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ROBUST REGULATION OF WORK LOADS IN PRODUCTION PLANNING

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Abstract: In production systems, the people responsible for scheduling decisions are obligated to identify how resources can be allocated best. Dividing the planning horizon into adjacent intervals facilitates work load regulation and the scheduling of operations according to available resources by helping to determine coherence between the operating plan and real shop capacity constraints. The methodology proposed in this paper is based on linear programming and allows for coherence and robust scheduling decisions and work load regulation, in aggregate terms, whitin a flexible planning horizon. *Copyright © 2002 IFAC*

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1. INTRODUCTION

In production systems, the people responsible for decisions on scheduling face the dual necessity of efficiently assigning tasks in a scheduling horizon, according to resources, and regulating or balancing required work loads in each planning horizon with shop capacity constraints. Dividing the horizon into adjacent intervals facilitates work load regulation and the scheduling of tasks according to available resources by helping to determine coherence between the operating plan and real shop capacity constraints (Thuriot and Torres, 1992). In the coherent mode, an updated work load plan to schedule operations efficiently is obtained through ideal manipulation of the margin for decision-making autonomy (Erschler, 1976). The remainder of this paper is organized as follows: The operating plan is defined in Section 2. The problem of dividing the temporal horizon is discussed in Section 3. Section 4 illustrates the adjacency property to be met by dividing the horizon to define the work load regulation problem. Section 5 contains the definitions of maximum and minimum load curves, and Section 6 lists the aggregate constraints of maximum capacity and minimum availability. Section 7 outlines the full aggregate regulation model developed through linear programming. In Section 8, there is a discussion of the computational results of applying the model to a test problem, using GAMS. The conclusions are offered in Section 9.

2. THE OPERATING PLAN

By calculating MRP requirements, it is possible to determine operating plans - earliest initiation and latest completion - in terms of aggregate shop resources (Johnson and Montgomery, 1974). A plan is comprised of a set of operations scheduled on the basis of resources. The operating plan is the set:

$$\pi = \{\omega_i / \omega_i = (C_i, F_i, D_i, q_i), 1 \le i \le n\} \quad (1)$$

With:

C_i = Earliest initiation date,

F_i = Latest completion date,

D_i = Duration,

q_i = Resource consumption or intensity of the scheduled operation w_i .

3. DIVISION OF THE TEMPORAL HORIZON

The quality of decisions on how work is organized (work load regulation, sequencing and arrangement of scheduled operations) can depend on how those in charge of the shop's course of operations handle the planning horizon. With the help of adequate decision-making tools, the people responsible can make robust decisions that will leave appropriate working margins for other decision-making centers. One such tool can be obtained by dividing the planning horizon into adjacent intervals. A "natural" division of the horizon would be limited to a division into intervals of equal length. These would correspond to the important periods in a collective work arrangement (hours, shifts, days, weeks, fortnights, etc.). Another, more general division, which is also compatible with "natural" divisions, considers the temporal horizon as capable of being divided into adjacent intervals, possibly of different length, with each scheduled task in the plan intersecting with at least one other interval in the divided horizon (Erschler and Thuriot, 1992). We refer to the set of temporal references in the divided horizon as the **adjacent structure**. Accordingly, the adjacent structure associated with the operating plan π is the set $\varepsilon(\pi) = \{T_j / T_j \in R; 0 \le j \le m\}$, in which R is the universal set of possible references and m is the total number of references. The elements of the adjacent structure $\varepsilon(\pi)$ are called **temporary references** and must have the following properties.

1. $T_0 < T_1 < K < T_m$
2. $T_0 \le \min_{\omega_i \in \pi} C_i$ (2)
3. $T_m \ge \max_{\omega_i \in \pi} F_i$

Properties 2 and 3 ensure that horizon $[T_0, T_m]$ covers all operations in the plan.

