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AGENTS COALITION IN COORDINATION PROCESS

Dang T-Tung., Frankovič B., Budinska I.

Institute of Informatics – Department Theory Control
Slovak Academy of Sciences,Dubravska c. 9, 84237, Bratislava, Slovakia.
Email: utrrtung@, utrrfran@, utrrbudi@savba.sk

Abstract: Coordination problem among agents is a very interesting area for studying. In this paper we will deal a problem how the agents can coordinate via coalitions, we will also analyse a problem of choosing an optimal structure for agents' coalitions. A method for reduction the searching space for finding the optimal coalition will be presented. Another problem is a coalition related various parameters, for it we will introduce and propose a method to resolve. *Copyright © 2002 IFAC.*

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1. INTRODUCTION

In a practice, mainly in the manufacturing, planning or market, very often occur a situation, where some subjects are coordinating with a purpose to resolve one common problem or are joining any group, where each helps another one with a goal to improve overall profits of the whole group. Such problem, for example, was presented in the work by Frankovič and Dang (2001), where authors introduced problem cooperation between planners or producers where each of them owns a set of resources, a set of tasks (example: a set of products composed from different tasks) necessary to execute and variety different goals. Cooperation between these agents was formulated and solved via sequential negotiation between individual agents in every group that the agents created plans by themselves and continue to negotiate still each agent was content with own plan or obtained results (for example: profits, utilization). But in this work, the problem how the agents could create these groups for negotiation was discussed only formally. To continue this problem, in this paper we focus to resolve problem of creating optimal coalitions and will propose a suitable method to search an optimal structure of these agents' groups. For simplicities we will focus essentially to such agent's groups where each agent helps another and not tries to damage another's execution. Such agents' group is named a coalition

Coalition formation among agents has been widely studied in some literatures as Kahan and Rapoport (1984). In their work, the main solution concepts focus to negotiation process among self-interested agents based on the game theorem and request negotiation between every pair of agents. The final solution that the agents consider as the best might be not absolutely optimal, but they only achieve any sub-optimal one, for example: Nash equilibrium. Furthermore, a process of formation agents' coalition has also been studied and discussed in works presented by Sandholm and Tohmé (1999). In these works the authors discussed about a problem of searching the optimal agents' coalition. Searching process for the optimal agents' coalition is based on heuristic approaches, and a motivation of each agent to join any coalition is to maximize monetary value, that each agent can receive in this coalition.

Similar to the agents' coalition we can see in various areas as parallel computing and distributed artificial intelligence, where to create an optimal structure of the agents' coalition (for example: an optimal structure for coordinative workstations, processors… with a purpose to improve qualities of final results) might apply a theory graph as in a work of Rauber and Runger (1998). In their work, agents' relations are presented via a graph, where each agent presents one node in the graph, and edges express relations between the agents. Using theory graph also does not have less complexity than another known heuristic methods and moreover, for arbitrary number of agents this problem theoretically is to be NP hard.

Furthermore, a lot of various applications in variety areas like economic, transport, policy, etc. also relate the problem of creating optimal coalitions, but they could also involve such problem as formulation coalitions between human societies. To enable to resolve it might need also knowledge of psychology or another one, but we do not focus to these applications and only deal such ones that we can meet in manufacturing, control or computer sciences.

The agents that we consider to use in this paper may be autonomy, or coordinative agents, but all of them have an interest to join coalition with another ones, however coalition may not bring all better results. Conflicts between agents in our model are omitted. These properties however are very often met in a practice, but to implement them to our model it is necessary another as for example psychology knowledge.

The rest of this paper is organized as follows: In section 2 we will introduce the main problem of creating agents' coalition. In section 3 we will present an method, which enables to reduce a searching process and can guarantee an optimal structure of agents' coalitions with an expected divergence. In another section we will deal a problem, where a target of each agent joint coalition is composed from various items. In a section 5 a short illustrate example will be shown.

2. COALITION STRUCTURE FOR AGENTS

Let **A={**A_1,..,A_n**}** be a set of n agents. We assume that, each agent has its plan, what it has to do and what will be a final result (it may be an expected profit, a production time or another, and can be expressed by any function).

Let **I**={1,..,n} be a set of index of agents. In some parts of this paper possible occurs a remark $i \in I$, it means that the agent A_i from a set A. For simplicities we shall use a note I instead of A and a mark $K \subseteq I$ denotes a subset created by agents of a set A and their index belongs to a set K. Now, we can define some basic definitions necessary for following using in this paper.

