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MULTIVARIABLE MPC PERFORMANCE ASSESSMENT, MONITORING AND DIAGNOSIS

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Abstract: This study focuses on performance assessment and monitoring of model predictive control systems. A methodology is proposed to determine a benchmark and monitor MPC performance on-line. A performance measure based on the ratio of historical and achieved performance is used for monitoring and a ratio of design and achieved performance is used for diagnosis. Case studies with linear and nonlinear models of an evaporator illustrate the methodology and limitations of linearity assumptions.

Keywords: Controller performance assessment and monitoring, MPC, Fault diagnosis

1. INTRODUCTION

Controller performance assessment (CPA) and monitoring (CPM) are necessary because many factors can cause abrupt or gradual performance deterioration of controllers. It is often difficult to monitor the performance and diagnose problems from raw data trends (Kozub 1997). A suitable performance criteria must be defined to determine the capability of a control system followed by the selection of a meaningful benchmark. Then, performance has to be monitored on-line to detect changes in controller performance. Values of performance measures are stochastic and statistical analysis tools have to be formulated to detect statistically significant changes. CPA and CPM methods proposed for model predictive control (MPC) systems include measuring the proximity of actual performance to optimal performance estimated by solving the LQG problem (Huang and Shah 1999), comparing actual controlled performance to historical performance using the expected value of the MPC cost function for a certain time window (Patwardhan *et al.* 1998) and comparing values of the objective function for the output of the plant model and the real plant output (Patwardhan *et al.* 1998, Zhang and Henson 1999). This study focuses on an integrated CPA, CPM and diagnosis of MPCs. Diagnosis is limited to distinguishing between root cause problems associated with the controller and other causes. Case studies based on an evaporator model are used to illustrate the methodology proposed.

2. MPC PERFORMANCE ASSESSMENT

MPC is based on real-time optimization of a cost (objective) function (Φ). CPA methods can be developed by using this cost function.

$$\Phi = \sum_{j=N_1}^{P} [\widehat{y}(t+j) - r(t+j)]^T Q [\widehat{y}(t+j) - r(t+j)] + \sum_{j=1}^{M} [\Delta u(t+j-1)]^T R [\Delta u(t+j-1)] \quad (1)$$

where $\widehat{y}(t)$, $r(t)$, and $\Delta u(t)$ are vectors of predicted output variables, reference trajectory, and change in manipulated variables at time t, respectively. Q and R are weighting matrices of relative importance of controlled and manipulated variables. P and M are the prediction and control horizons. A measure of success in reducing Φ is

$$J_{actual}(t) = \varepsilon^T(t) Q \varepsilon(t) + \Delta u^T(t) R \Delta u(t) \quad (2)$$

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where $\varepsilon(t)$ and $\Delta u(t)$ are vectors of controlled variable error and control moves, respectively. Because the cost function is a random variable influenced by measurement noise and disturbances, its expected value is a more suitable measure:

$$\begin{aligned} J_{ach} &= E[J_{actual}(t)] \\ &= E[\varepsilon^T(t)Q\varepsilon(t) + \Delta u^T(t)R\Delta u(t)] \end{aligned} \quad (3)$$

where $E[.]$ denotes expectation. Three CPA methods have been proposed for MPC: LQG benchmark (Huang and Shah 1999), historical performance benchmark (Patwardhan *et al.* 1998), and model-based performance benchmark (Patwardhan *et al.* 1998, Zhang and Henson 1999).

2.1 *LQG-Benchmark*

The achievable performance of a linear system characterized by quadratic costs and Gaussian noise can be estimated by solving the linear quadratic Gaussian (LQG) problem. The solution provides a benchmark, the tradeoff curve that displays the minimal achievable variance of the controlled variable versus the variance of the manipulated variable (Huang and Shah 1999). For the multivariable case, H_2 norms $\|G_Y\|_Q^2 = E(\varepsilon(t)^T Q\varepsilon(t))$ and $\|G_u\|_R^2 = E(\Delta u(t)^T R\Delta u(t))$ are plotted.

