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FUNDAMENTAL LIMITATIONS ON THE STEP RESPONSE OF SISO FEEDBACK SYSTEMS

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Abstract: This paper proposes new fundamental limitations on the step response of the unity-feedback system based on its transfer function. Using the definition of Laplace transform, it is shown that time-domain integral equalities have to be satisfied by the step response of SISO(single-input, single-output) stable system, and shown that the maximum magnitude of its step response has some lower bounds with respect to the achievable settling time of the systems. It is also shown that the closed-loop system with open-loop RHP(right half plane) poles and/or zeros has some limitations in the unity-feedback control scheme. These results can be used to check in advance whether desired design specifications are achievable or not. *Copyright © 2002 IFAC*

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1. INTRODUCTION

There are always fundamental limitations involved with any feedback control system. Many works have been done to clarify these limitations imposed by the inherent characteristics of the physical system. Most of them are formulated in the frequency domain for linear SISO(single-input, single-output) systems (Freudenberg and Looze, 1985; Horowitz and Liao, 1984; Middleton, 1991), and are extended to MIMO(multi-input, multi-output) systems (Chen, 2000) as well as nonlinear systems (Seron *et al.*, 1999). Time-domain limitations which are formulated by the time-domain integral equalities based on Laplace transform have been also developed (Goodwin *et al.*, 1999; Middleton, 1991; Kwon *et al.*, 2001; Kwon, 2002). As a result, it has been realized that nonminimum phase systems, compared with minimum phase systems, have more various fundamental limitations associated with the achievable closed-loop transfer function, closed-loop gain margin, loop transfer recovery, sensitivity or complementary sensitivity function, disturbance rejection, etc. (Qiu and Davison, 1993).

The fundamental limitations in linear filtering designs are also investigated as the counterparts to control theories (Goodwin *et al.*, 1995). A recent comprehensive survey of inherent design limitations is found in Freudenberg *et al.* (2000). The theory of fundamental design limitations provides the basic scientific background in the field of feedback controls for all control engineers who need the basic results from this area. These limitations may be used to check in advance whether desired design specifications are achievable or not.

In many cases, the fundamental limitations on the achievable performances are to cause trade-off relations between design specifications. Only for a few very special cases are analytic methods known to find the exact form of the trade-offs, and these trade-offs can be mostly computed by numerical convex problem methods (Barratt and Boyd, 1989). It has been studied that the limitations of an achievable performance with a linear

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controller can be computed numerically, where the performance reflects many practical constraints and qualities including a fast response to commands without excessive undershoot and overshoot phenomena, small and quick reactions to disturbances or noises, a lower actuator authority, and certain measures of robustness or insensitivity to unknown or unmodeled plant dynamics. (Boyd *et al.*, 1988).

One of the limitations is the step response extrema such as undershoot and overshoot phenomena. For a stable system with real poles and zeros, it is well-known that the step response extrema are perfectly characterized by the number and location of poles and zeros (Jayasuriya and Song, 1996; El-Khoury *et al.*, 1993; M$^{\underline{c}}$Williams and Sain, 1989). As a result, RHP(right half plane) real zeros lead to undershoot and LHP(left half plane) real zeros located right side of the dominant pole lead to overshoot in the step response of the system (León de la Barra, 1994; Kobayashi, 1993; M$^{\underline{c}}$Williams and Sain, 1989). In the unity-feedback control method, it has been shown that the plant with unstable real poles has overshoot in its closed-loop step response (Middleton, 1991). However, these results related with undershoot and overshoot has been studied for the systems with only real poles and zeros.

In this paper, the effects of open-loop unstable poles and RHP zeros in the unity-feedback system are investigated. It has derived new time-domain integral equalities on the step response of the closed-loop system, and it is shown that SISO stable systems have some lower bounds between the maximum magnitude of its step response and the achievable settling time. Moreover, it is shown that open-loop unstable real poles necessarily imply overshoot and RHP zeros necessarily imply undershoot in its step response.

