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A NEW VARIABLE STRUCTURE PID-CONTROLLER FOR ROBOT MANIPULATORS WITH PARAMETER PERTURBATIONS:AN AUGMENTED SLIDING SURFACE APPROACH

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Abstract: In this paper a new variable structure PID-controller is designed for stabilization of robot manipulator systems with parameter perturbations. The sufficient conditions for the existence of a sliding mode is considered. The techniques of matrix norm inequalities are used to cope with robustness issues. Some effective parameter-independent conditions are developed in a concise manner for the global asymptotic stability of the multivariable system using LMI's techniques and principle of Rayleigh's min/max matrix eigenvalue inequality. The stability conditions are derived by using the Lyapunov full quadratic form for the first time. The parameter perturbations of the robot motion are evaluated by introducing Frobenius norm. Simulation results have shown that the control performance of the robot system is satisfactory. *Copyright © 2002 IFAC*

Keywords: PID variable structure control, robot control, sliding mode, Lyapunov full quadratic form.

1. INTRODUCTION

In recent years, the variable structure principles are frequently used for the stabilization of robot motion (Hung, 1993), (Fu, 1987). Among some of the successful variable structure control applications, Bailey and Arapostathis developed a control law that ensures the stability of the intersection of the discontinuity surfaces to reduce the complexity of the design. In dealing with the set-point regulation problem of manipulators, in (Yeung, 1988), a new control algorithm is proposed which successfully utilizes the advantage of symmetric positive definiteness of the inertia matrix. The control law is successful in overcoming the strong dynamic coupling problem. In (Su, 1999) a model-based adaptive variable structure control scheme is successfully introduced for underactuated robots. The uncertainty bounds that are used depend only on the inertia parameters of the system. In (Chern, 1992), an integral variable structure control for robot manipulators is presented to achieve accurate servo-tracking in the presence of load variations, parameter variations and nonlinear dynamics. An integral sliding mode control of robot manipulators is considered by (Utkin, 1999). Integral sliding mode control approach for guided missile system is considered by (Jafarov, 2001). A survey paper (Sage, 1999) presents an overview of six different robust control schemes including current robot state coordinates for robot manipulators. Discontinuous min-max control term, combined with the linear control term or relay-saturation type control is used in robot control systems. In this paper a new PID-variable structure controller is designed for stabilization of robot manipulators with parameter perturbations.

2. DYNAMICS OF THE ROBOT MANIPULATOR

Introducing $\psi(t) = \int_0^t \theta(t)dt$, let an n-link manipulator be considered whose augmented dynamics is given by (Fu, 1987), (Bailey, 1987):

$$\dot{\psi} = \theta$$
$$\dot{\theta} = \omega \quad (1)$$
$$\dot{\omega} = M^{-1}(\theta)\left[-B(\theta,\dot{\theta})\dot{\theta} - f(\dot{\theta}) + u\right]$$

where

ψ	:	$n \times 1$	angular position integral vector
θ	:	$n \times 1$	angular position vector
ω	:	$n \times 1$	angular velocity vector
$M(\theta)$	:	$n \times n$	positive definite inertia matrix
$B(\theta,\dot{\theta})$	:	$n \times n$	coriolis and centripetal force matrix
$f(\dot{\theta})$	:	$n \times 1$	joint friction vector
$u(t)$	:	$n \times 1$	control input vector

The state vector of the dynamics is given as, $x = [\psi \quad \theta \quad \omega]^T$. Let $\psi_d(t)$, $\theta_d(t)$ and $\omega_d(t)$ denote the reference position integral, position and velocity. Then the deviation of the actual position integral, position and velocity from the reference counterparts are denoted by $\tilde{\psi} = \psi - \psi_d$, $\tilde{\theta} = \theta - \theta_d$, $\tilde{\omega} = \omega - \omega_d$, respectively. The position control case is considered: $\psi_d(t) = \theta_d t_s\,;\theta_d(t) = \theta_d\,;\omega_d(t) = 0$ where t_s is the settling time of the control process. The goal is to select a control law such that the system (1) will be globally asymptotically stable.

3. PID-CONTROL LAW

To achive this goal, the PID-variable structure control law is formed as follows

$$u(t) = u_{eq}(t) - \left[K_p\left\|\tilde{\theta}(t)\right\| + K_i\left\|\tilde{\psi}(t)\right\| + K_d\left\|\tilde{\omega}(t)\right\|\right]\mathrm{sign}(s(t)) \quad (2)$$

where K_p, K_i, K_d are scalar feedback gain parameters to be selected, $\|(\cdot)\| = \sqrt{(\cdot)^T(\cdot)}$ is the Euclidean norm, T is the transpose of vector or matrix, $\mathrm{sign}(s(t)) = [\mathrm{sign}(s_1(t)),...,\mathrm{sign}(s_n(t))]^T$ is the signum function vector and $u_{eq}(t)$ is the equivalent control. Let an augmented sliding surface is defined as follows

