15th Triennial World Congress, Barcelona, Spain

DEVELOPMENT OF HYBRID TIME PETRI NETS FOR SCHEDULING AND CONTROL OF MIXED BATCH/CONTINUOUS PROCESSES

Tianlong Gu, Parisa A. Bahri*, Peter Lee

School of Eng., Murdoch University (Rockingham Campus), Murdoch, WA 6150, Australia
*(*to whom all correspondence should be addressed, parisa@eng.murdoch.edu.au)*

Abstract: The scheduling of mixed batch/continuous process plants is difficult since the model and solution have to deal with both discrete and continuous operations. Motivated by the need for the effective scheduling and control of operations in sugar milling, a new type of hybrid Petri nets called Hybrid Time Petri Nets(HTPN) is proposed, which allows the formulation of variable cycle time and variable flow rate in a mixed batch/continuous process. Two smaller sugar milling problems are formulated and analysed by this HTPNs. *Copyright © 2002 IFAC*

Key words: Petri nets; batch modes; discrete event systems; hybrid systems; scheduling

1. INTRODUCTION

Mixed batch/continuous processes are one of the important processing modes in the process industries. The behaviours of this kind of process can no longer be considered as either continuous or discrete, and pose therefore a difficult problem with regards to formulation and solution algorithm. The sugar milling system is a typical system. The milling plant consists of a batch pan system, followed by a communal, limited storage facility before the remainder of the downstream process, which is generally considered to operate continuously. Furthermore, although the batch units have a fixed quantity to process, each batch may take a variable length of time to process, which is referred to as having variable cycle time. Constraints are specified by the limits on the cycle time of the pans, the capacity of the storage facility, as well as possible flow rate ranges through the continuous units. Variable cycle time and changeable flow rate make the formulation and solution algorithm extremely difficult (Nott, 1998).

Petri nets, as a graphical and mathematical tool, provides a powerful and uniform environment for modelling, analysis, and design of discrete event systems (Murata, 1989). One of the major advantages of using Petri nets is that the same methodology can be used for the modelling, qualitative and quantitative analysis, supervisory and coordination control, planning and scheduling, and hybrid system design in the life cycle of batch processes. Recently, Timed Petri Net-based approach has been examined to formulate complicated operations and to solve scheduling problems for multipurpose/multiproduct batch plants (Gu, and Bahri, 1999; Bahri, *et al*., 2000a). This approach has also been extended to deal with the scheduling problems of batch plants with variable batch sizes (Bahri, *et al*., 2000b). With the introduction of continuous places and transitions, continuous Petri net and hybrid Petri net were defined in David and Alla (1994). The continuous places can contain tokens of a positive real number, and continuous transitions can be continuously fired at some speed. Hence, the continuous part (continuous places and transitions) can model systems with continuous flows and the discrete part (discrete places and transitions) models the logic functions. A similar approach, hybrid flow net, was presented in Flaus and Ollganon (1997). This modelling tool presents a continuous flow net interacting with a Petri net according to a control interaction. Unfortunately, timed Petri net can not deal with both continuous flow and timed discrete operations of variable cycle time, and continuous Petri net, hybrid Petri net and hybrid flow net can not handle timed discrete operations and variable continuous flow rate.

The aim of this paper is to develop a new tool to formulate and analyse a complicated mixed batch/continuous process graphically and efficiently. This paper is organized as follows. In section 2, hybrid time Petri net (HTPN) is defined, including the enabling and firing rules. HTPN based formulation and behaviour analysis algorithms for mixed batch/continuous processes are presented in section 3. Next, two case studies of sugar milling systems are examined, and results are given in section 4. Finally, the paper concludes in section 5.

2. HYBRID TIME PETRI NETS

A hybrid time Petri net (HTPN) is a new type of hybrid Petri net, and also a particular kind of bipartite directed graphs with three types of objects: places, transitions, and directed arcs connecting places to transitions and transitions to places. The places and transitions have either discrete or continuous characteristics, and discrete places are stamped with time intervals, and continuous transitions with flow rate ranges. One discrete place with a time stamp is a time place, and one continuous transition with a flow rate range is a time transition.