4. THE PROPERTY OF ADJACENCY

Decisions on work regulation guarantee coherence between work load plans and resource capacity restraints. The work load regulation strategies we propose (Torres, 1995) assume a work load exchange between neighboring intervals in the adjacent structure. Implementation of these strategies is facilitated enormously by limiting these exchanges to a maximum of two neighboring intervals. Therefore, in our opinion, the temporal references must have the **property of adjacency:**

$$\forall \omega_i \in \pi, \exists j! 1 \le j \le m-1, T_{j-1} \le C_i < F_i \le T_{j+1} \quad (3)$$

5. WORK LOAD CURVES

The operating plan determines the maximum and minimum curves, which can be defined for each interval in the horizon. The work load curves set maximum and minimum limits on work (in units of time x unit of resource) according to the planning horizon. The maximum work load for an interval in the adjacent structure can be calculated by placing inside the interval all scheduled operational tasks with external rectangles that intersect the planning interval, then adding the surfaces remaining inside the interval (Figure I).

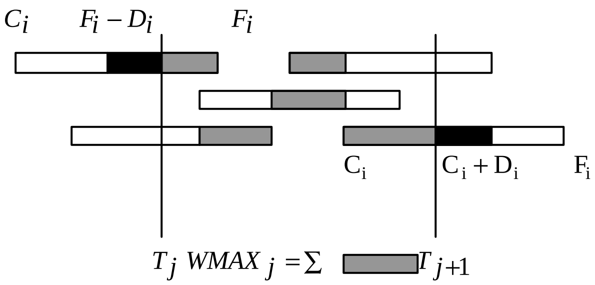


Fig. I. Maximum work load

Likewise, placing operational tasks as far outside the planning interval as possible gives us the minimum work load determined by the operating plan for the corresponding interval (Figure II).

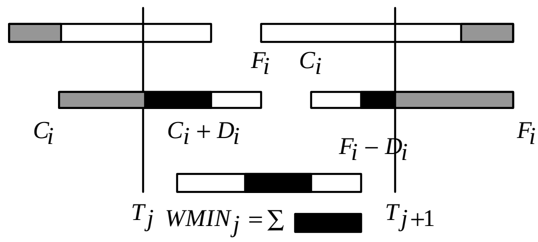


Fig. II. Minimum work load

The maximum and minimum work loads for each interval can be calculated as follows.

$$WMAX_j = \sum_{i \in \pi} \max\left(\min\left[D_i, F_i - T_j, T_{j+1} - C_i\right], 0\right)$$

$$WMIN_j = \sum_{i \in \pi} \max\left(\min\left[D_i, C_i + D_i - T_j, T_{j+1} - (F_i - D_i)\right], 0\right)$$

$$\forall T_j, T_{j+1} \in \varepsilon(\pi) \text{ with } 0 \le j \le m-1 \quad (4)$$

Therefore, the overall work load $MARG(\varepsilon(\pi), \pi)$ induced by the adjacent structure $\varepsilon(\pi)$ and operating plan π can be defined as follows.

$$MARG(\varepsilon(\pi), \pi) = \sum_{j=0}^{m-1} \left(WMAX_j - WMIN_j\right) \quad (5)$$

6. CAPACITY AND PROFITABILITY RESTRICTIONS

The operating plan is implemented using a set of resources with limited capacity. Assuming that any task or operation can be executed with any resource, the set of resources implies a restriction in terms of maximum capacity and minimum availability for each time interval between references. Maximum capacity corresponds to maximum available intensity and minimum availability to the intensity that guarantees a minimum profitability from available resources.

Therefore, the following is taken into account for each time interval $\lfloor T_j, T_{j+1} \rfloor$ between two consecutive references of the adjacent structure:

Q_j = Maximum capacity (in time x resource) of interval j

R_j = Minimum availability (profitability) (in time x resource) of interval j.

7. A MATHEMATICAL MODEL OF ROBUST AGGREGATE WORK LOAD REGULATION

Solving the problem of aggregate regulation consists of assigning a work load that satisfies the maximum capacity and minimum availability restrictions on the entire temporal horizon, by complying with coherence restrictions on each operation or task in the plan.