2.2 *Historical Benchmark*

This approach requires a priori knowledge that the performance was good during a certain time period according to some expert assessment (Patwardhan *et al.* 1998). For the selected input and output data, the historical benchmark J_{hist} is computed using Eqn (3) where $\varepsilon(t)$ and $\Delta u(t)$ are taken from the historical data set. The objective function for the performance achieved (J_{ach}) is calculated by using again Eqn (3) where $\varepsilon(t)$ and $\Delta u(t)$ are taken from any data set. The performance measure is the ratio $\gamma_{hist} = J_{hist}/J_{ach}$.

2.3 *Model based Performance Measure*

Design Case Approach. Patwardhan *et al.* (1998) propose the comparison of the achieved performance with the design case performance characterized by inputs and outputs given by the model. The design cost function J_{des} has the same form as Eqn (3) where $\varepsilon(t)^*$ and $\Delta u(t)^*$ are substituted for $\epsilon(t)$ and $\Delta u(t)$ to indicate the predicted deviations of model outputs from the setpoints (an estimate of the disturbance is included) and the optimal control moves, respectively. J_{ach} (Eqn 3) is the same as that in historical benchmark and is calculated using plant data. The deviation of the real plant performance (J_{ach}) from that of the model (J_{des}) is expressed by the ratio $\gamma_{des} = J_{des}/J_{ach}$.

Expectation Case Approach. Zhang and Henson (1999) have proposed an on-line comparison between expected and actual system performance. The expected performance is obtained when controller actions are implemented on the process model instead of the plant. Zhang and Henson (1999) compute the performances over a moving horizon P_C of past data. The actual performance is

$$J_{act}(t) = \sum_{j=1}^{P_C} \varepsilon^T(t+j-P_C)Q\varepsilon(t+j-P_C) \quad (4)$$

where $\varepsilon(t)$ is the vector of output deviation variables at time t. The expected controller performance ($J_{exp}(t)$) is computed using Eqn (4) where $\varepsilon(\cdot)$ is replaced by $\varepsilon^*(\cdot)$ and the ratio of expected over actual performance is defined as $I_{MPC}(t) = J_{exp}(t)/J_{act}(t)$. The ratios γ_{des} and I_{MPC} are very similar, and in general they will be smaller than 1 because of imperfect models, sensor and actuator noise or other uncertainties.

Zhang and Henson (1999) identified I_{MPC} as a stochastic variable and advocated statistical analysis to detect statistically significant changes in controller performance. Because the distribution function of this random variable is not known, confidence limits of I_{MPC} can not be obtained by using conventional techniques. An alternative approach based on time series analysis is pursued. I_{MPC} is assumed to be modeled by an autoregressive moving average (ARMA) process

$$A(q^{-1})I_{MPC}(t) = C(q^{-1})z(t) \quad (5)$$

where q^{-1} is the backward shift operator, $C(q^{-1})$ and $A(q^{-1})$ are monic polynomials and $z(t)$ is a zero-mean, uncorrelated, Gaussian noise signal. Collecting a sequence of I_{MPC} values when the controller performs as expected, A, C and the variance of z can be estimated. Zhang and Henson (1999) report that I_{MPC} is highly serially correlated and its AR part is of order 1. They propose $(1 - a_1 q^{-1})I_{MPC}(t) = z(t)$ and define

$$\Delta I_{MPC}(t) \equiv \frac{\hat{A}(q^{-1})}{\hat{C}(q^{-1})} I_{MPC}(t) \quad (6)$$

where $\hat{C}(q^{-1})$ and $\hat{A}(q^{-1})$ are estimated polynomials. The estimated noise variance is used to compute 95% confidence limits on $\Delta I_{MPC}(t)$.