Definition 1: A hybrid time Petri Nets (HTPN) is a Petri net HTPN= ($Pc \cup Pd$, $Tc \cup Td$, F, W, St, Sf) where:

1) Pd is a finite set of discrete places, and Pc a finite set of continuous places with $Pc \cap Pd = \varnothing$ and $P = Pc \cup Pd$; and
2) Td is a finite set of discrete transitions, and Tc a finite set of continuous transitions with $Tc \cap Td = \varnothing$ and $T = Tc \cup Td$; and
3) $F \subseteq (P \times T) \cup (T \times P)$ is a relation function that defines arcs between places and transitions; and
4) W: $(P \times Td) \cup (Td \times P) \rightarrow Z^+$ (positive integer) is a weight function; and
5) St: $Pd \rightarrow (R^+ \cup \{0\}) \times (R^+ \cup \{0\})$ is a time stamp function with the default $[0, \infty]$; and
6) Sf: $Tc \rightarrow (R^+ \cup \{0\}) \times (R^+ \cup \{0\})$ is a stamp function of flow rate ranges.

A place (transition) is called an input place (transition) to a transition (place) if there exists a directed arc connecting this place (transition) to the transition (place). A place (transition) is called an output place (transition) to a transition (place) if there exists a directed arc connecting the transition (place) to this place (transition). All the input (output) places (transitions) of transition *t* (place *p*) is referred to as input (output) place (transition) set denoted by ${}^{\bullet}t$, $t^{\bullet}$ (${}^{\bullet}p$, $p^{\bullet}$). Then, the relation function F can also be defined by ${}^{\bullet}t$ and $t^{\bullet}$ or by ${}^{\bullet}p$ and $p^{\bullet}$. Each place may potentially hold tokens of zero or a positive integer number or a positive real number. A hybrid time Petri net containing tokens is called as a marked hybrid time Petri net.

Definition 2: A marked hybrid time Petri net is defined by (HTPN, m_0), where HTPN is a hybrid time Petri net, and m_0 an initial marking satisfying the marking function m: $Pd \rightarrow (Z^+ \cup \{0\})$, and m: $Pc \rightarrow (R^+ \cup \{0\})$.

Pictorially, both discrete places and continuous places are depicted by circles, and discrete transitions by bars and continuous transitions by boxes. For discrete places, tokens are either pictured by small solid dots or marked by nonnegative integers, and continuous places only marked by nonnegative real numbers. The arc weight, time stamp and the flow rate stamp are written by the side of its corresponding object respectively (see Fig. 1).

In a marked HTPN, transitions may be enabled and fired. When a token arrives at a time place, it is to be reserved by the place, and to be available between the time stamp [l_1, l_2] of this place. One discrete transition is enabled only if there are enough available tokens in all of its input places, and one continuous transition is enabled if there are non-zero tokens in all the input places. The firing of one enabled discrete transition removes available tokens of its input places, and deposits non-reserved tokens to its output places. The firing of a continuous transition removes and deposits tokens continuously. The firing of either discrete or continuous transition renders the tokens being redistributed, and results in a new marking. The evolution of a marked THPN is governed by the following enabling and firing rules.

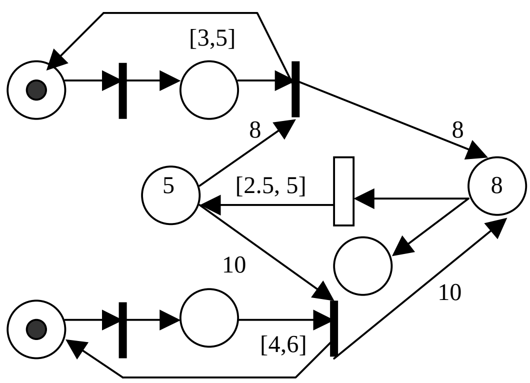


Fig. 1. HTPN model for a mixed batch/ continuous process.

Definition 3: (*Enabling rule*) A transition t is enabled if the tokens of each place p in $^{\bullet}t$ satisfy the conditions:

1) if $t \in$ Tc, there are enough available tokens so that $m(p) \geq W(p, t)$ for $\forall p \in (Pd \cap {}^{\bullet}t)$, and $m(p) > 0$ for $\forall p \in (Pc \cap {}^{\bullet}t)$; or
2) if $t \in$ Td, there are enough available tokens so that $m(p) \geq W(p, t)$ for $\forall p \in {}^{\bullet}t$.