Definition1: Let A_i, $i \in$ I is an *i-th* agent of a set **A**, then

1. $q_i^* \geq 0$ presents an expected value that agent A_i can receive if it works independently.
2. $q_i^K \geq 0$ presents an expected value that agent A_i can receive if it joins coalition **K** with |**K**|-1 another agents.
3. f_i presents an expected value that agent A_i can receive in coordination process inside **A**.

   $f_i = q_i^*$ if the agent works independently, or

   q_i^K if the agent joins coalition **K**.
4. $\boldsymbol{F}$ is an overall value of a set of agents **A** and it is defined as:

$$F = \sum_{i=1}^{n} f_i \qquad (1)$$

The first main subject of this paper is to find such structure of agents' coalitions to maximize a value $\boldsymbol{F}$. To simplify this task we assume some following properties of our model, which eliminate complicated problems that we can meet in a practice inside coalition of human.

Assumptions:

1. A value f_i is given in advance and is finite (may be by approximation, experiences or negotiation)
2. Agents within coalition coordinate their activities, and do not coordinate among coalitions; therefore a value q_i^K is dependent only on a structure of coalition **K**.

Another definitions needed for searching process are formulated as following:

Definition 2: Let a set $\mathbf{K} \subset \mathbf{I}$ be decomposed to m individual sets {K_1,..K_m} such: $\bigcup_{i=1}^{m} K_i$=**K**, and $K_i \cap K_j = \{0\}$ for $\forall i, j \in [1,..,m]$. Then, a maximal value $\mathbf{Q_K}$ of a set A_K={A_i| i$\in$**K**} is defined as following:

$$Q_K = \max_{m, K_1, \dots, K_m} (\sum_{K_i} \sum_{j \in K_i} f_j) \qquad (2)$$

A property of a function $\mathbf{Q_K}$ is presented in a following theorem.

Definition 3: A function f : Z $\rightarrow$R is a superadditive function in Z if f is bounded for all $z \in$Z, and is valid $f(z_1) + f(z_2) \leq f(z_1 + z_2)$ for all z_1, z_2 and $z_1 + z_2$ in Z.

Theorem 1: A function $Q: \mathbf{M_I} \rightarrow$ R defined by (2), where $\mathbf{M_I}$ is a set of all possible subsets $\mathbf{K} \subset \mathbf{I}$, is a superadditive function in $\mathbf{M_I}$.

$$\forall \mathbf{K}_1, \mathbf{K}_2 \in \mathbf{M_I}, \text{ such } \mathbf{K}_1 \cap \mathbf{K}_2 = \{0\},$$

$$Q_{K_1 \cup K_2} \geq Q_{K_1} + Q_{K_2}$$

Proof: A condition that a function Q is bounded can be easily verified by using assumption 1.

Let **K**=$\mathbf{K}_1 \cup \mathbf{K}_2$, it is clear that K$\in \mathbf{M_I}$. From definition 2 it is possible to write:

$$Q_K \geq \sum_{i=1,2} \sum_{j \in K_i} f_j$$

$$\geq \sum_{j\in K_1} f_j + \sum_{j\in K_2} f_j = Q_{K_1} + Q_{K_2}.$$

Theorem is proved □

Important consequences deduced from this theorem are:

Lemma 1: $\forall$ {$\mathbf{K_1},..,\mathbf{K_m}$}$\in$ $\mathbf{M_I}$ such $\mathbf{K_1} \subset \mathbf{K_2} \subset .. \subset \mathbf{K_m}$ is valid: $Q_{K_1} \leq ... \leq Q_{K_m}$

Proof: it is followed immediately from a theorem 1. □

Lemma 2: Q_I =max ***F***, where ***F*** is defined in (1).

Proof: From a definition 1 and by a suitable manner of rewriting an equation (1) it is obtained $F \leq Q_I$. From a theorem 1 it follows Q_I is bounded and a maximal top bound is max ***F***. The result, then, is Q_I =max ***F***. □

From a lemma 2 it follows that finding Q_I has an equivalent character to find max ***F***. On other hand, the finding a maximal value of ***F***, but, needs to explore a whole space of $\mathbf{M_I}$. In a Sandholm, *et al* (1999) was shown that a volume of $\mathbf{M_I}$ grows with exponential speed and $\cong O(n^n)$. Therefore, it is a very difficult task for the case with a lot of agents in a system, or at least to find a feasible solution it is necessary to reduce a searching space by optimal manners. It is clear that the result obtained after reduction of searching space may be not the best of all, but we can predict a range where the result belongs. One of these methods will be presented in next section.