The layout of this paper is organized as follows: In Section 2, we present the unity-feedback control system structure, and define some notations used in this paper. In Section 3, it is shown that there are new integral equalities on the time-domain representation of the step response. Based on these equalities, the fundamental limitations on the step response of unity-feedback system are formulated in Section 4. The concluding remarks are given in Section 5.

2. PRELIMINARIES

Let us consider the unity-feedback control system as shown in Fig. 1. It is the most commonly used system configuration with the controller placed in series with the controlled plant (Kuo, 1995). In Fig. 1, the symbols have the following meaning:

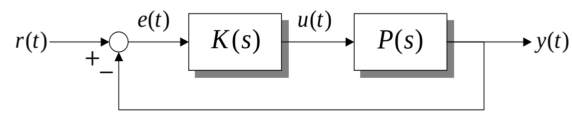

Fig. 1. The unity-feedback system.

$P(s)$: plant transfer function,
$K(s)$: controller transfer function,
$r(t)$: reference input,
$e(t)$: error signal,
$u(t)$: controller output or plant input,
$y(t)$: plant output.

Also, let us define the complementary sensitivity function by

$$T(s) \triangleq \frac{K(s)P(s)}{1+K(s)P(s)}, \tag{1}$$

which is also the closed-loop transfer function from the reference input $r(t)$ to the plant output $y(t)$. Hence, the output $Y(s)$, which is the Laplace transform of $y(t)$, can be written by

$$Y(s) = T(s)R(s), \tag{2}$$

where $R(s)$ is the Laplace transform of $r(t)$. Moreover, if p is an open-loop pole of the unity-feedback system, the closed-loop transfer function $T(s)$ always satisfies

$$T(p) = 1, \tag{3}$$

and equivalently, the unit step response yields

$$Y(p) = \frac{1}{p}. \tag{4}$$

Without loss of generality, we can restrict $T(s)$ on satisfying Assumption 1:

Assumption 1. Assume that $T(s)$ satisfy the conditions as follows:

(1) All poles of $T(s)$ have the real part less than $\gamma < 0$.
(2) $T(s)$ is relaxed at time 0.
(3) The DC gain of $T(s)$ is normalized by 1.
(4) There is no pole-zero cancellation.

Also, we will investigate some conditions such that the step response has the overshoot and undershoot in the sense of Definitions 1 and 2, respectively.

Definition 1. The step response is said to have overshoot if there is an open interval (a, b) such that

$$y(t) > 1, \ ^{\forall}t \in (a, b), \tag{5}$$

where $y(t)$ is the step response of $T(s)$.

Definition 2. The step response is said to have undershoot if there is an open interval (c, d) such that

$$y(t) < 0, \ ^{\forall}t \in (c, d), \tag{6}$$

where $y(t)$ is the step response of $T(s)$.

Note that Definition 2 includes Type A undershoot, *i.e.*, the initial undershoot, as well as Type B undershoot (Mita and Yoshida, 1981).

3. TIME-DOMAIN INTEGRAL EQUALITIES

This section is to investigate some equalities on the time-domain representation of the step response based on its Laplace transform. We will consider a closed-loop transfer function $T(s)$ as in Fig. 1, which satisfies Assumption 1.

Lemma 1 states new integral equalities related to the unit step response of $T(s)$.

Lemma 1. For $\mathrm{Re}\,[\chi] > 0$, the Laplace transform pair, $y(t)$ and $T(s)/s$, meets integral equalities as follows:

$$\int_0^\infty e^{-\sigma t}\cos(\omega t)y(t)dt = \mathrm{Re}\left[\frac{T(\chi)}{\chi}\right], \tag{7}$$

and

$$\int_0^\infty e^{-\sigma t}\sin(\omega t)y(t)dt = -\mathrm{Im}\left[\frac{T(\chi)}{\chi}\right], \tag{8}$$

where $\chi = \sigma + j\omega$.