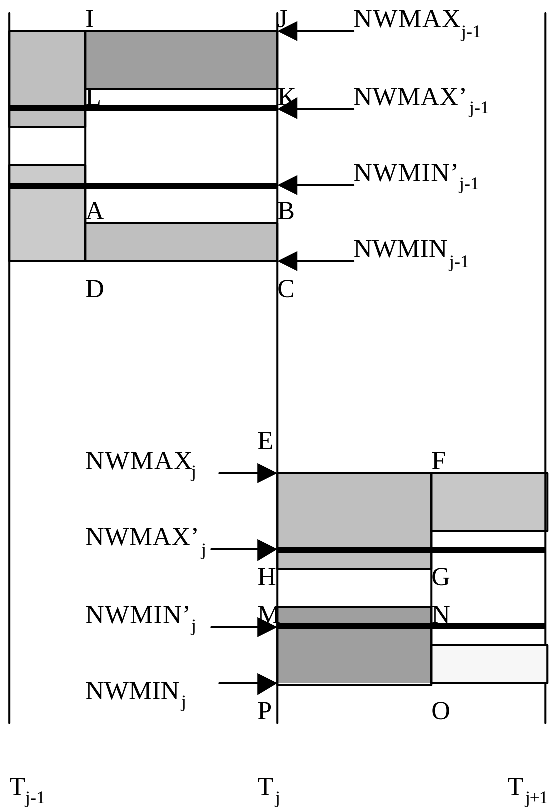
Compliance with the property of adjacency means that each task in the operating plan can be intersected by one temporal reference in the adjacent structure at the most. This property facilitates addressing the problem of aggregate regulation, since the work load for each task can be shifted coherently between adjacent intervals by shifting each task between its earliest initiation and latest completion date.

In aggregate terms, for each reference T_j, with $j \geq 1$, the total work load CT_j that can be shared between interval $\lfloor T_j, T_{j+1} \rfloor$ and its closest preceding neighbor $\lfloor T_{j-1}, T_j \rfloor$ is:

$$CT_j = \sum_{\substack{\omega_i \in \pi \\ C_i \leq T_j \leq F_i}} \left[CTB_{max}(\omega_i, j) - CTB_{min}(\omega_i, j) \right] \quad (6)$$

Where , $CTB_{max}(\omega_i, j)$ and $CTB_{min}(\omega_i, j)$ are the respective contributions to the maximum and minimum work load of interval $\lfloor T_j, T_{j+1} \rfloor$ that conducts the operation ω_i :

$$CTB_{max}(\omega_i, j) = \max\left(\min\lfloor D_i, F_i - T_j, T_{j+1} - C_i \rfloor, 0\right)$$
$$CTB_{min}(\omega_i, j) = \max\left(\min\left[D_i, C_i + D_i - T_j, T_{j+1} - F_i + D_i\right], 0\right) \quad (7)$$



Area of Rectangle ABCD=$WMIN_{der,j-1}$ =
Area of Rectangle EFGH=$WMAX_{izq,j}$
Area of Rectangle IJKL=$WMAX_{der,j-1}$ =
Area of Rectangle MNOP=$WMIN_{izq,j}$
Area of Rectangle EFOP=CT_j

Fig. III. Definitions of the variables of the Robust Work Load Regulation Model.

Based on the foregoing considerations, the following linear mathematical model of robust aggregate regulation can be proposed.

