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GPC FOR NON-UNIFORMLY SAMPLED SYSTEMS BASED ON THE LIFTED MODELS

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Abstract: In this paper, we study digital control systems with non-uniform updating and sampling patterns, which include multirate sampled-data systems as special cases. First, we derive lifted models in the state-space domain, and give a sufficient condition under which the lifted models preserve controllability and observability. The main obstacle for generalized predictive control (GPC) design using the lifted models is the so-called causality constraint. Taking into account this design constraint, we propose a new GPC algorithm, which results in optimal causal control laws for the non-uniformly sampled systems and applies immediately to multirate sampled-data systems.

Keywords: generalized predictive control (GPC), non-uniformly sampled systems, multirate control.

1. INTRODUCTION

Generalized predictive control (GPC) (Clarke *et al.* (1987); Rossiter (1993); Camacho and Bordons (1999)), has found wide applications in the process control industry. Most studies on GPC assume a single-rate sampling scheme, and the main purpose of this paper is to extend GPC algorithms to more general sampling and updating schemes. One extension from single-rate systems is the class of multirate systems. For the sampled-data system shown in Figure 1, a quite general multirate system is obtained by allowing S and H to operate at different rates, say, the sampling period for S is mh, and the updating period for H is nh, where m and n are integers and h is the *base period.*

The relevance and importance of multirate processes in the GPC/MPC (model predictive control) framework have been recognized by several researchers over the last decade, e.g., Lee *et al.* (1992), and Scattolini and Schiavoni (1995), just to name a few. Our approach to such problems is through the use of the lifting technique (Kranc (1957); Khargonekar *et al.* (1985)). However, the obstacle in using the lifting technique is the so-called *causality constraint* in the controller design (Chen and Qiu (1994)). To handle this causality constraint in the GPC design is one of the objectives of this paper.

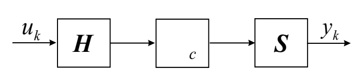

Fig. 1. A sampled-data system

Another objective of this paper is to study the non-uniformly sampled systems. There are three main reasons for such sampled systems to arise. First, in task-sharing situations, it is more reasonable and cost-effective to allow non-uniform sampling and updating operations. Second, the non-uniformly sampled systems are quite general

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and include multirate sampled-data systems as special cases. Third, there are advantages in non-uniformly sampled systems over uniformly sampled ones, e.g., the results by Kreisselmeier (1999).

Except the reference by Kreisselmeier (1999), research activities on non-uniformly sampled systems can also be found in the work by Salt *et al.* (1993), and Albertos and Salt (1999).

Briefly, the contributions in this paper are as follows:

- In Section 2, we derive a state-space lifted model for the non-uniformly sampled system in discrete time; a sufficient condition on sampling is given under which the state-space model is both controllable and observable.
- In Section 3, we present a causal solution to the GPC problem for non-uniformly sampled systems. To our best knowledge, this is the first causal and optimal solution proposed for the lifted models.
- Such a causal GPC solution in the lifted domain is *new* even in the special case of multirate systems in which the causality constraint has been the main difficulty in GPC/MPC design using the lifting framework, see, e.g., the work by Sheng *et al.* (2001).

For the rest of the paper, an illustrative example is presented in Section 4, and concluding remarks is given in Section 5.

2. MODELING OF NON-UNIFORMLY SAMPLED SYSTEMS

In the following, for the reason of simplicity, we will focus on the single-input, single-output (SISO) case. We also assume that the continuous-time process Σ_c in Figure 1 has the following state-space representation:

$$\Sigma_c : \begin{cases} \dot{x}(t) = Ax(t) + Bu(t), \\ y(t) = Cx(t) + Du(t), \end{cases} \tag{1}$$

where $x(t) \in \mathcal{R}^{n_c}$ is the state, $u(t) \in \mathcal{R}^1$ is the control input, and $y(t) \in \mathcal{R}^1$ is the output.