Proof. See in Kwon *et al.* (2002). □

Equations (7) and (8) in Lemma 1 have to be satisfied by the unit step response $y(t)$ for all complex value χ with a positive real part. In the sequel, we will mainly use the Eq. (7) in Lemma 1 since Eq. (8) is trivial when $\omega = 0$. However, Eq. (8) will imply that it has the undershoot or overshoot to the system satisfying some conditions in its step response.

For a system with RHP zeros, let us take the value of χ for those zeros in Lemma 1, then another integral equality on the relationship between RHP zeros and the step response of the system are formulated as follows:

Lemma 2. Let $T(s)$ have r_1 RHP real zeros at $s = z_i$ for $i = 1, 2, \cdots, r_1$ and $2r_2$ RHP complex conjugate zeros at $s = a_k \pm jb_k$ for $k = 1, 2, \cdots, r_2$. Then $y(t)$ has to satisfy

$$\int_0^\infty \left[E_r(t) + E_c(t)\Gamma_c(t)\right] y(t)dt = 0, \tag{9}$$

where $E_r(t)$ is a linear combination of $e^{-z_i t}$, $E_c(t)$ is a linear combination of $e^{-a_k t}$, and $\Gamma_c(t)$ is a linear combination of $\cos(b_k t)$ and/or $\sin(b_k t)$.

Proof. See in Kwon *et al.* (2002). □

Similarly to Lemma 2, it can be derived to the other integral equalities imposed by open-loop unstable poles as follows:

Lemma 3. If $Y(s)$ has open-loop unstable complex conjugate poles at $s = \alpha \pm j\beta$ on the complex plane, $y(t)$ has to satisfy

$$\int_0^\infty e^{-\alpha t}\cos(\beta t)y(t)dt = \frac{\alpha}{\alpha^2 + \beta^2}, \tag{10}$$

and

$$\int_0^\infty e^{-\alpha t}\sin(\beta t)y(t)dt = \frac{\beta}{\alpha^2 + \beta^2}. \tag{11}$$

Proof. Let us take $\chi = \alpha + j\beta$ in Lemma 1, and the result immediately follows from Eqs. (7) and (8) since $T(p) = 1$, which completes the proof. □

If $Y(s)$ has an open-loop RHP real pole at $s = p$ in the complex plane, Eq. (10) can be rewritten by

$$\int_0^\infty e^{-pt}y(t)dt = \frac{1}{p}, \tag{12}$$

which means that if the open-loop RHP real pole is close to the origin, the maximum overshoot has to be very large (Middleton, 1991).

4. FUNDAMENTAL LIMITATIONS ON THE STEP RESPONSE

Based on the results of the previous section, the performance limitations on the step response of the closed-loop system as shown in Fig. 1 are established in this section. Let us consider a situation in which the step response is equal to the DC gain after a finite time period, which is previously used in Goodwin *et al.* (1999). Although this assumption of an exact settling time would be unrealistic, corresponding results presented in this paper can be extended so that similar limitations hold under the less restrictive set of assumptions.

Definition 3. Let us define the exact settling time of the system as follows:

$$t_s = \inf\left\{\tau \ : \ y(t) = 1, \ ^{\forall}t \geq \tau\right\}, \tag{13}$$

where $y(t)$ is the step response of $T(s)$.

Let us firstly derive the condition under which the system has the undershoot phenomena in the sense of Definition 2.

Lemma 4. For the stable system with RHP complex conjugate zeros at $s = a \pm jb$ on the complex plane, the step response has the undershoot if

$$bt_s \leq \frac{\pi}{2} + \tan^{-1}\frac{a}{b}, \tag{14}$$

where t_s is the exact settling time.

