bmatrix} \boldsymbol{\psi}_i(D) \\ \vdots \\ \boldsymbol{\psi}_i(k) \\ \vdots \\ \boldsymbol{\psi}_i(N-1) \end{bmatrix}, \quad (6)$$

$$\mathbf{Y}_p^i = \begin{bmatrix} \beta_i(D) y_p(D+1) \\ \vdots \\ \beta_i(k) y_p(k+1) \\ \vdots \\ \beta_i(N-1) y_p(N) \end{bmatrix}. \quad (7)$$

The vector of parameters $\boldsymbol{\theta}_i$ can be obtained using the least squares method:

$$\boldsymbol{\theta}_i = \left(\mathbf{\Psi}_i^T \mathbf{\Psi}_i \right)^{-1} \mathbf{\Psi}_i^T \mathbf{Y}_p^i. \quad (8)$$

$\boldsymbol{\theta}_i$ consists of the elements a_{1i}, a_{2i}, b_i and r_i:

$$\boldsymbol{\theta}_i^T = \left[a_{1i} \quad a_{2i} \quad b_i \quad r_i \right]. \quad (9)$$

The steps presented in Eq. (5) to (9) should be repeated for all rules. Parameter vectors $\boldsymbol{\theta}_i$ can be joined to the parameter matrix $\mathbf{\Theta}$ as follows:

$$\mathbf{\Theta} = \left[\boldsymbol{\theta}_1 \quad \boldsymbol{\theta}_2 \quad \cdots \quad \boldsymbol{\theta}_K \right]. \quad (10)$$

For the purpose of deriving the control law, the model should be written in more compact form also called global linear model.

$$\begin{array}{l} y_p(k+1) = \tilde{a}_1(k) y_p(k) + \tilde{a}_2(k) y_p(k-1) + \\ + \tilde{b}(k) u(k-D) + \tilde{r}(k) \end{array}. \quad (11)$$

Global linear parameters $\tilde{a}_1$, $\tilde{a}_2$, $\tilde{b}$ and $\tilde{r}$ are given in the following equations:

$$\begin{aligned} \tilde{a}_1(k) &= \sum_{i=1}^{K} \beta_i(k) \mathbf{\Theta}_{1i} \\ \tilde{a}_2(k) &= \sum_{i=1}^{K} \beta_i(k) \mathbf{\Theta}_{2i} \\ \tilde{b}(k) &= \sum_{i=1}^{K} \beta_i(k) \mathbf{\Theta}_{4i} \\ \tilde{r}(k) &= \sum_{i=1}^{K} \beta_i(k) \mathbf{\Theta}_{5i} \end{aligned} . \quad (12)$$

3. FUZZY PREDICTIVE FUNCTIONAL CONTROL IN STATE SPACE DOMAIN

Model–based predictive control (MBPC) is a name of a several different control techniques. All are associated with the same idea. The prediction is based on the model of the process. The control action is determined in the way to minimize certain cost function, generally the difference between the predicted future behaviour and the reference trajectory expressed as

$$J(u,k) = \sum_{j=N_1}^{N_2} \left(y_m(k+j) - y_r(k+j) \right)^2 + \lambda \sum_{j=1}^{N_u} u^2(k+j) , \quad (13)$$

where $y_m(k+j)$, $y_r(k+j)$ and $u(k+j)$ are *j*–step ahead prediction of process output, reference trajectory and control signal. Parameter λ weights the relative importance of control signal, N_1, N_2 and N_u are boundaries of signal activity. The result of optimisation is vector of next samples of control action. Only the first element is used. At the next sample time the procedure is repeated.

Predictive functional control (PFC) is one of the MBPC methods. The method is suitable for any stable process. Originally was developed for linear systems (Richalet *et. al.*, 1978), (Richalet, 1993), but can also be used for nonlinear systems written as TS fuzzy model (Škrjanc, Matko, 2000), (Škrjanc, Matko, to be published). The method avoids the criterion minimization presented in Eq. (13). The control law is expressed explicitly, so the method is very robust and calculation of control law is time saving. The basic idea is to equalize the model output increment Δ_m and the objective increment Δ_p.