Definition 4: (*Firing rule*) The firing of an enabled transition t removes a certain amount of tokens from each input place p in $^{\bullet}t$, and adds another certain amount of tokens to each output place p in $t^{\bullet}$, where :

1) if $t \in$ Tc, $m'(p) = m(p) - W(p, t)$ for $\forall p \in ({}^{\bullet}t - t^{\bullet}) \cap Pd$, $m'(p) = m(p) + W(t, p)$ for $\forall p \in (t^{\bullet} - {}^{\bullet}t) \cap Pd$; $\forall p \in ({}^{\bullet}t - t^{\bullet})$, $m'(p) = m(p) - \phi\,\Delta$ for $\forall p \in ({}^{\bullet}t - t^{\bullet}) \cap Pc$, and $m'(p) = m(p) + \phi\,\Delta$ for $\forall p \in (t^{\bullet} - {}^{\bullet}t)\ ^{\bullet}) \cap Pc$, where ϕ is the instantaneous flow rate, and Δ a time difference; or
2) if $t \in$ Td, $\forall p \in ({}^{\bullet}t - t^{\bullet})$, $m'(p) = m(p) - W(p, t)$; $\forall p \in (t^{\bullet} - {}^{\bullet}t)$, $m'(p) = m(p) + W(t, p)$.

3. HTPN BASED FORMULATION AND ANALYSIS OF MIXED BATCH/CONTINUOUS PROCESSES

A sugar milling system is a mixed batch/continuous process plant. It consists of a batch pan system, followed by a communal, limited storage facility before the remainder of the downstream process, which is generally considered to operate continuously. Furthermore, although the batch units have a fixed quantity to process, each batch may take a variable length of time to process, which is referred to as having variable cycle time (Nott, 1998). Constraints of the physical system are specified by bounds on the cycle time of the pans, the limited storage facility as well as possible flow rates through the continuous units. A smaller system having the important characteristics of the sugar milling process is shown in Fig. 2, where the storage tank receives discrete inputs from two batch units, and sends continuous flow to the continuous process.

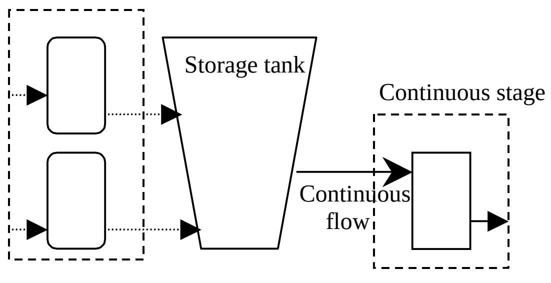


Fig. 2. Simple system of mixed batch/continuous process

3.1 Formulation via HTPN

Essentially, the mixed batch/continuous process consists of three parts: a discrete batch stage, an intermediate storage facility and a continuous stage. The key elements are batch operations with variable cycle time and storage tank with discrete batch and continuos variable flow rate. The global THPNs model can be developed through formulating each part and integrating them using algorithm 1.

Definition 5: Batch operation with variable cycle time $[l_1, l_2]$ is defined by: $P_d = \{p_r, p\}$, $T_d = \{t_s, t_f\}$, $^{\bullet}(p) = \{t_s\}$, $(p)^{\bullet} = \{t_f\}$, $(p_r)^{\bullet} = \{t_s\}$, and $St(p) = [l_1, l_2]$.

Definition 6: Storage tank with discrete batch feed from units $j \in J$, continuous output of variable flow rate $[f_1, f_2]$, storage restriction $[S_1, S_2]$ and initial storage amount S_0 is defined by: $P_c = \{p_{c1}, p_{c2}\}$, $T_c = \{t_c\}$, $T_d = \{t_j \mid j \in J\}$, $^{\bullet}(p_{c1}) = \{t_j \mid j \in J\}$, $(p_{c1})^{\bullet} = \{t_c\}$, $^{\bullet}(p_{c2}) = \{t_c\}$, $(p_{c2})^{\bullet} = \{t_j \mid j \in J\}$, $Sf(t_c) = [f_1, f_2]$, $m_0(p_{c1}) = (S_0 - S_1)$, $m_0(p_{c2}) = (S_2 - S_1)$, and $W(t_j, p_{c1}) = W(p_{c2}, t_j) = B_j$ ($j \in J$), where B_j is the batch size of unit j.