$$s = C_1\tilde{\psi} + C_2\tilde{\theta} + \tilde{\omega} \quad (3)$$

where C_1, C_2 are design matrices. In order to investigate how the design matrices can be chosen to establish a stable sliding surface, let $\dot{s}(t) = 0$ be represented in state-space form as follows: $\begin{bmatrix}\dot{\tilde{\theta}}(t)\\ \dot{\tilde{w}}(t)\end{bmatrix} = \begin{bmatrix}0_{n\times n} & \mathrm{I}_{n\times n}\\ -C_1 & -C_2\end{bmatrix}\begin{bmatrix}\tilde{\theta}(t)\\ \tilde{w}(t)\end{bmatrix}$ with n as the degree of the robot arm system. From the state-space representation, it is inferred that $\begin{bmatrix}0_{n\times n} & \mathrm{I}_{n\times n}\\ -C_1 & -C_2\end{bmatrix}$ must have stable eigenvalues. Differentiating the sliding surface along the state trajectory of system (1), (2), the equivalent control is derived using the equivalent method procedure:

$$\dot{s} = C_1\dot{\tilde{\psi}} + C_2\dot{\tilde{\theta}} + \dot{\tilde{\omega}} = 0 \Rightarrow u(t) = u_{eq}(t) \quad (4)$$

Substituting (1) into (4) and rewriting (4) by using the given desired trajectory of $\omega_d(t) = 0$ and $\dot{\omega}_d(t) = 0$, (5)

$$\dot{s} = C_1\tilde{\theta} + C_2\tilde{\omega} + M^{-1}(\theta)\left[-B(\theta,\dot{\theta}) + f(\dot{\theta}) + u_{eq}(t)\right]$$

Then the equivalent control including the accessible manipulator parameters is obtained as follows,

$$u_{eq}(t) = B(\theta,\dot{\theta})\dot{\theta} + f(\dot{\theta}) - M(\theta)C_1\tilde{\theta} - M(\theta)C_2\tilde{\omega} \quad (6)$$

After selecting the sliding mode control (2) with sliding surface (3) and determining the corresponding equivalent control which can be easily implemented by the digital microprocessors, the next step is to choose the design parameters, such that the sufficient conditions for the existence of a multivariable sliding condition are satisfied and the closed-loop sliding system will be globally asymptotically stable.

4. SLIDING CONDITIONS

Here, the sufficient conditions for the existence of the sliding mode in the robot-manipulator system is analyzed.

Lemma 1: The stable sliding mode on *s(t)=0* always exists in uncertain system (1) driven by variable structure controller (2), if the following conditions hold

$$K_d \geq \frac{\eta}{n}\max_{\theta}\|M(\theta)\|_F,\, K_i \geq \|C_1\|_F K_d,\, K_p \geq \|C_2\|_F K_d \quad (7)$$

where $\|(\cdot)\|_F = \sqrt{\mathrm{trace}((\cdot)^T(\cdot))}$ is the Frobenius norm and n is the degree of the robot arm system.

Proof: Let a Lyapunov function be chosen as

$$V(t,\theta,\omega) = \frac{1}{2}\left[s(t)^T\mathrm{sign}(s(t))\right]^2 \quad (8)$$

Time derivative of V along the state trajectory of system (1), (2), (3), and (6) is given by

$$\begin{aligned}\dot{V} &= \left[s(t)^T\mathrm{sign}(s(t))\right]\left[\mathrm{sign}(s(t))\right]^T\dot{s}(t)\\ &= \left[s(t)^T\mathrm{sign}(s(t))\right]\Big[C_1\psi + C_2\tilde{\theta} + M^{-1}(\theta)\big[-B(\theta,\dot{\theta})\\ &\quad - f(\dot{\theta}) + B(\theta,\dot{\theta}) + f(\dot{\theta})) - M(\theta)C_1\psi - M(\theta)C_2\tilde{\theta}\big]\\ &\quad - \left[K_p\left\|\tilde{\theta}(t)\right\| + K_i\left\|\tilde{\psi}(t)\right\| + K_d\left\|\tilde{\omega}(t)\right\|\right]M^{-1}(\theta)\mathrm{sign}(s(t))\Big]\\ &= -\left[s(t)^T\mathrm{sign}(s(t))\right]\left[\mathrm{sign}(s(t))\right]^T\Big[K_p\left\|\tilde{\theta}(t)\right\|\\ &\quad + K_i\left\|\tilde{\psi}(t)\right\| + K_d\left\|\tilde{\omega}(t)\right\|\Big]M^{-1}(\theta)\mathrm{sign}(s(t))\end{aligned} \quad (9)$$

Note that, if $K_p, K_i, K_d > 0$ then $\dot{V} < 0$ because $M^{-1}(\theta) > 0$. But the η-reaching condition can be used (Hung et al., 1993), (Asada, 1986). For the multivariable case η-reaching condition is given by

$$\begin{aligned}\dot{V} &= \left[s(t)^T\mathrm{sign}(s(t))\right]\left[\mathrm{sign}(s(t))\right]^T\dot{s}(t) \leq -\eta\|s(t)\|^2\\ &\leq -K_d\left[s(t)^T\mathrm{sign}(s(t))\right]\left[\mathrm{sign}(s(t))\right]^T\Big[\|C_1\|_F\left\|\tilde{\psi}(t)\right\|\\ &\quad + \|C_2\|\left\|\tilde{\theta}(t)\right\| + \left\|\tilde{\omega}(t)\right\| + \left(\frac{K_i}{K_d} - \|C_1\|_F\right)\left\|\tilde{\psi}(t)\right\|\end{aligned}$$

$$+\left(\frac{K_p}{K_d}-\|C_2\|_F\right)\left\|\widetilde{\theta}(t)\right\|\Bigg]M^{-1}(\theta)\mathrm{sign}(s(t)) \le -\eta\|s(t)\|^2 \tag{10}$$