Proof. Let $E_r(t) = 0$, $E_c(t) = e^{-at}$ and $\Gamma_c(t) = \sin(bt)$ in Lemma 2, and the step response $y(t)$ has to satisfy

$$\begin{aligned} 0 &= \int_0^\infty e^{-at}\sin(bt)y(t)dt \\ &= \int_0^{t_s} e^{-at}\sin(bt)y(t)dt + \int_{t_s}^\infty e^{-at}\sin(bt)dt. \end{aligned} \tag{15}$$

since the system has RHP complex conjugate zeros at $s = a \pm jb$. From Eq. (15), we can obtain the relation

$$\begin{aligned} &\int_0^{t_s} e^{-at}\sin(bt)y(t)dt \\ &\quad = -\frac{e^{-at_s}}{a^2+b^2}\left[a\sin(bt_s) + b\cos(bt_s)\right] \\ &\quad = -\frac{e^{-at_s}}{a^2+b^2}\cos\left(bt_s - \tan^{-1}\frac{a}{b}\right). \end{aligned} \tag{16}$$

Hence, if $bt_s \leq \pi/2 + \tan^{-1}(a/b)$, then $y(t)$ will take both positive and negative signs since the right side of Eq. (16) is always negative, and $e^{-at}\sin(bt) > 0$ for all $t \in [0, t_s]$, which completes the proof. □

It is noted that the value of $\tan^{-1}(a/b)$ is given by

$$0 \leq \tan^{-1}\frac{a}{b} \leq \frac{\pi}{2} \tag{17}$$

since $a > 0$ and $b > 0$. If $a \geq b$, Eq. (14) leads to the result

$$bt_s \leq \frac{3}{4}\pi. \tag{18}$$

Corollary 4 also implies that the system with RHP real zeros, *i.e.*, the case of $b = 0$, always has the undershoot in the step response without any relation with the settling time. As a matter of fact, it is well-known that SISO LTI continuous-time systems with an odd number of RHP real zeros have the initial undershoot on the step type reference input (León de la Barra, 1994; Mita and Yoshida, 1981). However, the corresponding result has not presented for the system with RHP complex conjugate zeros related to the undershoot phenomena in the step response.

Also, let us define the maximum magnitude of the step response using the L_∞ norm as follows:

Definition 4. Define the L_∞ norm by

$$\|y\|_\infty = \operatorname{ess\,sup}|y(t)|, \ ^\forall t \geq 0, \tag{19}$$

where $y(t)$ is the step response.

Under this definition, the lower bounds of $\|y\|_\infty$ derived by using Lemma 1 can be formulated as follows:

Theorem 1. If $\omega t_s \leq \pi/2$, the maximum magnitude of the unit step response $y(t)$ has a lower bound as follows:

$$\|y\|_\infty \geq \max_{\sigma\in\mathbb{R}^+,\omega\in\mathbb{R}}\left[\frac{\mathcal{A}(\chi)}{\mathcal{M}(\chi)}\right] \tag{20}$$

with

$$\mathcal{A}(\chi) = \mathrm{Re}\left[\frac{T(\chi)}{\chi}\right] - \mathcal{H}(\chi), \tag{21}$$

$$\mathcal{M}(\chi) = \frac{\sigma}{\sigma^2+\omega^2} - \mathcal{H}(\chi), \tag{22}$$

where t_s is the exact settling time, $\chi = \sigma + j\omega$ and

$$\mathcal{H}(\chi) = \frac{e^{-\sigma t_s}}{\sigma^2+\omega^2}\left[\sigma\cos(\omega t_s) - \omega\sin(\omega t_s)\right]. \tag{23}$$

Moreover, if $\omega t_s \leq \pi$, the maximum magnitude of $y(t)$ has a lower bound as follows:

$$\|y\|_\infty \geq \max_{\sigma\in\mathbb{R}^+,\omega\in\mathbb{R}}\left[\frac{\mathcal{B}(\chi)}{\mathcal{N}(\chi)}\right] \tag{24}$$

with

$$\mathcal{B}(\chi) = \mathrm{Im}\left[\frac{T(\bar{\chi})}{\bar{\chi}}\right] - \mathcal{I}(\chi), \tag{25}$$

$$\mathcal{N}(\chi) = \frac{\omega}{\sigma^2+\omega^2} - \mathcal{I}(\chi), \tag{26}$$

where $\bar{\chi} = \sigma - j\omega$ and

$$\mathcal{I}(\chi) = \frac{e^{-\sigma t_s}}{\sigma^2+\omega^2}\left[\sigma\sin(\omega t_s) + \omega\cos(\omega t_s)\right]. \tag{27}$$