Algorithm 1: (Constructing THPNs model for mixed batch/continuous process)

s1: For the batch operation i at unit j, introduce discrete transition t_{sij} and t_{fij} for the events of starting and finishing this operation respectively, discrete place p_{rij} for the availability of its feed, and discrete time place p_{ij} with time stamp $[l_1, l_2]$ and initial marking $m_0(p_{ij})=0$ for the processing activity, where time stamp $[l_1, l_2]$ is the corresponding variable cycle time;

s2: Connect discrete transition t_{sij}, t_{fij} , discrete place p_{rij} and discrete time place p_{ij} through $^{\bullet}(p_{ij}) = \{t_{sij}\}$, $(p_{ij})^{\bullet} = \{t_{fij}\}$ and $(p_{rij})^{\bullet} = \{t_{sij}\}$;

s3: For unit j, introduce discrete place p_j with initial marking $m_0(p_j)= 1$. For each operation i at unit j, create the links $^{\bullet}(t_{sij}) = \{p_j\}$ and $(t_{fij})^{\bullet} = \{p_j\}$;

s4: In terms of the product recipe, add the link $(t_{fhk})^{\bullet} = \{p_{rij}\}$ if batch operation i at unit j receives its feed from batch operation h at unit k;

s5: Introduce continuous places p_{c1}, p_{c2} and p_{c3} with $m_0(p_{c1}) = (S_0 - S_1)$ and $m_0(p_{c2}) = (S_2 - S_1)$, and continuous time transition with $Sf(t_c) = [f_1, f_2]$, where $[f_1, f_2]$, $[S_1, S_2]$ and S_0 are the variable flow rate, storage restriction and initial storage amount respectively;

s6: Create links between continuous places p_{c1}, p_{c2} and continuous time transition t_c, with $(p_{c1})^{\bullet} = \{t_c\}$, $^{\bullet}(p_{c2}) = \{t_c\}$and $^{\bullet}(p_{c3}) = \{t_c\}$, and arcs between discrete transition t_{fij} and continuous places p_{c1}, p_{c2} with $W(t_{fij}, p_{c1}) = W(p_{c2}, t_{fij}) = B_j$ for each batch operation i at unit j, whose output is the feed of storage tank, where B_j is the batch size of unit j.

THPNs model for the above example can be created by algorithm 1, as depicted in Fig. 1.

3.2 Behaviour Analysis Algorithm

Once a mixed batch/continuous process is formulated by THPNs, its dynamical behaviour evolution is totally described by markings and their related parameters of this model. Due to continuous firing flow and time stamp, there are actually infinite reachable markings in a THPNs. For scheduling purposes, this renders it impossible to use the reachablility tree technique as in traditional Petri nets. Therefore, the concept of regional states is proposed. In this framework, the behaviour analysis algorithm is defined as follows.

Definition 7: (*Regional state*) A regional state is defined by rs = (m^d, m^c, Φ, [τ_1, τ_2]), where m^d is a partial marking vector of discrete places, m^c a partial marking vector of continuous places at time τ_2, Φ a flow rate vector of continuous transitions and [τ_1, τ_2] a time interval. They satisfy: during [τ_1, τ_2], the marking of discrete place m^d remains unchanged; and during [τ_1, τ_2], the flow rates of continuous transitions Φ stay constant.

Algorithm 2: (Behaviour analysis algorithm for mixed batch/continuous process)

s1: Input the parameters of HTPNs model and time horizon TH, initialize rs = (0, 0, 0, [0, 0]), and put rs into LIST;

s2: retrieve the regional state rs = (m^d, m^c, ϕ, [τ_1, τ_2]) from LIST, let rs as the current state, and put rs into LIST0;

s3: if τ_2 < TH, then for $\forall t \in ({}^\bullet t_c)^\bullet \cap$ Td, calculate the possible earliest firing time of discrete transition t in view of all its input discrete place denoted by $\tau_m(t)$; else Stop;