Choosing the gain parameters $K_p, K_i,$ and K_d such that $K_i \ge \|C_1\|_F K_d$ and $K_p \ge \|C_2\|_F K_d$ and noticing $\|C_1\|_F\|\widetilde{\psi}(t)\| + \|C_2\|_F\|\widetilde{\theta}(t)\| + \|\widetilde{\omega}(t)\| \ge \|s(t)\|$ and $s(t)^T\mathrm{sign}(s(t)) \ge \|s(t)\|$, the inequality in (10) can be arranged as follows,

$$[\mathrm{sign}(s(t))]^T\left[-nK_d\lambda_{\min}(M^{-1}(\theta))+\eta\right]\mathrm{sign}(s(t))\|s(t)\|^2 \le 0 \tag{11}$$

Therefore, the inequality in (11) implies

$$\eta - \frac{nK_d}{\lambda_{\max}(M(\theta))} < \eta - \frac{nK_d}{\max\limits_{\theta}\|M(\theta)\|_F} \le 0 \tag{12}$$

If conditions (7) are satisfied, then sliding inequality (11) reduces to $\dot{V} < 0$ for all $s(t) \ne 0$ and we conclude that the sliding motion is always generated on the switching surface $s(t) = 0$.

The global asymptotic stability with respect to the state coordinates of the robot-manipulator system will be analyzed now.

5. STABILIZATION OF THE CLOSED-LOOP SYSTEM

The following theorem summarizes the stability results, which are based on full quadratic Lyapunov function method with LMI's techniques.

Theorem 1: Suppose that the conditions (7) of Lemma 1 hold, then the uncertain multi-input system (1) driven by discontinuous sliding mode controller (2), (3), (6) is globally asymptotically stable, if the following conditions hold

$$R_3 > 0, \quad \min_{\theta,\dot{\theta}}\lambda(R_1) > \frac{\max\limits_{\theta,\dot{\theta}}\|P_3\|_F^2}{\min\limits_{\theta,\dot{\theta}}\lambda(R_3)}$$

$$K_d > \frac{1}{\phi_0\lambda_{\min}(C_1)}\left[\frac{\max\limits_{\theta,\dot{\theta}}\|P_1\|_F^2}{\min\limits_{\theta,\dot{\theta}}\lambda(R_1) - \dfrac{\max\limits_{\theta,\dot{\theta}}\|P_3\|_F^2}{\min\limits_{\theta,\dot{\theta}}\lambda(R_3)}} + \tfrac{1}{2}\left(\|\Phi_{11}\|_F^2 + \|\Phi_{12}\|_F^2 + \|\Phi_{13}\|_F^2\right)\max_{\theta,\dot{\theta}}\left\|\dot{M}(\theta,\dot{\theta})\right\|_F\right] \tag{13}$$

$$K_i \ge \|C_1\|_F K_d, \quad K_p \ge \|C_2\|_F K_d$$

where

$\phi_{13}, \phi_{23}, \phi_{33}$ are appropriate scalars to be chosen, $\Phi_{13} = \phi_{13}\mathrm{I}_{n\times n}$, , $\Phi_{23} = \phi_{23}\mathrm{I}_{n\times n}$, $\Phi_{33} = \phi_{33}\mathrm{I}_{n\times n}$, $\phi_0 = \phi_{13}^2 + \phi_{23}^2 + \phi_{33}^2$, $C_1 = \frac{1}{\phi_0}\left(\Phi_{13}^T\Phi_{11} + \Phi_{23}^T\Phi_{12}^T + \Phi_{33}^T\Phi_{13}^T\right)$, $C_2 = \frac{1}{\phi_0}\left(\Phi_{13}^T\Phi_{12} + \Phi_{23}^T\Phi_{22} + \Phi_{33}^T\Phi_{23}^T\right)$,

$$P_1 = \tfrac{1}{2}\left(\Phi_{11}^T M\Phi_{11} + \Phi_{12}M\Phi_{12}^T + \Phi_{13}M\Phi_{13}^T - \phi_0 C_1^T MC_1\right)$$
$$-\tfrac{1}{2}\left(\Phi_{11}^T\dot{M}\Phi_{11} + \Phi_{12}\dot{M}\Phi_{12}^T + \Phi_{13}\dot{M}\Phi_{13}^T\right)C_1^{-1}C_2$$

$$R_1 = P_1^T C_1^{-1}C_2 + C_2^T(C_1^T)^{-1}P_1 + \tfrac{1}{2}\left(\phi_0 C_2^T MC_1 + \phi_0 C_1^T MC_2 - \Phi_{11}^T M\Phi_{12} - \Phi_{12}^T M\Phi_{11} - \Phi_{12}M\Phi_{22} - \Phi_{22}^T M\Phi_{12}^T - \Phi_{13}M\Phi_{23}^T - \Phi_{23}M\Phi_{13}^T\right)$$
$$+\tfrac{1}{2}C_2^T(C_1^T)^{-1}\left(\Phi_{11}^T\dot{M}\Phi_{11} + \Phi_{12}\dot{M}\Phi_{12}^T + \Phi_{13}\dot{M}\Phi_{13}^T\right)C_1^{-1}C_2 - \tfrac{1}{2}\left(\Phi_{12}^T\dot{M}\Phi_{12} + \Phi_{22}^T\dot{M}\Phi_{22} + \Phi_{23}\dot{M}\Phi_{23}^T\right),$$