3. COMPREHENSIVE TECHNIQUE FOR MPC PERFORMANCE MONITORING

The LQG benchmark is limited to a special group of MPCs characterized by M=P and lack of feedforward components and constraints. Since M and P are two independent and important tuning parameters and the incorporation of constraints and feedforward control are two important advantages of MPC over conventional controllers, the LQG benchmark is not applicable to the probably more interesting group of MPC implementations. The essential step in obtaining the LQG benchmark is the calculation of various control laws for (P=M). This is a case study for a special type of MPC (unconstrained, no feedforward) and a special parameter set ($M = P$). Using the same information (plant and disturbance model, covariance matrices of noise and disturbances), case studies can be conducted for any type of MPC and the influence of any parameter can be examined. These case studies can be automated and the corresponding value of the cost function can be reported as a function of the underlying parameter set. This approach is used in the work reported.

3.1 *A Benchmark Obtained from Case Studies*

The tuning parameters of the MPC include P, M and α (parameter for calculating the reference trajectory for given set points). In addition, weight matrices and input constraints can be used to adjust the aggressiveness of the controller. An optimal parameter set and the corresponding cost function J are computed for given constraints, and weight and covariance matrices. The minimal value of J can be used as a benchmark and a measure of performance is given by γ_{hist}.

3.2 *Alternative Approach in Statistical Monitoring of Historical Benchmark*

ΔI_{MPC} can be monitored by using residuals charts (Zhang and Henson 1999). Use of traditional statistical process monitoring (SPM) charts for autocorrelated variables may yield erroneous results. An alternative SPM method for autocorrelated data develops a time series model, generates residuals between predicted and measured values, and monitors the residuals. The residuals should be approximately normally and independently distributed with zero mean and constant variance if the time series model provides an accurate description of process behavior. Therefore, standard SPM charts such as $\overline{x}$-chart can be used for monitoring the residuals.

For on-line monitoring, γ_{hist} is computed at each sampling time. J_{ach} is calculated over a moving horizon P_C of past data.

$$J_{ach} = \frac{1}{P_C}\Big[\sum_{j=1}^{P_C}(\varepsilon^T(t+j-P_C)Q\varepsilon(t+j-P_C) + \Delta u^T(t+j-P_C)R\Delta u(t+j-P_C))\Big] \quad (7)$$

where $\varepsilon(t)$ is the vector of control errors. The performance measure $\gamma_{hist}(t)$ at time t is

$$\gamma_{hist}(t) = \frac{J_{hist}}{J_{ach}(t)} \quad (8)$$

γ_{hist} is a random variable, SPM can be used to detect statistically significant changes. Since $\gamma_{hist}(t)$ is highly autocorrelated, residuals based SPM is used to monitor it. If an AR model is used to model $\gamma_{hist}(t)$:

$$A(q^{-1})\gamma_{hist}(t) = \epsilon(t) \quad (9)$$

where $\epsilon(t)$ is a zero-mean, uncorrelated, Gaussian noise signal. Expand Eqn (9) to estimate $\gamma_{hist}(t)$

$$\gamma_{hist}(t) = -(a_1 q^{-1} + ... + a_{na}q^{-na})\gamma_{hist}(t) + \epsilon(t) \quad (10)$$

Estimates of a_i are obtained from analysis of process data, and estimates $\hat{\gamma}_{hist}(t)$ are computed using Eqn (10). The residuals are

$$e_{\gamma_{hist}}(t) = \gamma_{hist}(t) - \hat{\gamma}_{hist}(t) \ . \quad (11)$$

3.3 *Monitoring of Model-Based Performance Measure*

Two model-based performance measures are proposed in the literature. γ_{des} ((Patwardhan *et al.* 1998)) accounts for the control effort and seems to be in closer agreement with MPC methodology. Therefore γ_{des} is used as model-based performance measure after modifying the cost functions for on-line monitoring. $J_{des}(k)$ and $J_{ach}(t)$ are computed using Eqn (7) with ε and ε^*, respectively. The performance measure $\gamma_{des}(t)$ is

$$\gamma_{des}(t) = \frac{J_{des}(t)}{J_{ach}(t)} \quad (12)$$

A residuals based SPM similar to monitoring $e_{\gamma_{hist}}$ is developed for monitoring $\gamma_{des}(t)$.