s4: choose discrete transition t_e satisfying $\tau_m(t_e) = \min \{\tau_m(t) \mid t \in ({}^\bullet t_c)^\bullet \cap \text{Td}\}$;

s5: for the shared continuous place p_s of transitions $\forall t \in ({}^\bullet t_c)^\bullet \cap$ Td, compute $\Delta 1 = \tau_m(t_e) - \tau_2$ **;** $\Delta 2 = W(p_s, t_e) - m^c(p_s)$; and $\Delta 3 = \Delta 1 * f_2$, where [f_1, f_2] is the flow range stamp of continuous transition t_c;

s6: if $\Delta 2 \geq \Delta 3$, then calculate $\Delta = \Delta 2 / f_2$ and update rs by $\tau_1 := \tau_2$; $\tau_2 := \tau_1 + \Delta$; $\phi = f_2$; for $\forall p \in {}^\bullet t_e \cap$ Pd, $m^d(p) := m^d(p) - 1$; $m^c(p_s) := m^c(p_s) - \Delta 2$; $m^c({}^\bullet t_c) := m^c({}^\bullet t_c) + \Delta 2$; put rs = (m^d, m^c, ϕ, [τ_1, τ_2]) into LIST; Goto **s2**;

s7: update rs by $\phi = \min(m^c({}^\bullet t_c)/\Delta 1, f_2)$; $\tau_1 := \tau_2$; $\tau_2 := \tau_1 + \Delta 1$; for $\forall p \in {}^\bullet t_e \cap$ Pd, $m^d(p) := m^d(p) - 1$; $m^c(p_s) := m^c(p_s) + \phi * \Delta 1 - W(p_s, t_e)$; $m^c({}^\bullet t_c) := m^c({}^\bullet t_c) - \phi * \Delta 1 + W(p_s, t_e)$; put rs = (m^d, m^c, ϕ, [τ_1, τ_2]) into LIST; Goto **s2**.

4. CASE STUDY

Two case studies are presented in this section to show the applicability of the proposed approach in scheduling of mixed batch/continuous systems.

Case Study 1: The example in Fig. 2 is revisited. The related parameters are: batch size of unit 1 = 8, batch size of unit 2 = 10, cycle time for unit 1 = [3, 5], cycle time for unit 2 = [4, 6], storage range = [2, 15], initial storage amount = 10, flow rate limit = [2.5, 5.0] and time horizon = 25. From algorithm 2, a feasible schedule is solved. The Gannt chart for batch operation stage is shown in Fig.3(a), and the flow rate graph for the continuous stage and storage level changes are given in Fig.3 (b) and (c) respectively.

Case study 2 : A similar system is also tested, where three batch units are included. The plant data are: batch size of unit 1 = 10, batch size of unit 2 = 10, batch size of unit 3 = 10, cycle time for unit 1 = [3, 5], cycle time for unit 2 = [4, 6], cycle time for unit 3 = [4, 6], storage range = [2, 20], initial storage amount = 10, flow rate limit = [2.5, 10.0] and time horizon = 25. The HTPNs model is shown in Fig. 4. The Gannt chart for batch operation stage, flow rate graph for the continuous stage and storage level changes are presented in Fig.5 (a), (b) and (c) respectively.

5. CONCLUSIONS

Hybrid time Petri nets is a new modelling and analysis tool for systems with both discrete and continuous behaviours. This is motivated by the need for effective operations of a sugar milling process. The new type of hybrid Petri nets, HTPNs, is defined through introducing a time duration stamp with a discrete place and a flow range stamp with continuous transition, and henceforth allows the formulation of variable cycle time and variable flow rate in mixed batch/continuous processes. In order to simulate practical systems, the concept of regional states is proposed and a regional state based behaviour analysis algorithm is developed. Through the analysis of HTPNs model, the feasible schedule for mixed batch/continuous process can be found efficiently.

The HTPNs have great potential for handling complicated operations in mixed batch/ continuous processes. It is expected that other related problems of both discrete and continuous characteristics such as supervisory control might be solved easily in the framework of HTPNs. Further work is under way to cover both fundamental aspects and applications. The fundamental issues regarding HTPNs are structure properties, state evolution equation and efficient analysis technique. The application problems include the incorporation of different complicated storage policies into the formulation, addressing the algorithms for optimal scheduling and implementing the integration of scheduling and control.