$$P_2 = \tfrac{1}{2}\Phi_{11}^T M\Phi_{12} + \Phi_{12}M\Phi_{22} + \Phi_{13}M\Phi_{23}^T - \phi_0 C_1^T MC_2\Big) + \tfrac{1}{2}\left(\phi_0 C_1^T\dot{M}C_1 - \Phi_{11}^T\dot{M}\Phi_{11} - \Phi_{12}\dot{M}\Phi_{12}^T - \Phi_{13}\dot{M}\Phi_{13}^T\right)C_1^{-1}$$

$$R_2 = P_2^T C_1^{-1} + (C_1^T)^{-1}P_2 + \tfrac{1}{2}(C_1^T)^{-1}\left(\Phi_{11}^T\dot{M}\Phi_{11} + \Phi_{12}\dot{M}\Phi_{12}^T + \Phi_{13}\dot{M}\Phi_{13}^T - \phi_0 C_1^T\dot{M}C_1\right)C_1^{-1}$$

assuming P_1 is nonsingular,

$$P_3 = \tfrac{1}{2}\left(\Phi_{12}^T M\Phi_{12} + \Phi_{22}^T M\Phi_{22} + \Phi_{23}M\Phi_{23}^T - C_2^T MC_2\right) + \tfrac{1}{2}C_2^T(C_1^T)^{-1}\left(\phi_0 C_1^T\dot{M}C_1 - \Phi_{11}^T\dot{M}\Phi_{11} - \Phi_{12}\dot{M}\Phi_{12}^T - \Phi_{13}\dot{M}\Phi_{13}^T\right)C_1^{-1} - P_1^T C_1^{-1} - C_2^T(C_1^T)^{-1}P_2 + R_1P_1^{-1}P_2$$

$$R_3 = R_2 - P_2^T(P_1^T)^{-1}R_1P_1^{-1}P_2 + P_3^T P_1^{-1}P_2 + P_2^T(P_1^T)^{-1}P_3$$

Proof: Let a positive definite full quadratic form of Lyapunov function candidate be introduced as

$$V(\widetilde{\psi},\widetilde{\theta},\widetilde{\omega}) = \begin{bmatrix}\widetilde{\psi}\\ \widetilde{\theta}\\ \widetilde{\omega}\end{bmatrix}^T Q^T M'(\theta)Q\begin{bmatrix}\widetilde{\psi}\\ \widetilde{\theta}\\ \widetilde{\omega}\end{bmatrix} \tag{14}$$

where

$$Q = \begin{bmatrix}\Phi_{11} & \Phi_{12} & \Phi_{13}\\ \Phi_{12}^T & \Phi_{22} & \Phi_{23}\\ \Phi_{13}^T & \Phi_{23}^T & \Phi_{33}\end{bmatrix}, M'(\theta) = M(\theta)\begin{bmatrix}\mathrm{I} & 0 & 0\\ 0 & \mathrm{I} & 0\\ 0 & 0 & \mathrm{I}\end{bmatrix}$$

Q is a full rank design matrix and I is an nxn identity matrix. Taking the time derivative of (14) along the trajectory of (1), (2), (3), (6), and choosing C_1 and C_2 as

$$C_1 = \left(\Phi_{13}^T M\Phi_{13} + \Phi_{23}^T M\Phi_{23} + \Phi_{33}^T M\Phi_{33}\right)^{-1}\left(\Phi_{13}^T M\Phi_{11} + \Phi_{23}^T M\Phi_{12}^T + \Phi_{33}^T M\Phi_{13}^T\right)$$

$$C_2 = \left(\Phi_{13}^T M\Phi_{13} + \Phi_{23}^T M\Phi_{23} + \Phi_{33}^T M\Phi_{33}\right)^{-1}\left(\Phi_{13}^T M\Phi_{12} + \Phi_{23}^T M\Phi_{22} + \Phi_{33}^T M\Phi_{23}^T\right)$$

$\Phi_{13} = \phi_{13}\mathrm{I}, \Phi_{23} = \phi_{23}\mathrm{I}$, $\Phi_{33} = \phi_{33}\mathrm{I}, \Phi_{13}, \Phi_{23}$, Φ_{33} are scalars and remembering from sliding conditions that $K_p, K_i,$ and K_d are chosen as $K_i \ge \|C_1\|_F K_d$ and $K_p \ge \|C_2\|_F K_d$ and using the triangular inequality

$$\|C_1\|_F\|\widetilde{\psi}\| + \|C_2\|_F\|\widetilde{\theta}\| + \|\widetilde{\omega}\| \ge \|C_1\widetilde{\psi} + C_2\widetilde{\theta} + \widetilde{\omega}\| = \|s(t)\|$$

where $s(t)^T\mathrm{sign}(s(t)) > \|s(t)\|$. After rearrangings, finally it is obtained