3. APPROXIMATE COALITION STRUCTURE BY MERGING AGENTS

Exist various methods for reduction of searching space. For example sequentially search in hierarchical levels with using or combination of various heuristic-searching methods, parallel searching methods with deduction interspaces, or another were presented as well in Bonacina (2000). In this section is presented a method to reduce a searching space by merging agents. This method can be explained by a following definition.

Definition 4: An agent $A_{i@j}$ is a merge of an agent A_i and A_j with following properties:

1. $q^*_{i@j} = q_i^{i\cup j} + q_j^{i\cup j}$,
2. $q^K_{i@j} = q_i^K + q_j^K$ when both agents A_i and A_j participate in a coalition **K**,
3. $\forall\, m\in \mathbf{I}$, $m\neq i,j$. If all agents A_i, A_j, and $A_m \in \mathbf{K}$, then, q_m^K is not changed. □

On other words, both agents A_i and A_j are merged and replaced by a new agent with all properties, as if both agents always join a common coalition.

By merging two agents, a number of agents after that is (*n*-1), and the searching space is reduced to $\cong O((n-1)^{n-1})$. As mentioned above, an optimal result obtained after reduction may be not the best solution of all, but with a certain difference. About a quality of the new optimal solution is valid a following theorem.

Theorem 2: Let denote: F_{new} is a maximal overall value of a new set of agents after merging the agent A_i and A_j, and

$$\Delta_1 = \max_{j\notin K\subset I}\{Q_{i\cup K} - Q_K - q_i^*\},$$

$$\Delta_2 = \max_{i\notin K\subset I}\{Q_{j\cup K} - Q_K - q_j^*\}$$

then, $F_{new} \geq \max\ F$ - min{ Δ_1, Δ_2 }.

Proof: Let a F have a maximal value in such structure of agents' coalitions:

$$\mathbf{I}=\mathbf{K}_1 \cup ... \cup \mathbf{K}_m, \quad \text{where } \forall x\neq y\in[1,..,m]\ \mathbf{K}_x \cap \mathbf{K}_y=\{0\} \tag{3}$$

Then, from lemma 2 it follows:

$$\max\ F = Q_I = \sum_{x=1}^{m} Q_{Kx}, \tag{4}$$

By a similar way we can obtain:

$$F_{new} = \sum_{x=1}^{r} Q_{K_x^n}, \tag{5}$$

where

$$\{\mathbf{I}\cup (i@j)\backslash(i\cup j)\} = \mathbf{K}_1^n \cup ... \cup \mathbf{K}_r^n. \tag{6}$$

Two cases could possible happen: The first case is: if exists any K_x, x$\in$ [1,*m*] from (3) such that, both *i,j* $\in K_x$. From a definition 4 about the merged agent, then, it is deduced that must exist a $\mathbf{K}_x^n$, x$\in$ [1,*r*] from (6), which involves both *i* and *j*. Because a new agent $A_{i@j}$ has all properties as if both the agents A_i and A_j always participate in the same coalition.

The second case: if $i\in \mathbf{K}_x$, $j\in \mathbf{K}_y$, $x\neq y$. Let $\mathbf{K}_x = i \cup \mathbf{K}_{x1}$, $\mathbf{K}_y = j\cup \mathbf{K}_{y1}$, From an equation (5) it follows:

$$F_{new} = \sum_{x=1}^{r} Q_{K_x^n} \geq Q_{i@j\cup K_{x1}\cup K_{y1}} + \sum_{z=1,z\neq x,y}^{m} Q_{K_z} \tag{7}$$

because a set of {$i@j \cup K_{x1} \cup K_{y1}$, $\bigcup_{z\in[1,m],z\neq x,y} K_z$} fulfils a condition (6).