3.4 *Combination to a Comprehensive Approach*

Tools for CPM and diagnosis are available for four types of MPCs by obtaining benchmarks for constrained cases and controllers including feedforward (ff), and establishing statistical analysis on $\gamma_{hist}(t)$ and $\gamma_{des}(t)$ (Table 1). CPM is based on $\gamma_{hist}(t)$. When controller performance is declared poor, $\gamma_{des}(t)$ is used for diagnosis.

Table 1. Uses of Performance Measures

Controller	CPA	CPM	Diagnosis
unconstrained, no ff	LQG	$\gamma_{hist}(t)$	$\gamma_{des}(t)$
unconstrained, ff	case study	$\gamma_{hist}(t)$	$\gamma_{des}(t)$
constrained, no ff	case study	$\gamma_{hist}(t)$	$\gamma_{des}(t)$
constrained, ff	case study	$\gamma_{hist}(t)$	$\gamma_{des}(t)$

4. DIAGNOSIS

Some root causes affect the design case while others do not. For instance, increases in unmeasured disturbances, actuator faults, or increase in model mismatch do not influence design case performance. Accordingly, J_{des} remains constant while

J_{ach} increases, decreasing $\gamma_{des}(t)$. Root cause problems such as input saturation or change in measured disturbance affect J_{des} as well. This leads to small changes in $\gamma_{des}(t)$, if the effects are quantitatively equal (assuming a good process model). If degradation in performance is indicated, diagnosis starts by looking at $\gamma_{des}(t)$. If it has not changed significantly, the reason for the degradation affects both J_{des} and J_{ach} to the same extent. Thus, the cause belongs to group I listed below. If $\gamma_{des}(t)$ shows a degradation, the cause belongs to group II.

Subgroups are defined to further distinguish between root cause problems in group I. First, all *changes in the controller* (e.g. tuning parameters, estimator, constraints) are assumed to be performed manually. Since the action taken is known, the root cause of the effect does not need to be identified by diagnosis tools (Subgroup Ia). The remaining two root cause problems (*changes in measured disturbances* and *input saturation*) make up subgroup Ib. Additional information is needed to distinguish between them. Input saturation can be determined by visual inspection of manipulated variables.

Diagnosis of Group II. Distinguishing between performance degradation due to *changes in unmeasured disturbances* and *changes in process dynamics*, is a model validation issue. In an idealized case where disturbances are white noise, if the model is perfect, the innovation sequence is white noise as well (Brian *et al.* 1979). Imperfect models color the innovation sequence. This can be detected using standard methods.

If changes in the controller are done manually and need not be diagnosed, the diagnosis sequence is described in Fig. 1. The performance is monitored over time using γ_{hist}. Once a degradation is detected, γ_{des} is used to distinguish between root cause problems of groups I and II. Saturation of manipulated variables is used to distinguish between problems resulting from constraints and increases in measured disturbances.

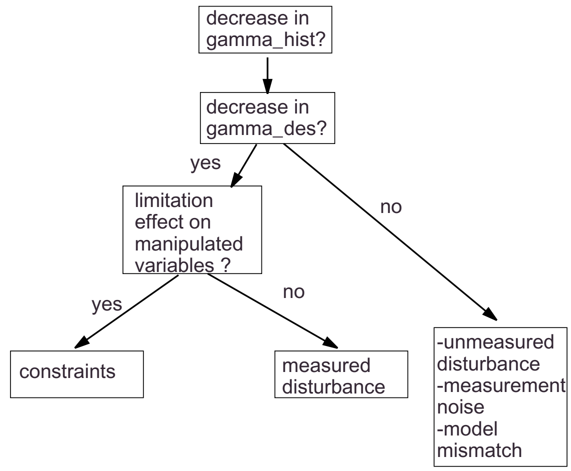

Fig. 1. Diagnosis Logistics

5. MPC PERFORMANCE MONITORING FOR CONTROLLING AN EVAPORATOR

The techniques for CPA, CPM, and diagnosis are applied to MPC of an evaporator model described by Newell and Lee (1988). First the initial assessment is made and a historical benchmark is found. Then, CPM and diagnosis are performed simultaneously for two cases differing by the use of linear and nonlinear plant models. The impact of linearity assumption and other effects resulting from nonlinearity are discussed.