$$\dot{V} \le \widetilde{\psi}^T\left[\tfrac{1}{2}\left(\Phi_{11}^T\dot{M}\Phi_{11} + \Phi_{12}\dot{M}\Phi_{12}^T + \Phi_{13}\dot{M}\Phi_{13}^T\right)_1 - \phi_0 C_1^T K_d C_1\right]\widetilde{\psi}$$
$$+\widetilde{\psi}^T\left[\left(\Phi_{11}^T M\Phi_{11} + \Phi_{12}M\Phi_{12}^T + \Phi_{13}M\Phi_{13}^T\right) + \left(\Phi_{11}^T\dot{M}\Phi_{12} + \Phi_{12}\dot{M}\Phi_{22} + \Phi_{13}\dot{M}\Phi_{23}^T\right) - 2\phi_0 C_1^T K_d C_2 - \phi_0 C_1^T MC_1\right]\widetilde{\theta}$$

$$
\begin{aligned}
&+\tilde{\psi}^T\Big[\big(\Phi_{11}^T M\Phi_{12}+\Phi_{12}M\Phi_{22}+\Phi_{13}M\Phi_{23}^T\big)+\big(\Phi_{11}^T\dot{M}\Phi_{13}\\
&+\Phi_{12}\dot{M}\Phi_{23}+\Phi_{13}\dot{M}\Phi_{33}\big)-2\phi_0C_1^TK_d-\phi_0C_1^TMC_2\Big]\tilde{\omega}\\
&+\Phi_{22}^T\dot{M}\Phi_{22}+\Phi_{23}\dot{M}\Phi_{23}^T\big)-\phi_0C_2^TK_dC_2-\phi_0C_2^TMC_1\Big]\tilde{\theta}\\
&+\tilde{\theta}^T\Big[\big(\Phi_{11}^TM\Phi_{13}+\Phi_{12}M\Phi_{23}+\Phi_{13}M\Phi_{33}^T\big)+\big(\Phi_{12}^TM\Phi_{12}\\
&+\Phi_{22}^TM\Phi_{22}+\Phi_{23}M\Phi_{23}^T\big)+\big(\Phi_{12}\dot{M}\Phi_{13}+\Phi_{22}^T\dot{M}\Phi_{23}\\
&+\Phi_{23}\dot{M}\Phi_{33}\big)-2\phi_0C_2^TK_d-\phi_0C_2^TMC_2-\phi_0C_1^TM\Big]\tilde{\omega}\\
&+\tilde{\omega}^{T^T}\Big[\big(\Phi_{12}^TM\Phi_{13}+\Phi_{22}^TM\Phi_{23}+\Phi_{23}M\Phi_{33}\big)+\tfrac{1}{2}\big(\Phi_{13}^T\dot{M}\Phi_{13}\\
&+\Phi_{23}^T\dot{M}\Phi_{23}+\Phi_{33}^T\dot{M}\Phi_{33}\big)-\phi_0K_d-\phi_0MC_2\Big]\tilde{\omega}\\
&=-\begin{bmatrix}\tilde{\psi}\\ \tilde{\theta}\\ \tilde{\omega}\end{bmatrix}^T\begin{bmatrix}H_{11} & H_{12} & H_{13}\\ H_{12}^T & H_{22} & H_{23}\\ H_{13}^T & H_{23}^T & H_{33}\end{bmatrix}\begin{bmatrix}\tilde{\psi}\\ \tilde{\theta}\\ \tilde{\omega}\end{bmatrix} \qquad (15)
\end{aligned}
$$