Proof. If $\omega t_s \leq \pi/2$, Eq. (7) in Lemma 1 can be modified as follows:

$$\begin{aligned} &\mathrm{Re}\left[\frac{T(\chi)}{\chi}\right] \\ &= \int_0^{t_s} e^{-\sigma t}\cos(\omega t)y(t)dt + \int_{t_s}^\infty e^{-\sigma t}\cos(\omega t)dt \\ &\leq \|y\|_\infty \int_0^{t_s} e^{-\sigma t}\cos(\omega t)dt \\ &\qquad + \frac{e^{-\sigma t_s}}{\sigma^2+\omega^2}\left[\sigma\cos(\omega t_s) - \omega\sin(\omega t_s)\right], \end{aligned} \tag{28}$$

which implies Eq. (20). Equation (24) can be also derived from Eq. (8) in Lemma 1 when $\omega t_s \leq \pi$, which completes the proof. □

Theorem 1 states the trade-off relations between the maximum magnitude of the step response and the achievable settling time. Although it is difficult to compute the lower bounds in Theorem 1 by analytic methods, they can be easily changed to simple form (Kwon *et al.*, 2002). For example, if we take the imaginary part of χ for 0, we can obtain the lower bound without any relation with ω as follows:

$$\|y\|_\infty \geq \max_{\sigma\in\mathbb{R}^+}\left[\frac{T(\sigma)-e^{-\sigma t_s}}{1-e^{-\sigma t_s}}\right]. \qquad (29)$$

When the exact settling time t_s approaches ∞, it is obvious that Eq. (29) becomes

$$\|y\|_\infty \geq \max_{\sigma\in\mathbb{R}^+}\left[T(\sigma)\right]. \qquad (30)$$

It can be seen that as σ goes to 0, the lower bound of Eq. (29) goes to the DC gain and as σ goes to ∞, it goes to 0 for strictly proper systems. Hence, all unity-feedback systems have some lower bounds such as Eqs. (29) and (30) as well as Eqs. (20) and (24) in Theorem 1. Note that the lower bound of Eq. (30) does not concern with the exact settling time t_s.

Theorem 1 has very important information related to the overshoot on the unit step response of the unity feedback system. It can be summarized as follows:

Corollary 1. A stable unity-feedback system $T(s)$ must have the overshoot in its unit step response if there exists $\chi = \sigma + j\omega$ satisfying one of two conditions as follows:

(1) $\mathrm{Re}\left[\frac{T(\chi)}{\chi}\right] > \frac{\sigma}{\sigma^2+\omega^2}$ and $\omega t_s \leq \pi/2$,

(2) $\mathrm{Im}\left[\frac{T(\bar{\chi})}{\bar{\chi}}\right] > \frac{\omega}{\sigma^2+\omega^2}$ and $\omega t_s \leq \pi$,

where t_s is the exact settling time.

Proof. It follows from Theorem 1 since the unit step response has the overshoot if and only if $\|y\|_\infty > 1$. □

Another result related to the overshoot can be induced by Eq. (29) as follows:

Corollary 2. A stable unity-feedback system with open-loop RHP real poles must have overshoot in the unit step response.