Newell and Lee (1988) have developed two mathematical models: a simplified mechanistic nonlinear model, and a linear state space model in deviation variables. The system is has 3 controlled variables (separator level L_2, product composition X_2, and operating pressure P_2), 3 manipulated variables (product flowrate F_2, steam pressure P_{100}, and cooling water flowrate F_{200}), and 5 disturbances (circulation flowrate F_3, feed flowrate F_1, feed composition X_1, feed temperature T_1, and cooling water inlet temperature T_{200}).

5.1 *Initial Assessment of Control System Capability*

The capability of the MPC for controlling the evaporator is assessed by conducting simulations using the linear evaporator model. The weight matrices are $W = diag(0.5/m, 1.0/\%, 0.5/kPa)$ and $R = diag(0.2min/kg, 2.0/kPa, 0.5kg/min)$. Noise is assumed white and is generated such that the standard deviation of each measurement is approximately 1% of its original value under normal operating conditions. The uncontrolled inputs are a combination of white noise sequences whose standard deviations are about 1% of their original value and a pseudo random binary signal that adds step changes to the disturbance. The magnitude of step changes is about 1% of the original value of the variables. A Kalman filter is used for state estimation.

Case Studies for Initial Performance Assessment. Case studies are performed to find an optimal achievable performance and the corresponding tuning parameters based on known plant and disturbance models, and estimates of the noise and disturbances. P and M are the only tuning parameters since α is irrelevant (no setpoint change) and the weight matrices and constraints are given. Simulations are performed for $P = 1, 15$ and $M = 1, P$. The optimal J_{ach} is obtained for M=P=1 which becomes the reference case. This is surprising because stability problems usually exist for these values. F_3 is used as measured disturbance and the corresponding reduced value of the cost function (J_{min}=0.06) as the historical benchmark. After identifying the benchmark and design case tuning parameters, the ARMA models needed for CPM are built and an $\bar{x}$-chart with 2σ limits is applied to prediction residuals.

Six cases have been considered to test the CPM and diagnosis techniques:

(1) Increase in unmeasured disturbances F_1 and X_1 at $t = 300\ min$. The disturbance data sequence of these variables are multiplied by 4. Hence, the variance and the size of the step disturbance increases.
(2) Increase in measured disturbance F_3 at t=300 min. The disturbance data sequence of this variable is increased by a factor 4.
(3) Increase in measurement noise at t=300 min. The noise sequence is increased by a factor 4.
(4) Change to a less sophisticated state estimator as an example of an online tuning attempt. The default state estimator (DMC State Estimator with an identity matrix relating unmeasured disturbances and states) of Matlab MPC Toolbox is used.
(5) Increase in model mismatch at t=300 min. Some elements of matrix B of the state space model are changed: $b_{1,3}$=0, $b_{3,3}$=0.00753. The control system has turned out to be fairly robust concerning changes in B. To get an effect that causes a large decrease in performance, matrix B is multiplied by 0.5.
(6) Decrease the saturation limit of P_{100} at t=300 min. The upper limit of P_{100} is decreased from 295 kPa to 195 kPa.

5.2 *CPM with Linear Plant Model*

The cases described above are assumed to occur one by one. The effects on γ_{hist}, γ_{des} and on manipulated variables are discussed as appropriate.

"In Control" Situation. Figure 2 shows γ_{hist} and the prediction residuals $e_{\gamma_{hist}}$. γ_{des} and $e_{\gamma_{des}}$ have similar trends. $P_C = 75$, hence γ_{hist} and γ_{des} can be calculated for $t > 75\ min$. The step change in performance measures at $t = 75\ min$ is statistically relevant, leading to a violation of control limits. Apart form this initialization effect, the residuals are in statistical control.