$$\Delta_p = \Delta_m . \quad (14)$$

Model output increment is the predicted increment of model output over the next *H* samples

$$\Delta_m = y_m(k+H) - y_m(k) . \quad (15)$$

The objective increment is the difference between predicted trajectory and present process output:

$$\Delta_p = y_r(k+H) - y_p(k) . \quad (16)$$

To derive the *H*–step ahead prediction the model has to be transformed to the state space domain:

$$\mathbf{x}_m(k+1) = \tilde{\mathbf{A}}_m \mathbf{x}_m(k) + \tilde{\mathbf{B}}_m u(k) + \tilde{\mathbf{R}}_m , \quad (17)$$

$$y_m(k) = \tilde{\mathbf{C}}_m \mathbf{x}_m(k) . \quad (18)$$

If the state vector $\mathbf{x}_m(k)$ is chosen as

$$\mathbf{x}_m(k) = \begin{bmatrix} y_m(k) \\ y_m(k-1) \end{bmatrix} , \quad (19)$$

than the matrices $\tilde{\mathbf{A}}_m$, $\tilde{\mathbf{B}}_m$, $\tilde{\mathbf{R}}_m$ and $\tilde{\mathbf{C}}_m$ become

$$\tilde{\mathbf{A}}_m = \begin{bmatrix} \tilde{a}_1 & \tilde{a}_2 \\ 1 & 0 \end{bmatrix} , \quad (20)$$

$$\tilde{\mathbf{B}}_m = \begin{bmatrix} \tilde{b} \\ 0 \end{bmatrix} , \quad (21)$$

$$\tilde{\mathbf{R}}_m = \begin{bmatrix} \tilde{r} \\ 0 \end{bmatrix} , \quad (22)$$

$$\tilde{\mathbf{C}}_m = \begin{bmatrix} 1 & 0 \end{bmatrix} . \quad (23)$$

Presuming the constant control signal over the whole horizon, the *H*–step ahead prediction can be obtained:

$$\begin{aligned} y_m(k+H) = \tilde{\mathbf{C}}_m \Big(& \tilde{\mathbf{A}}_m^H \mathbf{x}_m(k) + \\ & + \left(\tilde{\mathbf{A}}_m^H - \mathbf{I} \right) \left(\tilde{\mathbf{A}}_m - \mathbf{I} \right)^{-1} \left(\tilde{\mathbf{B}}_m u(k) + \tilde{\mathbf{R}}_m \right) \Big) \end{aligned} . \quad (24)$$

The reference trajectory is normally output of the reference model, which is usually the same order as the controlled process to assure natural response. When formed in state space domain, the following can be written:

$$\mathbf{x}_r(k+1) = \tilde{\mathbf{A}}_r \mathbf{x}_r(k) + \tilde{\mathbf{B}}_r w(k) , \quad (25)$$

$$y_r(k) = \tilde{\mathbf{C}}_r \mathbf{x}_r(k) . \quad (26)$$

The parameters should be chosen to fulfil the following criterion, which assures the gain factor of the reference model equal to one:

$$\mathbf{C}_r \left(\mathbf{I} - \mathbf{A}_r \right)^{-1} \mathbf{B}_r = 1 . \quad (27)$$

The *H*–step ahead prediction of the reference model can be derived in the similar way like the process model prediction:

$$\begin{aligned} y_r(k+H) = \tilde{\mathbf{C}}_r \Big(& \tilde{\mathbf{A}}_r^H \mathbf{x}_r(k) + \\ & + \left(\tilde{\mathbf{A}}_r^H - \mathbf{I} \right) \left(\tilde{\mathbf{A}}_r - \mathbf{I} \right)^{-1} \tilde{\mathbf{B}}_r w(k) \Big) \end{aligned} . \quad (28)$$

Combining the Eq. (14), (15), (16), (24) and (28) the control law can be derived:

$$u(k) = \frac{C_1 \mathbf{x}_r(k) + C_2 w(k) - y_p(k) - C_3 \mathbf{x}_m(k) - C_4 + y_m(k)}{C_5} . \quad (29)$$

Coefficients C_1 and C_2 are constant, other are variable:

$$
\begin{aligned}
C_1 &= \mathbf{C}_r \mathbf{A}_r^H \\
C_2 &= \mathbf{C}_r \left(\mathbf{A}_r^H - \mathbf{I}\right)\left(\mathbf{A}_r - \mathbf{I}\right)^{-1} \mathbf{B}_r \\
C_3 &= \tilde{\mathbf{C}}_m \tilde{\mathbf{A}}_m^H \\
C_4 &= \tilde{\mathbf{C}}_m \left(\tilde{\mathbf{A}}_m^H - \mathbf{I}\right)\left(\tilde{\mathbf{A}}_m - \mathbf{I}\right)^{-1} \tilde{\mathbf{R}}_m \\
C_5 &= \tilde{\mathbf{C}}_m \left(\tilde{\mathbf{A}}_m^H - \mathbf{I}\right)\left(\tilde{\mathbf{A}}_m - \mathbf{I}\right)^{-1} \tilde{\mathbf{B}}_m
\end{aligned}
\qquad (30)
$$