where

$$
\begin{aligned}
&H_{11}=\phi_0C_1^TK_dC_1-\tfrac{1}{2}\big(\Phi_{11}^T\dot{M}\Phi_{11}+\Phi_{12}\dot{M}\Phi_{12}^T\\
&+\Phi_{13}\dot{M}\Phi_{13}^T\big)=K_{d1},\ H_{12}=K_{d1}C_1^{-1}C_2\\
&+\tfrac{1}{2}\big(\Phi_{11}^T\dot{M}\Phi_{11}+\Phi_{12}\dot{M}\Phi_{12}^T+\Phi_{13}\dot{M}\Phi_{13}^T\big)C_1^{-1}C_2\\
&+\tfrac{1}{2}\phi_0C_1^TMC_1-\tfrac{1}{2}\big(\Phi_{11}^TM\Phi_{11}+\Phi_{12}M\Phi_{12}^T+\Phi_{13}M\Phi_{13}^T\big)\\
&-\tfrac{1}{2}\big(\Phi_{11}^T\dot{M}\Phi_{12}+\Phi_{12}\dot{M}\Phi_{22}+\Phi_{13}\dot{M}\Phi_{23}^T\big),\ H_{13}=K_{d1}C_1^{-1}\\
&+\tfrac{1}{2}\big(\Phi_{11}^T\dot{M}\Phi_{11}+\Phi_{12}\dot{M}\Phi_{12}^T+\Phi_{13}\dot{M}\Phi_{13}^T\big)C_1^{-1}\\
&+\tfrac{1}{2}\big(\phi_0C_1^TMC_2-\Phi_{11}^TM\Phi_{12}-\Phi_{12}M\Phi_{22}-\Phi_{13}M\Phi_{23}^T\big)\\
&-\tfrac{1}{2}\phi_0C_1^T\dot{M},\ H_{22}=C_2^T(C_1^T)^{-1}K_{d1}C_1^{-1}C_2\\
&+\tfrac{1}{2}C_2^T(C_1^T)^{-1}\big(\Phi_{11}^T\dot{M}\Phi_{11}+\Phi_{12}\dot{M}\Phi_{12}^T+\Phi_{13}\dot{M}\Phi_{13}^T\big)C_1^{-1}C_2\\
&+\tfrac{1}{2}\big(\phi_0C_2^TMC_1+\phi_0C_1^TMC_2-\Phi_{11}^TM\Phi_{12}-\Phi_{12}^TM\Phi_{11}\\
&-\Phi_{12}M\Phi_{22}-\Phi_{22}^TM\Phi_{12}^T-\Phi_{13}M\Phi_{23}^T-\Phi_{23}M\Phi_{13}^T\big)\\
&-\tfrac{1}{2}\big(\Phi_{12}^T\dot{M}\Phi_{12}+\Phi_{22}^T\dot{M}\Phi_{22}+\Phi_{23}\dot{M}\Phi_{23}^T\big)\\
&H_{23}=C_2^T(C_1^T)^{-1}K_{d1}C_1^{-1}+\tfrac{1}{2}C_2^T(C_1^T)^{-1}\big(\Phi_{11}^T\dot{M}\Phi_{11}\\
&+\Phi_{12}\dot{M}\Phi_{12}^T+\Phi_{13}\dot{M}\Phi_{13}^T\big)C_1^{-1}+\tfrac{1}{2}\big(C_2^TMC_2-\Phi_{12}^TM\Phi_{12}\\
&-\Phi_{22}^TM\Phi_{22}-\Phi_{23}M\Phi_{23}^T\big)-\tfrac{1}{2}\phi_0C_2^T\dot{M},\\
&H_{33}=(C_1^T)^{-1}K_{d1}C_1^{-1}+\tfrac{1}{2}(C_1^T)^{-1}\big(\Phi_{11}^T\dot{M}\Phi_{11}+\Phi_{12}\dot{M}\Phi_{12}^T\\
&+\Phi_{13}\dot{M}\Phi_{13}^T\big)C_1^{-1}-\tfrac{1}{2}\phi_0\dot{M}
\end{aligned}
$$

Now the next step is to choose K_d that makes H nonnegative. Since H is a partioned matrix, a proposition given in (Parlakci, et al., 2001) will be used to search for positive definiteness.

The following inqualities are set up to choose appropriate K_d's that make H positive definite,

$$H_{11}>0,\quad H_{22}-H_{12}^TH_{11}^{-1}H_{12}>0 \qquad (16)$$

$$
\begin{aligned}
&H_{33}-H_{13}^TH_{11}^{-1}H_{13}-\big(H_{23}^T-H_{13}^TH_{11}^{-1}H_{12}\big)\big(H_{22}\\
&-H_{12}^TH_{11}^{-1}H_{12}\big)^{-1}\big(H_{23}-H_{12}^TH_{11}^{-1}H_{13}\big)>0 \qquad (17)
\end{aligned}
$$

The inequalities in (16) and (17) are reduced to

$$\lambda_{\min}(K_{d1})>0\,,\ R_1-P_1^TK_{d1}^{-1}P_1>0 \qquad (18)$$

where P_1 and R_1 are as given in (13).

Establishing conditions such that $R_1>0$ is satisfied and $K_{d1}>0$ from the first condition,

$$R_1-P_1^TK_{d1}^{-1}P_1>\lambda_{\min}(R_1)-\lambda_{\max}(K_{d1}^{-1})\max_{\theta,\dot{\theta}}\|P_1\|_F^2 \qquad (19)$$

Therefore, the stability condition given in (16) is converted to

$$\lambda_{\min}(K_{d1})>\frac{\max_{\theta,\dot{\theta}}\|P_1\|_F^2}{\lambda_{\min}(R_1)} \qquad (20)$$

Finally, let us now consider the stability condition given in (17),

$$H_{33}-H_{13}^TH_{11}^{-1}H_{13}=R_2-P_2^TK_{d1}^{-1}P_2 \qquad (21)$$

where P_1 and R_1 are as given in (13).

Suggesting a change of variable results in as follows

$$H_{22}-H_{12}^TH_{11}^{-1}H_{12}=R_1-P_1^TK_{d1}^{-1}P_1=K_{d2} \qquad (22)$$

then assuming P_1 is nonsingular

$$
\begin{aligned}
H_{33}-H_{13}^TH_{11}^{-1}H_{13}&=R_2-P_2^T(P_1^T)^{-1}R_1P_1^{-1}P_2\\
&+P_2^T(P_1^T)^{-1}K_{d2}P_1^{-1}P_2 \qquad (23)
\end{aligned}
$$

Now, the inequality in (17) is reduced to

$$R_3-P_3^TK_{d2}^{-1}P_3>0 \qquad (24)$$

Similarly as in the second condition, establishing conditions such that $R_3>0$ is satisfied and since $K_{d2}>0$ from the second condition,

$$R_3-P_3^TK_{d2}^{-1}P_3>\lambda_{\min}(R_3)-\lambda_{\max}(K_{d2}^{-1})\max_{\theta,\dot{\theta}}\|P_3\| \qquad (25)$$

Hence, the last stability condition is obtained as follows

$$\lambda_{\min}(K_{d2})>\frac{\max_{\theta,\dot{\theta}}\|P_3\|_F^2}{\lambda_{\min}(R_3)} \qquad (26)$$

Note that each condition when satisfied, guarantees to satisfy the former, in other words it is only needed to satisfy the third condition without worrying about the first and second conditions.