$$\text{Max} \sum_{j=0}^{m-1} \left(WMAX'_j - WMIN'_j \right) \quad (8.0)$$
$$WMAX'_j = WMAX_j - \left(WMAX_{izq,j} + WMAX_{der,j} \right),$$
$$1 \leq j \leq m-2 \quad (8.1)$$
$$WMIN'_j = WMIN_j + \left(WMIN_{izq,j} + WMIN_{der,j} \right),$$
$$1 \leq j \leq m-2, \quad (8.2)$$
$$WMAX'_0 = WMAX_0 - WMAX_{der,0} \quad (8.3)$$
$$WMIN'_0 = WMIN_0 + WMIN_{der,0} \quad (8.4)$$
$$WMAX'_{m-1} = WMAX_{m-1} - WMAX_{izq,m-1} \quad (8.5)$$
$$WMIN'_{m-1} = WMIN_{m-1} + WMIN_{izq,m-1} \quad (8.6)$$
$$WMAX_{izq,j} + WMIN_{izq,j} \leq CT_j, 1 \leq j \leq m-1 \quad (8.7)$$
$$WMAX_{izq,j} = WMIN_{der,j-1}, 1 \leq j \leq m-1 \quad (8.8)$$
$$WMIN_{izq,j} = WMAX_{der,j-1}, 1 \leq j \leq m-1 \quad (8.9)$$
$$R_j \leq WMIN'_j, 0 \leq j \leq m-1 \quad (8.10)$$
$$WMAX'_j \leq Q_j, 0 \leq j \leq m-1 \quad (8.11)$$
$$WMIN'_j \leq WMAX'_j, 0 \leq j \leq m-1 \quad (8.12)$$
$$WMAX_{izq,j}, WMAX_{der,j}, WMIN_{izq,j}, WMIN_{der,j} \geq 0$$

The target function (8.0) corresponds to the overall work load. Restrictions(8.1), (8.2), (8.3), (8.4), (8.5) and (8.6) pertain to the maximum and minimum balances for the work load after regulation, according to Figure 3. Restriction (8.7) represents the work load that can be shared between adjacent intervals. Equations (8.8) and (8.9) are called "communicating vessel restrictions" and show conservation of the world load after regulation. Equations (8.10) and (8.11) correspond respectively to the minimum availability and maximum capacity restrictions. Equation (8.12) ensures coherence of the work load balance after regulation.

In Figure III, regulation of the work load between intervals $\lfloor T_{j-1}, T_j \rfloor$ y $\lfloor T_j, T_{j+1} \rfloor$ corresponds to an exchange of work loads represented by the pair of restrictions (8.8) and (8.9). $WMIN_{izq,j} = WMAX_{der,j-1}$ is the total work load shifted between the two intervals by modifying the earliest initiation date of operations whose temporal window $[C_i, F_i]$ intersects with T_j. Similarly, $WMIN_{izq,j} = WMAX_{der,j-1}$ correspond to the total work load shifted between the two intervals by modifying the earliest completion dates of the operations whose temporal window intersects with T_j.

In Figure III, $NWMAX_j$, $NWMIN_j$, are assumed to be the maximum and minimum work load intensity levels in the initial plan and $NWMAX'_j$, $NWMIN'_j$ correspond to the same levels of intensity after regulation.

There is a direct relationship between work load levels and levels of work load intensity.

$$
\begin{aligned}
WMAX_j &= (T_{j+1} - T_j) NWMAX_j \\
WMIN_j &= (T_{j+1} - T_j) NWMIN_j \\
WMAX'_j &= (T_{j+1} - T_j) NWMAX'_j \\
WMIN'_j &= (T_{j+1} - T_j) NWMIN'_j
\end{aligned} \qquad (9)
$$

8. NUMERICAL APPLICATION OF THE LINEAR PROGRAMMING

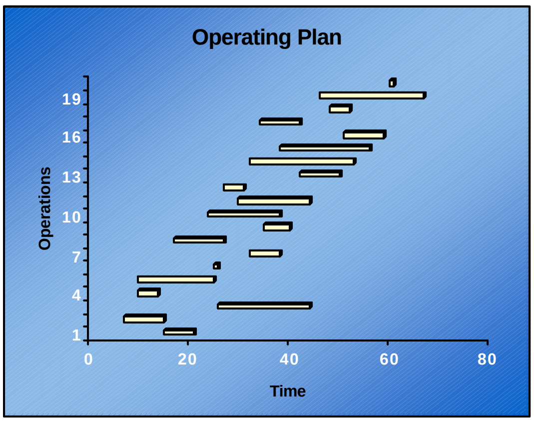


Fig. IV. Plan for 20 operations

To apply the linear programming model, we suggest the following problem with the 20 operations in Figure IV, as represented in Table 1.