4. CSTR CASE STUDY

Continuous stirred–tank reactor (CSTR) process consists of an irreversible, exothermic reaction, A → B, in a constant volume reactor cooled by a single coolant stream which can be modelled by the following equations:

$$
\begin{aligned}
\dot{C}_A(t+d) &= \frac{q}{V}\left[C_{A0} - C_A(t+d)\right], \\
&\quad -k_0 C_A(t+d) e^{-\frac{E}{RT(t)}}
\end{aligned}
\qquad (31)
$$

$$
\begin{aligned}
\dot{T}(t) &= \frac{q}{V}\left[T_0 - T(t)\right] - \frac{(-\Delta H) k_0 C_A(t+d)}{\rho\ C_p} \times e^{-\frac{E}{RT(t)}} + \\
&+ \frac{\rho_c C_{pc}}{\rho\ C_p V} q_C(t) \times \left[1 - e^{-\frac{hA}{q_c(t)\rho_c C_{pc}}}\right]\left[T_{c0} - T(t)\right]
\end{aligned}
\qquad .(32)
$$

Table 1 - Nominal CSTR parameters values

Measured product conc.	C_A	0.1 mol/l
Reactor temperature	T	438.54 K
Coolant flow rate	q_c	103.41 l/min
Process flow rate	q	100 l/min
Feed concentration	C_{A0}	1 mol/l
Feed temperature	T_0	350 K
Inlet coolant temperature	T_{c0}	350 K
CSTR volume	V	100 l
Heat transfer term	hA	7×10^5 cal min^{-1} K^{-1}
Reaction rate constant	k_0	7.2×10^{10} min^{-1}
Activation energy term	E/R	1×10^4 K
Heat of reaction	ΔH	2×10^5 cal/mol
Liquid densities	ρ, ρ_c	1×10^3 g/l
Specific heats	C_p, C_{pc}	1 cal^{-1} K^{-1}

The measured concentration has a time delay $d = 0.5\text{min}$. The objective is to control the measured concentration of A C_A by manipulating the coolant flow rate q_c . This model is a modified version of the first of a two – tank CSTR example by Henson and Seborg (1990). In the original model the time delay was zero. The nominal parameter values appear in Table 1.

A discrete static compensator was added to stabilize the process at higher concentration values:

$$
\Delta u_{ff} = K_{ff}\left[T(k) - T(k-1)\right], \qquad (33)
$$

where K_{ff} was chosen to be 3. Sampling time of compensator was 0.1 min.

4.1 Process idetification

The process was identified as a discrete second order process. The sampling time was 0.1 min. Strong nonlinearity forces us to use large number of rules. In our case there were six rules with the same shape of membership function.

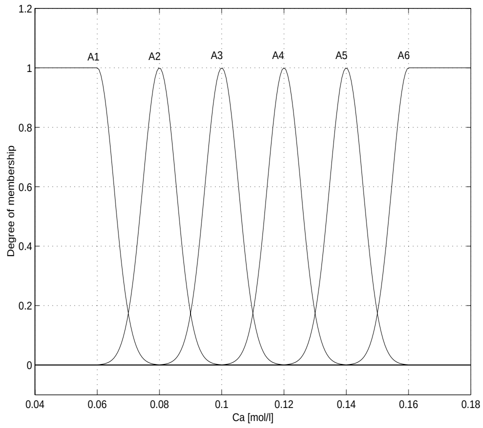

Fig. 1 - Membership functions

If the shape and place of membership functions would be optimised using clustering, the number could be reduced, but then each membership function would be harder to evaluate, so that would not be a great benefit. For the antecedent variable the present and not delayed concentration $C_A(t)$ was used. That can be done because the data are already prepared before the identification. When applying the model to the controller, present concentration is obtained using the prediction model. After the membership functions A^1 to A^6 shown in Fig. 1 were determined the following fuzzy model was obtained:

$$
\begin{aligned}
&R^1: \text{if } C_A \text{ is } A^1 \text{ then } C_A(k+1) = \\
&1.3783 C_A(k) - 0.4510 C_A(k-1) + 0.000142 q_c(k-5) - 0.0082 \\
&R^2: \text{if } C_A \text{ is } A^2 \text{ then } C_A(k+1) = \\
&1.4565 C_A(k) - 0.5138 C_A(k-1) + 0.000189 q_c(k-5) - 0.0137 \\
&R^3: \text{if } C_A \text{ is } A^3 \text{ then } C_A(k+1) = \\
&1.5445 C_A(k) - 0.6062 C_A(k-1) + 0.000230 q_c(k-5) - 0.0176 \\
&R^4: \text{if } C_A \text{ is } A^4 \text{ then } C_A(k+1) = \\
&1.6994 C_A(k) - 0.7426 C_A(k-1) + 0.000219 q_c(k-5) - 0.0185 \\
&R^5: \text{if } C_A \text{ is } A^5 \text{ then } C_A(k+1) = \\
&1.7533 C_A(k) - 0.7951 C_A(k-1) + 0.000277 q_c(k-5) - 0.0251 \\
&R^6: \text{if } C_A \text{ is } A^6 \text{ then } C_A(k+1) = \\
&1.8519 C_A(k) - 0.8828 C_A(k-1) + 0.000238 q_c(k-5) - 0.0223
\end{aligned}
\qquad .(34)
$$