2.1 *The sampling and updating scheme*

Non-uniformly sampled systems are characterized by the fact that both the control updating instants (when u_k occur) and the sampling instants (when y_k occur) need not be equally spaced in time; however, for tractability we assume that the whole sampling and updating pattern is periodic over a larger interval T, known as the *frame period* (Salt *et al.* (1993); Albertos and Salt (1999)). The notation and arrangement with the sampling and updating are as follows:

- Over the k-th period $[kT, (k+1)T)$, we assume the control signal u is updated non-uniformly m times at time instants $kT + t_i$, $i = 1, 2, \cdots, m$. Without loss of generality, we can take $t_1 = 0$ and arrange $t_1 < t_2 < \cdots < t_m < T$.
- Over the period $[kT, (k+1)T)$, there are n_i ($n_i \geq 0$) output samples available within the time interval $[kT + t_i, kT + t_{i+1})$, $i = 1, 2, \cdots, m$ (denoting $t_{m+1} = T$); these n_i output samples occur at time instants $kT + t_i^j$, $j = 1, 2, \cdots, n_i$. Without loss of generality, we arrange these t_i^j in the following order:

$$t_i \leq t_i^1 < t_i^2 < \cdots < t_i^{n_i} < t_{i+1}.$$

Thus during each period of T, the control signal u is updated m times, and the output signal y is sampled $p = n_1 + n_2 + \cdots + n_m$ times, all non-uniformly. Such a sampling and updating scheme is briefly illustrated in Figure 2. We remark that

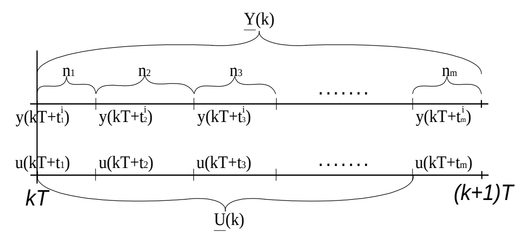

Fig. 2. The non-uniform sampling and updating scheme

the non-uniform sampling and updating scheme introduced is quite general: Compared with the one used by Albertos and Salt (1999), we do not assume that the sampling and updating instants are integer multiples of some based period.

2.2 *Lifted models and the causality constraint*

Looking at the system discussed earlier in discrete time, if we group every m input values and every p output samples together, we will have a $p \times m$ LTI system operating over period T; this is the idea of lifting. Let

$$v = \{v_0, v_1, v_2, \cdots\}, \tag{2}$$

$$\underline{v} = \left\{ \begin{bmatrix} v_0 \\ v_1 \\ \vdots \\ v_{n-1} \end{bmatrix}, \begin{bmatrix} v_n \\ v_{n+1} \\ \vdots \\ v_{2n-1} \end{bmatrix}, \cdots \right\}, \tag{3}$$

where v_k is a discrete-time signal, and $\underline{v}$ is an n-fold signal, with n a positive integer. The map from v to $\underline{v}$ is defined as the lifting operator L_n. The inverse lifting operation, L_n^{-1}, is from $\underline{v}$ to v, defined in the obvious way.

Note that all the lifted signals in this paper are underlined. The following notation is used throughout the paper: Continuous-time signals evolve over time t closed by the round brackets, e.g., $u(t)$ and $y(t)$ in (1); discrete-time signals evolve over time k (integer valued) which appears in subscripts, e.g., v_k in (2). This convention applies to non-uniformly sampled signals, e.g., y_k is $y(t)$ sampled at the k-th sampling instant.

For the discussed non-uniformly sampled system, lifting u_k by L_m and y_k by L_p, we get $\underline{u}_k$ and $\underline{y}_k$, corresponding to inputs and outputs over the interval $[kT, (k+1)T)$:

$$\underline{u}_k = \left[\, u(kT+t_1)\;\; u(kT+t_2)\;\; \cdots\;\; u(kT+t_m)\,\right]^T,$$

$$\underline{y}_k = \begin{bmatrix} n_1 \left\{ \begin{array}{c} y(kT+t_1^1) \\ \vdots \\ y(kT+t_1^{n_1}) \end{array} \right. \\ \vdots \\ n_m \left\{ \begin{array}{c} y(kT+t_m^1) \\ \vdots \\ y(kT+t_m^{n_m}) \end{array} \right. \end{bmatrix}.$$

Then the lifted system, Σ_l, from $\underline{u}_k$ to $\underline{y}_k$, has m inputs and p outputs. Furthermore, it admits a state-space realization in terms of the given model Σ_c in (1); hence Σ_l is LTI, an advantage of lifting. See Proposition 1 below.