Next we can derive:

$$Q_{i@j\cup K_{x1}\cup K_{y1}} \geq \max\{ Q_{i@j\cup K_{x1}} + Q_{K_{y1}},\ Q_{i@j\cup K_{y1}} + Q_{K_{x1}} \} \tag{8}$$

Then,

$$Q_{i@j\cup K_{x1}} + Q_{K_{y1}} = Q_{j\cup K_x} + Q_{K_{y1}} \geq$$

$$\geq Q_{K_x} + q_j^* + Q_{K_{y1}} =$$

$$=(Q_{K_x} + Q_{K_y}) - (Q_{K_y} - q_j^* - Q_{K_{y1}}) \geq$$

$$\geq (Q_{K_x} + Q_{K_y}) - \Delta_2 \qquad (9)$$

Similarly it is obtained:

$$Q_{i@j\cup K_{y1}} + Q_{K_{x1}} \geq (Q_{K_x} + Q_{K_y}) - \Delta_1 . \qquad (10)$$

By substitute (9)+(10) to (8), after that (8) to (7) and by comparing with (4) the theorem will be proved. □

Theorem 2 is useful to predict a difference between results obtained by approximation and the global optimal solution, which can be obtained by exploring the whole space of all possible variants.

For users it is the best if is happening the first case as introduced above, because a new sub-optimal solution is the same like the global one, even thought, after reduction by merging agents, but unfortunately we do not know exactly in advance which pair of agents A_i and A_j could be appropriate for merging. A Simple method to select them is a random choice of an agents' set or another more fair method is choosing sequentially two different agents in each time.

Another property deduced from the theorem 2 is that, these values Δ_1 and Δ_2 are fully independent, thence each of them can be computed by every agent independently. A consequence from that is: before collecting pair agents to merge, every agent can compute and propose its value Δ and on a basis of these values a pair of two agents with the minimum Δ will be selected.

The merging process furthermore could continue, but it means a difference between new results and the global optimal solution will increase. After each merge the theorem 2 can be used to predict an expected deviation. For illustration we shall apply this approach to a case with 10 agents (n=10) and compare with the well-known genetic algorithm. Here, we assumed that $\forall i \in I, q_i^K, q_i^*$ lie in an interval [0,100] and these values were generated randomly. The results are depicted below, where Com=complex searching, C1=A_1@A_2, C2=A_1@A_4, C3=A_1@A_6, GA=genetic algorithm, C4 = A_1@A_2@A_3, Δ= calculate values Δ before choosing agents to merge.

Table 1: an example with 10 agents

	Com.	C1	C2	C3	GA	C4	Δ
Step	159124	9353	19917	17281	15000	1261	35212
Q_I	802	789	799	802	789	677	802

From these results we can observe that with small number of agents, too much agents selected to merge will cause worse results, therefore, better is to compute values Δ before choosing agents. For arbitrary n the obtained results showed that this method is better than GA, however, we need to find a feasible method to compute these Δ. We will discuss more about this problem in future works.

4. AGENTS' COALITION WITH MORE PARAMETERS

In previous sections we have introduced a problem of searching an optimal structure of agents coalition to maximize an overall value of this set of agents. A value that each agent expects to improve when joins coalition is only a function with one parameter, in a practice very often meet a situation where the agents negotiate about some subjects composed from various items, thence, the expected function when agent joins coalition with another ones is also composed from as many as these items. For example we can mention a case, which was studied in a paper of Frankovič and Dang (2001), where the subject of negotiation is to improve a quality of a production plan, and the goal of creating coalition is to increase profits, reduce production time, cost, or another. In such complicated case a fuzzy theory is a suitable way to solve this problem, where the fuzzy theory is used to present individual variants that the agent can execute and by using it the space of potential solutions could be reduced.

Because of short framework of this paper the basic definitions of fuzzy theory are omitted. The readers can read in any literature about fuzzy theory.

It is assumed that each agent has its plan, what it has to do. An expected value of this plan is a function composed from a certain set of items. Instead of a value q_i^* we will use a function $q_i^*(\alpha_1,...,\alpha_p)$ where p is a number of items.

It is easy to verify that now a searching space grows $\cong O(n+p)^{n+p}$, therefore, to find the global optimal structure of agents' coalition with more than 20 agents, 5 items ($n \geq 20$, $p \geq 5$) might be not effective.

It is assumed that, each agent can have a set plan that it is possible to apply by joining to coalition with another ones. Let denote $\mathbf{S}_i$ as a set of possible plans for the agent A_i, $i \in \mathbf{I}$. We can prove that $|\mathbf{S}_i| = 2^{n-1}$, where $|\mathbf{S}_i|$ denotes a number of members of a set $\mathbf{S}_i$.