4.2 FPFC applied to the CSTR process

The obtained fuzzy model was used as internal model of the control algorithm. Besides the process model, the controller parameters are also the reference model and the prediction horizon.

The reference model was chosen to have two discrete poles placed at 0.82. The predictive horizon H is normally chosen to fulfil the criterion:

$$N \le H \le \frac{T_r}{2T_s}, \qquad (35)$$

where N is order of process, T_r is raising time and T_s is the sampling time. Large H means the process has more time to equalize with the reference trajectory what results in the smaller gain factor of the controller. The benefit is in the case of great noise disturbance of measured signal or varying dead–time. The disadvantage of large H is the great deviation of global linear model parameters from the process parameters because the response prediction is based on the present degrees of membership. In our case the predictive horizon was chosen to be $H = 4$.

The FPFC was compared to the conventional PI controller. The parameters K_p and T_i were optimised using the ITAE criterion, to give good response for set–point change from 0.1 to 0.15 mol/l. The controller gain was 148 $l^2mol^{-1}min^{-1}$ and the integral time was 0.76 min.

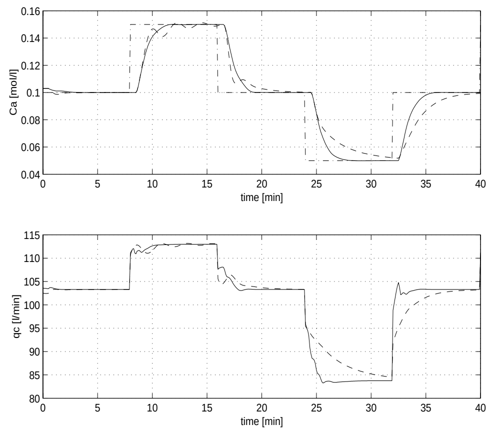

Fig. 2 - Step response in comparison with PI control

The obtained predictive controller has been evaluated by the reference step response and the disturbance rejection. The simulation results of set–point tracking are presented in Fig. 2. The upper diagram presents the set–point with the dash–dot line, the FPFC response is plotted with the solid line and the PI controller response with the dashed line. The lower graph presents control signal. The solid line corresponds to the FPFC and the dashed to the PI controller. The set–point was changed from 0.1 to 0.15, back to 0.1, then to 0.05 and again to 0.1 mol/l. The changes were made every 8 minutes.
The disturbance rejection can be seen from Fig. 3. The unmeasured feed concentration changes from 1 mol/l to 0.95 mol/l at 8 min and back to 1 mol/l at 24 min. The unmeasured coolant temperature decreases from 350°C to 340°C at 16 min and back to 350°C at 32 min. The responses associating the FPFC are plotted with solid line and those which associating the PI control with the dashed line.

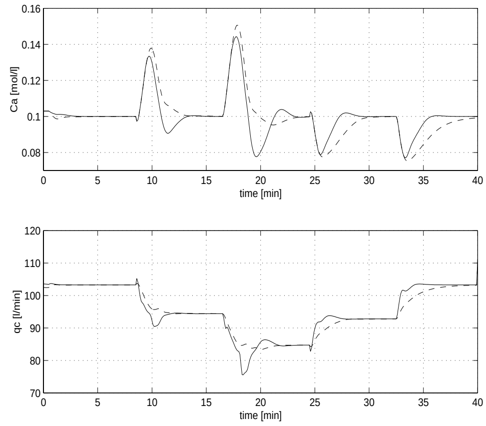

Fig. 3 - Disturbance rejection in comparison with PI controller

5. CONCLUSION

The FPFC algorithm is presented. The development of a new fuzzy predictive scheme was motivated by the unsatisfactory results obtained by using PI controller. The new controller is computationally efficient and offers great robustness when designed with model inaccuracies. It is tuned by placing the reference model poles and choosing the predictive horizon *H*. The simplicity of design and its great performance means great progress in comparison to the conventional control techniques.

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