Increase in Unmeasured Disturbances at $t = 300$ min causes controller performance degradation as indicated by a decrease in γ_{hist} and the "out of control" signals for the residual (Fig. 3). γ_{des} and $e_{\gamma des}$ show similar changes indicating that the problem belongs to group II as expected.

Increase in Measured Disturbances decreases γ_{hist}. Because γ_{des} does not decrease, the cause of degradation belongs to group I. The trend of manipulated variables is observed and performance degradation due to constraints is ruled out since manipulated variables are not saturated.

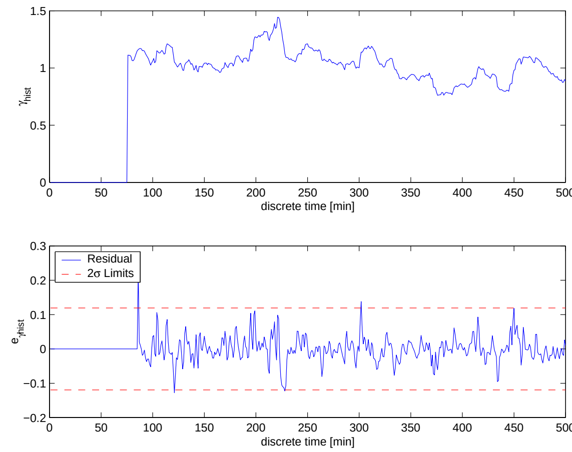

Fig. 2. γ_{hist} in an "In Control" Situation

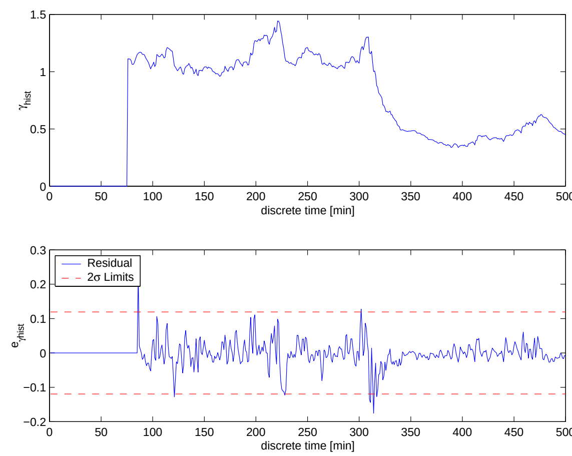

Fig. 3. Effect of Increase in Unmeasured Disturbances on γ_{hist}

Increase in Measurement Noise has a negative effect on performance reducing both γ_{hist} and γ_{des}. Because γ_{des} is affected as well, the root cause is identified as belonging to group II.

Change in the State Estimator has a negative effect on γ_{hist}, but γ_{des} is not affected indicating that the change in the estimator affects similarly the estimation accuracy of the design and achieved performance cases.

Increase in Model Mismatch. A change in the matrix relating the manipulated and controlled variables reduces γ_{hist}. Because γ_{des} is affected in a similar manner, the underlying problem is diagnosed correctly to belong to group II.

Decrease of the Saturation Limit. The saturation limit of P_{100} is set to zero at $t = 300\ min$. γ_{hist} indicates a performance degradation. γ_{des} does not decrease, hinting that the source cause of the degradation belongs to group I. To distinguish between measured disturbances, increase in the measurement noise and input saturation, the trend of the manipulated variables is observed (Fig. 4). The effect of input saturation can be seen clearly between $t = 300\ min$ and $t = 350$

min. After $t = 350$ *min* the MPC being aware of this limit tries to stay at the operation point by rearranging the use of manipulated variables.