Proof. Since the unit step response has the overshoot if and only if $\|y\|_\infty$ is larger than 1, it can be seen from Eq. (29) that $\|y\|_\infty > 1$ if and only if $T(\sigma) > 1$ for some $\sigma > 0$, where $T(s)$ is given by Eq. (1). Without loss of generality, let us denote $K(s) = N(s)/D(s)$ and $P(s) = B(s)/A(s)$. Then $T(\sigma)$ is larger than 1 if and only if $A(\sigma)D(\sigma) < 0$ since $A(\sigma)D(\sigma) + B(\sigma)N(\sigma) > 0$ for some $\sigma > 0$. If $T(s)$ has open-loop RHP real poles, there exist $\sigma > 0$ such that $A(\sigma)D(\sigma) < 0$, and equivalently, its step response must have the overshoot, which completes the proof. □

As a matter of fact, Corollary 2 is the special case of Corollary 1 with $\omega = 0$. As a result, if the plant $P(s)$ has unstable real poles, any controller does not make the step response without the overshoot in the unity-feedback scheme as shown in Fig. 1.

Finally, let us consider that $T(s)$ has open-loop unstable real poles and RHP zeros. In this case, we can obtain another lower bound of the maximum magnitude of the step response imposed by those poles and zeros in the unity-feedback system.

Theorem 2. Let $T(s)$ have RHP complex conjugate zeros at $s = a \pm jb$ and an open-loop unstable real pole at $s = p$ on the complex plane. If $a > p$, the maximum magnitude of the unit step response has a lower bound as follows:

$$\|y\|_\infty \geq \max\left[\frac{a^2+b^2}{a^2+b^2-ap}, \frac{a^2+b^2}{a^2+b^2-bp}\right]. \qquad (31)$$

Proof. From Theorem 1, the unit step response $y(t)$ has to satisfy

$$\int_0^\infty e^{-at}\cos(bt)y(t)dt = 0, \qquad (32)$$

since $T(s)$ has RHP complex zeros at $s = a \pm jb$. Moreover, $y(t)$ has to also satisfy Eq. (12) since $T(s)$ has the RHP real pole at $s = p$. The difference of Eq. (12) and Eq. (32) is given by

$$\int_0^\infty \left[e^{-pt} - e^{-at}\cos(bt)\right] y(t)dt = \frac{1}{p}. \qquad (33)$$

If $a > p$, it can be rewritten by

$$\begin{aligned}\frac{1}{p} &\leq \|y\|_\infty \int_0^\infty \left[e^{-pt} - e^{-at}\cos(bt)\right] dt \\ &= \|y\|_\infty\left[\frac{1}{p} - \frac{a}{a^2+b^2}\right],\end{aligned} \qquad (34)$$

which implies a lower bound as follows:

$$\|y\|_\infty \geq \frac{a^2+b^2}{a^2+b^2-ap}. \qquad (35)$$

The difference of Eq. (12) and

$$\int_0^\infty e^{-at}\sin(bt)y(t)dt = 0, \qquad (36)$$

yields another lower bound as follows:

$$\|y\|_\infty \geq \frac{a^2+b^2}{a^2+b^2-bp}. \qquad (37)$$

Finally, Eq. (31) follows from the Eqs. (35) and (37), which completes the proof. □

When $T(s)$ has an RHP real zero at $s = z$ and open-loop unstable real pole at $s = p$, the lower bound of Eq. (31) can be rewritten by

$$\|y\|_\infty \geq \frac{z}{z-p}, \qquad (38)$$

provided that $z > p$. It means that the overshoot is extremely large when $T(s)$ has the open-loop real pole located in the left vicinity of the RHP real zero.

5. CONCLUSION

This paper has presented some new results on time-domain integral equalities related to the step response of SISO stable system, which are formulated by the complex value χ having a real part larger than 0. Based on these equalities, it has shown that the unity-feedback system as shown in Fig. 1 has some lower bounds on the maximum magnitude of its step response with respect to the achievable settling time, which is given by Theorem 1. Moreover, it has shown that RHP zeros necessarily imply the undershoot and open-loop unstable real poles necessarily imply the overshoot in the step response, which are given Corollaries 4 and 2, respectively.

The results presented in this paper are formulate on the time-domain, which are the counterparts of the fundamental limitations on the frequency-domain. The results will provide some guidelines for designing the feedback controller of any system.

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