Since there exist a nonlinear relationship between K_{d1} and K_{d2}, we can express the stability condition in terms of K_{d1},

$$\lambda_{\min}(K_{d2})>\lambda_{\min}(R_1)-\frac{\max_{\theta,\dot{\theta}}\|P_1\|_F^2}{\lambda_{\min}(K_{d1})}>\frac{\max_{\theta,\dot{\theta}}\|P_3\|_F^2}{\lambda_{\min}(R_3)} \qquad (27)$$

Assuming $\lambda_{\min}(R_1)>\dfrac{\max_{\theta,\dot{\theta}}\|P_3\|_F^2}{\lambda_{\min}(R_3)}$ holds, it can be implied that

$$
\begin{aligned}
&\lambda_{\min}(K_{d1})>\phi_0^2\lambda_{\min}^2(C_1)K_d-\tfrac{1}{2}\Big(\|\Phi_{11}\|_F^2+\|\Phi_{12}\|_F^2\\
&+\|\Phi_{13}\|_F^2\Big)\max_{\theta,\dot{\theta}}\|\dot{M}(\theta,\dot{\theta})\|_F>\frac{\max_{\theta,\dot{\theta}}\|P_1\|_F^2}{\lambda_{\min}(R_1)-\dfrac{\max_{\theta,\dot{\theta}}\|P_3\|_F^2}{\lambda_{\min}(R_3)}} \qquad (28)
\end{aligned}
$$

In view of (32) if conditions in (13) are satisfied, then (19) reduces to

$$\dot{V}(\tilde{\theta},\tilde{\omega})\le-\begin{bmatrix}\tilde{\theta}\\ \tilde{\omega}\end{bmatrix}^T H\begin{bmatrix}\tilde{\theta}\\ \tilde{\omega}\end{bmatrix}<0 \qquad (29)$$

Hence Theorem1 is proved.

6. SIMULATION RESULTS

For simulation purposes the problem of controlling a two-link SCARA type manipulator shown in Fig.1 can be considered. Denoting $\psi = [\psi_1 \quad \psi_2]$ where $\psi_1 = \int \theta_1 dt$, $\psi_2 = \int \theta_2 dt$, $\theta = [\theta_1 \quad \theta_2]$ and $u = [u_1 \quad u_2]$ the matrices, $M(\theta)$, $B(\theta,\dot{\theta})$ and vector $f(\dot{\theta})$ can be defined as below,

$$M(\theta) = \begin{bmatrix} p_1 + 2p_3\cos(\theta_2) & p_2 + p_3\cos(\theta_2) \\ p_2 + p_3\cos(\theta_2) & p_2 \end{bmatrix}$$

$$B(\theta,\dot{\theta}) = \begin{bmatrix} -\dot{\theta}_2 p_3 \sin(\theta_2) & -(\dot{\theta}_1 + \dot{\theta}_2) p_3 \sin(\theta_2) \\ \dot{\theta}_1 p_3 \sin(\theta_2) & 0 \end{bmatrix}$$

$$f(\dot{\theta}) = \begin{bmatrix} K_1\dot{\theta}_1 + K_2\,\mathrm{sgn}(\dot{\theta}_1) \\ K_1\dot{\theta}_2 + K_2\,\mathrm{sgn}(\dot{\theta}_2) \end{bmatrix}$$

where the inertial parameters are set as p_1, p_2, p_3. For a numerical example, we the mass, inertia and friction parameters are selected appropriately to yield $p_1 = 3.2, p_2 = 0.11, p_3 = 0.17, K_1 = 1, K_2 = 10, c = 1s^{-1}$ The reference trajectory for the position control problem is chosen as, $\theta_1(t) = \theta_d, \theta_2(t) = 0, t > 0$ where $\theta_d = 0.7199\,\mathrm{rad}$. Moreover, the initial state of the manipulator is set to be as $\theta_1(0) = 0.0005\,\mathrm{rad}$, $\theta_2(0) = 0.05\,\mathrm{rad}$, $\dot{\theta}_1(0) = 0\,\mathrm{rad\,s^{-1}}$ and $\dot{\theta}_2(0) = 0\,\mathrm{rad\,s^{-1}}$. The required min/max values of the inertia matrice are computed as follows: $\min_{\theta} \lambda(M(\theta)) = 0.0905$, $\max_{\theta} \|M(\theta)\|_F = 3.5229$, and $\max_{\theta,\dot{\theta}} \|\dot{M}(\theta,\dot{\theta})\|_F = 0.0655 \max(\dot{\theta}_2)^2$.

Fig. 2 shows the position and velocity errors which perfectly converge to zero. Furthermore, the rate of convergence is good which is an important issue in some applications. For a comparison it can be said that the convergence rates of the system trajectories to the switching surfaces are less than the stabilization rates of the sliding manifolds. Fig. 3 represents the switching surfaces which converge perfectly and smoothly as well. Finally looking at Fig. 4 which represents the applied control inputs, it is observed that the control tends to go to zero after the sliding mode occurs and the system becomes stable. There exists a chattering in the applied control. If required, saturation function can be used in the control law instead of pure signum function. The simulations given in Fig. 5,6 have shown that by utilizing a saturation function the performance continues to be good.