REFERENCE

Bahri, P. A., T. Gu, and P. L. Lee. (2000a). Petri-Net Based Modelling and Scheduling of Batch Processing Plants—A Heuristic Algorithm . In: *International Symposium on Advanced Control of*

Chemical Processes, June 14-16, 2000, Pisa, Italy, 467-472.

Bahri, P. A., T. Gu, and P. L. Lee. (2000b), A Novel Approach for Batch Processes Scheduling with Variable Batch Sizes. *CHEMECA 2000*, June 9-12, 2000, Perth, Australia.

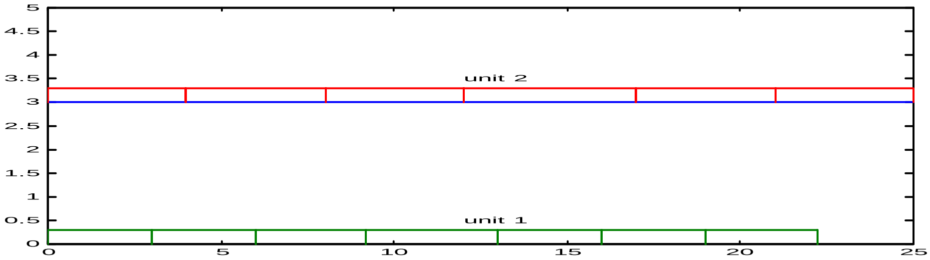
David, R. and H. Alla. (1994), Petri Nets for Modeling of Dynamic Systems: A survey". *Automatica*, **30**, 175-20.

Flaus, J.-M. and G. Ollagnon. (1997). Hybrid flow nets for hybrid processes modeling and control. In: *Proc. of the Int. Workshop on Hybrid and Real-Time Systems*, Grenoble, France, March,213-217.

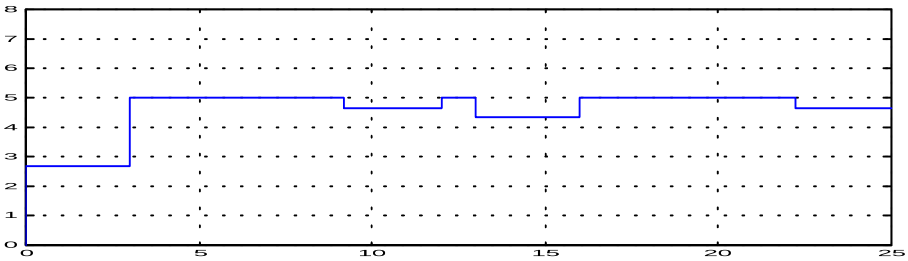
Gu, T. and P. A. Bahri. (1999). Timed Petri-net representation for short term scheduling of multi-product batch plants. In: *Proceedings of the 1999 American Control Conference*, San Diego, USA, 4092-4096.

Murata, T. (1989). Petri nets: properties, analysis and applications. *Proceedings of the IEEE*, **77**, 541-580.

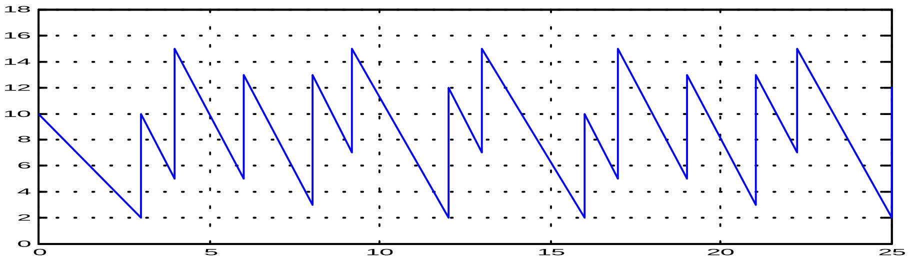
Nott, H. P. (1998). *Modelling Aalternatives for schedling mixed batch/continuous process plants with variable cycle time*. Ph D Thesis, Murdoch University,Australia.