Moreover, each plan $pl \in \mathbf{S}_i$ can have various parameters. Example: $pl_j \in \mathbf{S}_i$ has a set of parameters $q_{ij} = \{\alpha_1^{ij},...,\alpha_p^{ij}\}$, then, each agent A_i has a set of vectors $\{q_{ij}\}_{j \in [1,2^{n-1}]}$, q_{ij} is defined as above. Each vector q_{ij} presents one variant that the agent A_i can use. From these values we can propose a following approach:

Method for searching optimal structure for agents' coalition:

1.Step: Each agent A_i, $i \in$ **I,** from a set $\{q_{ij}\}$ can create a fuzzy set $M(q_{ij})=\{M^{i}_{\alpha_r}\}=\{\varphi^{ij}_r\}_{r,j}$ where $r \in [1,p]$, $j \in [1,2^{(n-1)}]$. Different methods using to create $M(q_{ij})$ are presented in Novak (1990).

2.Step: From an obtained set $M(q_{ij})$ every agent can choose the best variant for its execution. The result will be a fuzzy set $\mathbf{M}_i=\{\beta^i_j\}_j$ where $j \in [1,2^{(n-1)}]$. For value β^i_j may be computed by two ways:

* $\beta^i_j = \inf_r \{\varphi^{ij}_r\}$ - optimistic decision, or

** $\beta^i_j = \sup_r \{\varphi^{ij}_r\}$ - pessimistic decision.

3.Step: From a set of $\mathbf{M}_i$ is created a new fuzzy set $\mathbf{M}=\bigcup_{i \in I} M_i=\{\delta_x\}_x$. Where a operation $\bigcup_{i \in I} M_i$ is defined as following:

x: $\bigcup_{i=1,..n} S_i \rightarrow N$,

$\delta_x=(\sum_{i=1}^{n}\beta^i_j)/n$ if agents collect such set of plans-equivalently with collecting such set of coalitions, for which is valid:

* $\{\mathbf{K}_1 \cup \ldots \cup \mathbf{K}_m\}=\mathbf{I}$, and exactly $|\mathbf{K}_1|$ agents collect $\mathbf{K}_1$,…and $|\mathbf{K}_m|$ agents collect $\mathbf{K}_m$.

** $\sum_{r=1}^{m}|K_r|=n$.

In another case $\delta_x=0$.

4.Step: From a set **M** search x, where δ_x=max. Then, a set of coalitions that the agents choose in this case is the optimal. □

For a set **M** is a valid following theorem.

Theorem 3: $|\mathbf{M}|$ is equal to dimension of the searching space for Q_I.

Proof: From a step 3 is easy to see that every alternative, which is possible to examine in searching process Q_I fulfils conditions presented in a step 3. Another variant, which does not belong to the searching space for Q_I, for it, afterward, are not valid these conditions (* and **). In a contrary way will be a confrontation. □

From theorem 3 it is possible to deduce that, the searching space by applying this approach can be reduced to $\cong O(n^n)+O(p*2^{n-1})$ if two first steps each agent can compute independently and parallel.

A step 3 is similar to a task introduced in a section 2. To resolve this step, thence, we can apply the method presented in section 3 to reduce a complexity of the given problem. Another related problems with this approach might be methods for computing value β^i_j in a step 2. Because each agent can possess different knowledge a can choose a different method to make decision, therefore, the final solution may be various, however with the same set of date.

5. AN EXAMPLE

In this section we will show a short example to illustrate the previous presented approach. Let be given two agents A1 and A2, each of them has its plan, but can join to coalition. The common coalition may improve some parameters of its plan, but in other hand may fail some another ones.

Let these plans have 3 parameters: $\alpha_1, \alpha_2, \alpha_3$ and there are given following values (expressed in Tab. 2 and 3):

Table 2: Expected values for the agent A1

	α_1	α_2	α_3
$q^*_1(\alpha)$	5	14	9
$q^{1\cup 2}_1(\alpha)$	7	16	6

Table 3: Expected values for the agent A2

	α_1	α_2	α_3
$q^*_2(\alpha)$	8	10	4
$q^{1\cup 2}_2(\alpha)$	4	12	6

By applying a step 1 from a section 4 will obtain following sets:

Table 4: Results after a step 1- $M(q_{1j})$

$M(q_{1j})_{j=1,1\cup 2}=\varphi^{1j}_r$	$M^1_{\alpha_1}$	$M^1_{\alpha_2}$	$M^1_{\alpha_3}$
1	5/12	14/30	9/15
$1\cup 2$	7/12	16/30	6/15

For example: for $\varphi^{1,1}_1$, it means j=1, r=1, the first parameter when the agent A_1 works independently.