Table 1: Plan for 20 Operations

Operation	C(i)	D(i)	F(i)
1	15	2	21
2	7	4	15
3	26	13	44
4	10	4	14
5	10	10	25
6	25	1	26
7	32	1	38
8	17	6	27
9	35	1	40
10	24	11	38
11	30	9	44
12	27	4	31
13	42	7	50
14	32	16	53
15	38	18	56
16	51	5	59
17	34	6	42
18	48	3	52
19	46	20	67
20	60	1	61

The linear programming model was programmed in GAMS, using the adjacent structure {7, 15, 25, 38, 56, 67}, with capacity and profitability restrictions of 56 and 8 units of resource x unit of time, respectively. The following results were obtained for robust aggregate regulation as shown in Table 2:

Table 2: Maximum and Minimum Work Loads

Interval	t(j)	t(j+1)	wmax	wmin	wmax'	wmin'
1	7	15	13	8	13	8
2	15	25	19	11	19	11
3	25	38	50	29	50	44
4	38	56	75	53	56	53
5	56	67	15	11	15	15

These results pertain to the following work load amounts, as shown in Table 3, when shifted between time intervals $WMAX_{izq,j}, WMAX_{der,j}, WMIN_{izq,j}, WMIN_{der,j}$:

Table 3: Work Loads When Shifted Between Time Intervals

Intervalo	Wmaxder	Wmaxizq	Wminder	Wminizq
1	0	0	5	0
2	0,999999	5	4,000001	0
3	4	4,000001	1	0,999999
4	3	1	0	4
5	0	0	0	3

The results of the robust regulation model produce an ideal work load of 22 units of resource x unit of time.

The final amounts produced by the model are shown in Figure V.

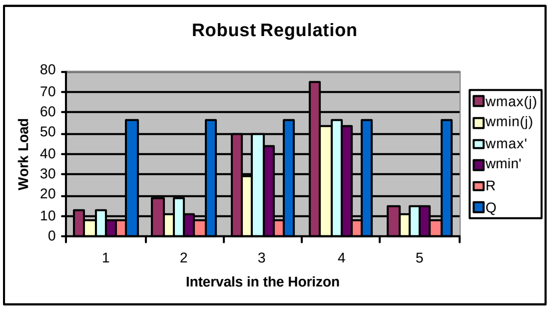


Fig. V. Robust Aggregate Regulation for the Plan with 20 Operations in Table 1

In interval 5, robust regulation leaves no operational work load margin. In the other intervals, the overall work load margin after regulation in units of resource x unit of time is not nil. According to the results of Table 3, effective work load exchanges at aggregate level must be done between intervals 3, 4 and 5.

The shadow prices of capacity and profitability restrictions, as illustrated in Table 4, can be determined by analyzing the dual model. Accordingly, it is possible to conclude that the critical restriction pertains to the capacity of interval 4, which has the greatest maximum work load. An increase of 1 in this capacity will result in an increase of 2 in the optimum cost. This analysis also leads to the conclusion that the ideal interval of variation in maximum capacity would be $[53,+\infty)$, and that cost increases will occur in interval [53,75] if capacity is increased.

Table 4: Shadow Prices of Capacity and Profitability Restrictions

Interval	Capacity	Profitability
1	0	0
2	0	0
3	0	0
4	2	0
5	0	0

9. CONCLUSIONS

The linear programming model proposed in this paper allows for solving the problem of aggregate work load regulation on a temporal horizon through division into adjacent intervals that comply with the property of adjacency. The solution to the regulation problem is based on optimizing the plan's overall margin of autonomy, taking into account the coherence restrictions on operations (earliest initiation dates, latest completion dates and duration) and on resources (maximum work load capacity and minimum work load availability). The model was applied satisfactorily to a randomly generated problem, using GAMS software. Future extensions of this work will attempt to define, once again, the aggregate regulation problem without taking into account the adjacency restriction for dividing the horizon and considering additional sequence restrictions on the operations in the plan, the effective availability of resources and more complex operations using more than one resource.

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