(a) Gannt chart for batch operations



(b) Flow rate of continuous stage



(c) Storage capacity

Fig. 3. Simulation analysis results of case study 1

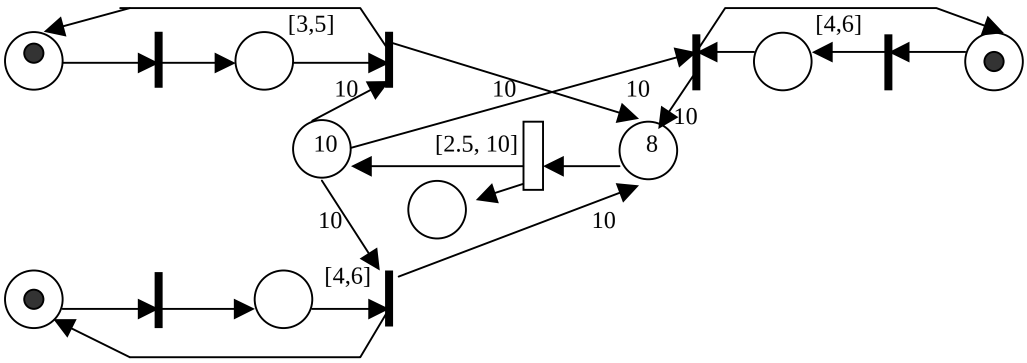
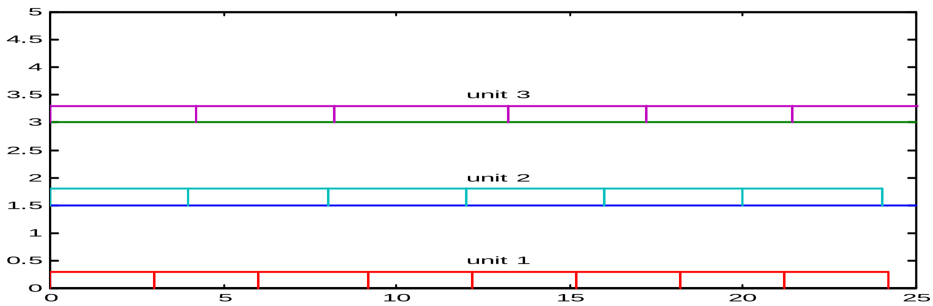
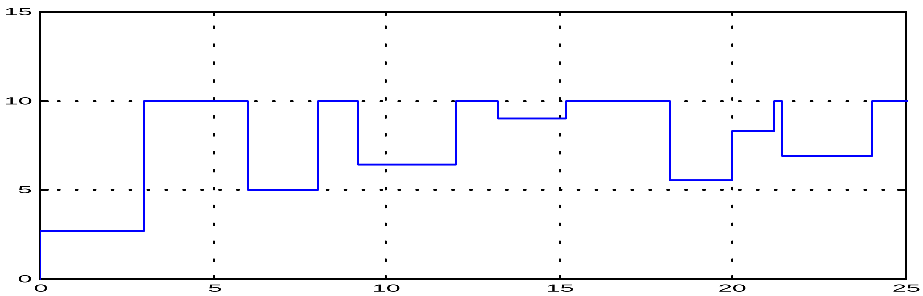


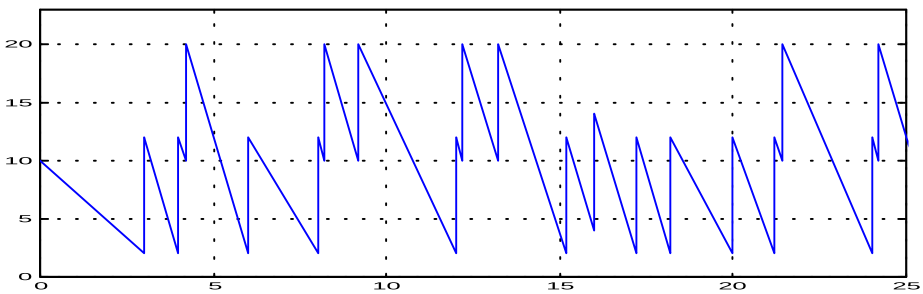
Fig. 4. HTPN model for mxed batch/continuous process with three batch units



(a) Gannt chart for batch operations



(b) Flowrate of continuous stage



(c) Storage capacity

Fig. 5. Simulation analysis results of case study 2