8. CONCLUSION

A new variable structure PID control law is successfully designed for the stabilization of robot manipulators with parameter perturbations by using the Lyapunov full quadratic form function. The compact form of the sliding and stability results are elaborately derived by using LMI's techniques and Rayleigh's principle of min/max eigenvalues. By this way, the robustness and stability of the control law is ensured in large. The proposed design procedure is applied to the control of a two-link robot arm and simulations are performed. The results of the simulations clearly indicate that the practical results confirm the theoretical conclusions.

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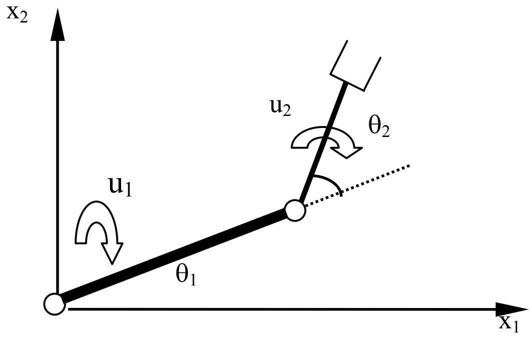

Fig. 1. Joint positions of the manipulator

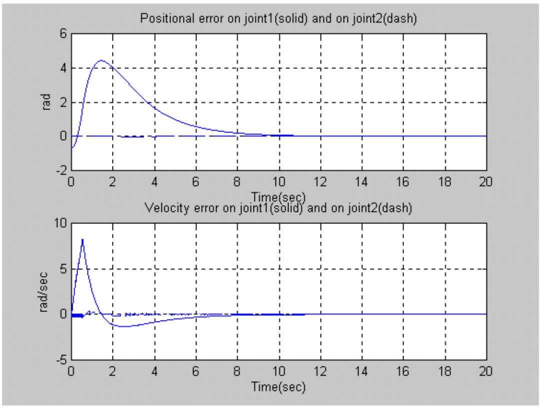

Figure 2. Positional and velocity errors on the joints

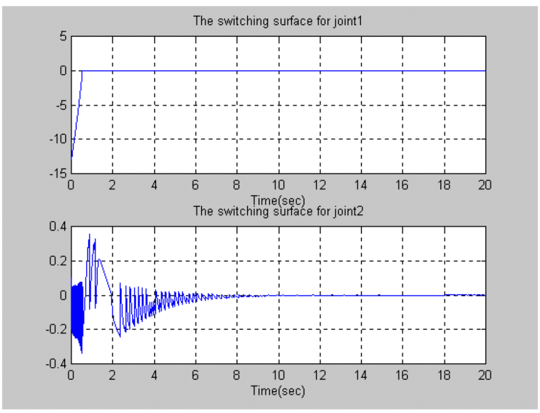

Figure 3. The switching surfaces and applied control inputs for the joints

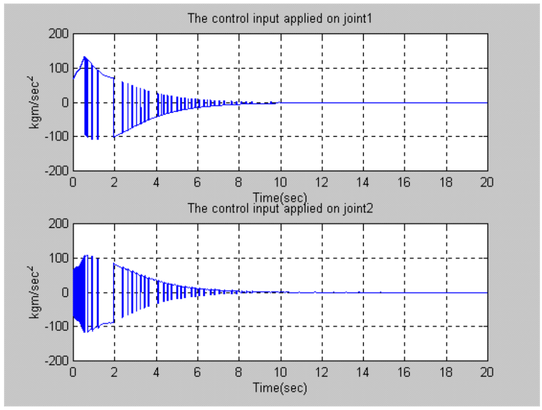

Figure 4. Control inputs applied on the joints

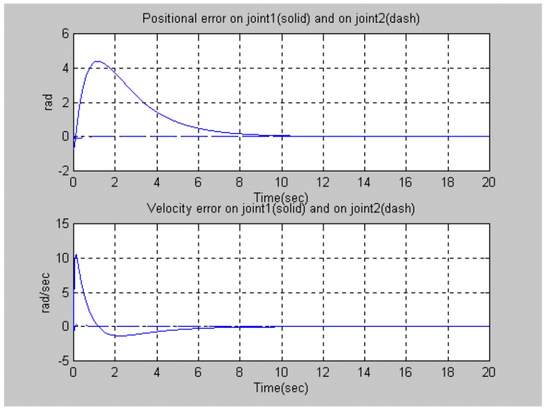

Figure 5. Positional and velocity errors on the joints with saturation in the control law

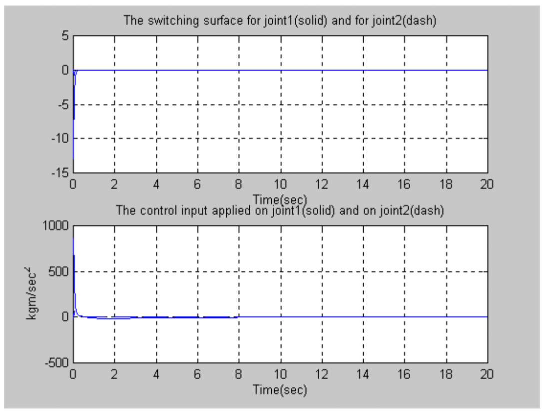

Figure 6. Switching surfaces and applied control inputs for the joints with saturation in the control law