$\varphi^{1,1}_1 = q^*_1(\alpha_1)/(q^*_1(\alpha_1)+q^{1\cup 2}_2(\alpha_1))$=5/12.

Similarly,

$\varphi^{1,1\cup 2}_1 = q^{1\cup 2}_2(\alpha_1)/(q^*_1(\alpha_1)+q^{1\cup 2}_2(\alpha_1))$=7/12.

Table 5: Results after a step 1- $M(q_{2j})$

$M(q_{2j})_{j=2,1\cup 2}=\varphi^{2j}_r$	$M^2_{\alpha_1}$	$M^2_{\alpha_2}$	$M^2_{\alpha_3}$
2	8/12	10/22	4/10
$1\cup 2$	4/12	12/22	6/10

In a step 2: let assume that both the agents are pessimistic, the results will be:

Table 6: After a step 2 – $\mathbf{M}_1$

β_j^1	1	$1 \cup 2$
$\mathbf{M}_1$	9/15	7/12

For example: $\beta_{1\cup 2}^1 = \sup_{r=1,2,3} \{\varphi_r^{1j}\}$, where $j = 1 \cup 2$

$\beta_{1\cup 2}^1 = \sup$ {7/12; 16/30; 6/15}= 7/12.

Table 7: After a step 2 – $\mathbf{M}_2$

β_j^2	2	$1 \cup 2$
$\mathbf{M}_2$	8/12	12/22

After a step 3:

Table 8: The final matrix **M**

	1, 2	$1 \cup 2$
M	(9/15+8/12)/2	(7/12+12/22)/2

From the results in Tab. 8 it is easy to see that both the agents will choose the first variant and will work independently.

Certainly, exist a lot of other methods to create matrix $M(q_{1j})$ and $M(q_{2j})$, furthermore, exist various mechanisms for computing β_j^1 and β_j^2, these features, then, could influence to the final solution. For example, in the previous example, if one of the agents is optimistic, and for computing β_j^1 or β_j^2 is applied an another method (a function sup() is changed to a function inf()), afterwards, the final solution shows that both agents will choose the second variant and will join coalition together.

As shown above, a dimension of matrix $M(q_{1j})$ and $M(q_{2j})$ grows linearly with parameter *p*, hence, the method presented in section 4 can be applied for cases with a higher sum of parameters (*p*), but in a case, when a number of agents increases, dimension of these matrix will increase with exponential speed. Therefore, it is a need to find further methods, which are able to apply for cases with a greater number of agents.

6.CONCLUSION

In this paper we have presented one interesting problem that is often met in a practice, mainly in a market, or evenly in a human society. Coalition among agents is one of methods that enable the agents to coordinate. Coordination process among the agents can relate a lot of problems, as example: communication, negotiation between self-interest agents, and coordination in unknown environment connected with learning agents, or all another. All these features can be appeared and implemented in every coalition structure. In our model we have been still dealing a problem with simple agents, without unknown parameters and all agents are motivated to improve qualities of their execution, not only execution of one agent but also of another ones, with a purpose to improve qualities of the whole agents' society. It is a reason, for that we have introduced a criterion function ***F*** in a definition 1.

The contribution of this paper we can summarize briefly as following: We have presented a method for reduction a searching space, and a predicted difference of the optimal solution can be computed by using theorem in a section 3. Another problem, which has been presented and for it has been proposed a feasible solution, is a coalition with various parameters. Our method from section 4 shows that, complexity of searching space by applying this method can be reduced from $\cong O(n+p)^{n+p}$ to $\cong O(n^n) + O(p * 2^{n-1})$. It is sufficient to enable to find the suboptimal solution. A short example to understand these previous theories has been shown in a section 5. Various results can be appeared, dependently on knowledge of each agent. A lot of problems, which while have not been introduced and resolved in this paper as: optimal method for creating fuzzy set $M(q_{1j})$ and $M(q_{2j})$, problem with self-interest agents, negotiation between agents before choosing coalition also can reduce a searching space. About these problems we will discuss in future works.

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