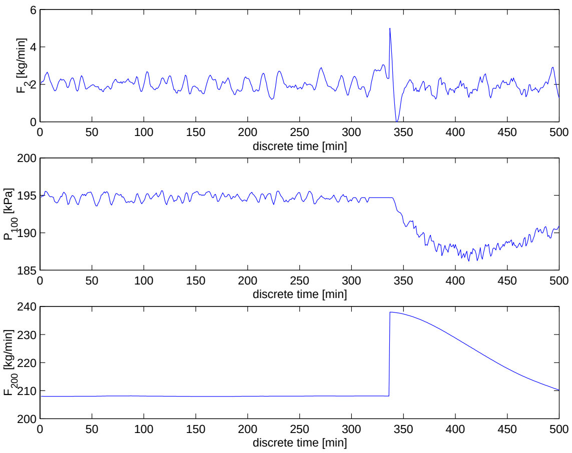

Fig. 4. Effect of Input Saturation on Manipulated Variables

5.3 *CPM with Nonlinear Plant Model*

The nonlinear plant model is used to test the historical benchmark and optimal tuning obtained earlier. Tuning parameter values (M=P=1) were questioned earlier. Wild oscillations occur in controlled variables when these parameters are used with the nonlinear plant (Fig. 5). Effects of changes in M and P on γ_{hist} (Fig. 6) indicate that P=15 and M=1 is better choice yielding $J_{hist} = 0.162$.

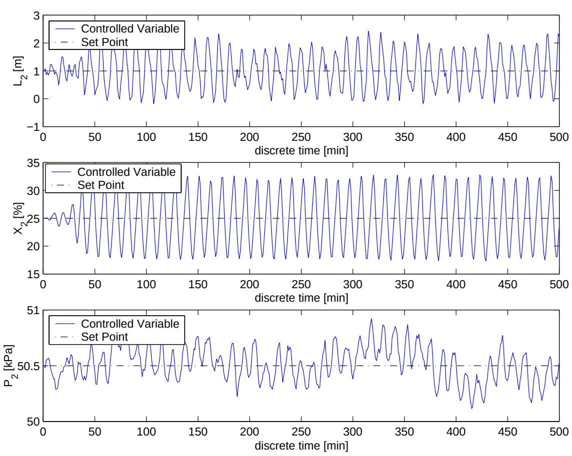

Fig. 5. Controlled Variables, M=P=1, NL model

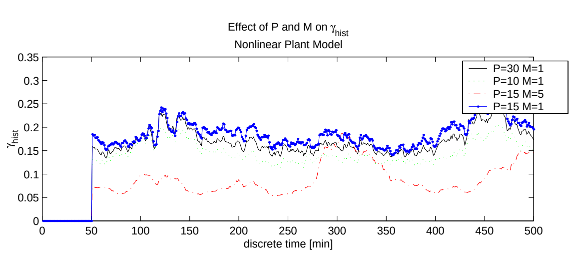

Fig. 6. γ_{hist} for Various M and P, NL model

"In Control" Situation. All measures are in statistical control with P=15 and M=1. $\gamma_{hist}(t) \approx 0.5$ and $\gamma_{des} \approx 0.6$ as compared to 1 in linear model case, indicating a model mismatch between the internal (linear) model and nonlinear plant. Table 2 summarizes the responses of the three indicators in case studies with linear and nonlinear models. Bold letters indicate differences between linear and nonlinear plant model results and d denotes decrease, i increase, n not affected, s saturated, and - not considered.

Table 2. CPM and Diagnosis Results

Case	γ_{hist}	γ_{des}	MVs exceed limit
1	d	d	-
2	d	n (**i**)	n
3	d	d	-(**n**)
4	d	n (**d**)	-
5	d	d	-
6	d	n (**i**)	s

For example, an increase in measured disturbance reduces γ_{hist}. In contrast to the linear model case, γ_{des} shows a statistically significant increase, indicating a reduction of the difference between J_{des} and J_{ach}. The diagnosis is identical, degradation due to constraints is ruled out by lack of saturation of manipulated variables.

6. CONCLUSIONS

For the linear model, assumption of known plant and disturbance model is valid. The integrated CPA, CPM and diagnosis techniques perform well in monitoring and diagnosis of MPC performance. Studies with the nonlinear plant model illustrate that use of the linearized model for obtaining a benchmark is not suitable.

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