Proposition 1. A state-pace model for the lifted system Σ_l is given by

$$\Sigma_l : \left\{ \begin{array}{rcl} x_{k+1} &=& \underline{A}x_k + \underline{B}\,\underline{u}_k, \\ \underline{y}_k &=& \underline{C}x_k + \underline{D}\,\underline{u}_k, \end{array} \right. \tag{4}$$

where $x_k := x(kT)$, and

$$\underline{A} = \mathrm{e}^{AT}, \quad \underline{B} = \left[\, B_1\;\; B_2\;\; \cdots\;\; B_m \,\right], \tag{5}$$

$$\underline{C} = \begin{bmatrix} C_1 \\ C_2 \\ \vdots \\ C_m \end{bmatrix}, \quad \underline{D} = \begin{bmatrix} D_1^1 & & & \\ D_2^1 & D_2^2 & & \\ \vdots & \vdots & \ddots & \\ D_m^1 & D_m^2 & \cdots & D_m^m \end{bmatrix}, \tag{6}$$

with

$$B_i = \int_{T-t_{i+1}}^{T-t_i} \mathrm{e}^{A\tau} B d\tau, \quad C_i = \begin{bmatrix} C\mathrm{e}^{At_i^1} \\ \vdots \\ C\mathrm{e}^{At_i^{n_i}} \end{bmatrix},$$

$$D_i^i = \begin{bmatrix} D_i^i(1) \\ \vdots \\ D_i^i(n_i) \end{bmatrix}, \quad D_i^i(k) = \int_0^{t_i^k - t_i} C\mathrm{e}^{A\tau} B d\tau + D,$$

$$D_i^j = \begin{bmatrix} D_i^j(1) \\ \vdots \\ D_i^j(n_i) \end{bmatrix}, \quad D_i^j(k) = \int_{t_i^k - t_{j+1}}^{t_i^k - t_j} C\mathrm{e}^{A\tau} B d\tau.$$

Here $i = 1, 2, \cdots, m$ in the definition of B_i, C_i and D_i^i; $i = 2, \cdots, m,\ 1 \leq j < i$ in the definition of D_i^j; and $k = 1, \cdots, n_i$ in the definition of D_i^i and D_i^j.

Note that the upper triangular blocks in $\underline{D}$ in (6) are zero; this represents the so-called *causality constraint* in Σ_l: Certain blocks in the direct feedthrough term must be zero to satisfy causality. For model Σ_l derived from the continuous-time model Σ_c, this constraint is automatically satisfied. However, in the lifted controller design, this constraint on the controller poses a difficult problem.

Proposition 2. The direct feedthrough terms in the lifted controllers are $m \times p$ matrices mapping $\underline{y}_k$ to $\underline{u}_k$, and must satisfy the *causality constraint*, which takes the following block lower triangular structure:

$$\begin{bmatrix} \overbrace{a_1\;\; 0\;\; \cdots}^{n_1} & & & \\ \overbrace{\times\; \cdots\; \times}^{n_1} & \overbrace{a_2\;\; 0\;\; \cdots}^{n_2} & & \\ \ddots & \ddots & & \\ \cdots & \overbrace{\times\; \cdots\; \times}^{n_{m-1}} & \overbrace{a_m\;\; 0\;\; \cdots}^{n_m} \end{bmatrix}. \tag{7}$$

Here the upper triangular blocks are all zero, $\times$ means a designable element (no restriction). If $t_i^1 = t_i$, $a_i = \times$; otherwise, $a_i = 0$.

This causality constraint must be satisfied by all lifted controllers. We will handle this constraint in the GPC design for non-uniformly sampled systems later in this paper.

2.3 *Controllability and observability*

For the lifted model Σ_l in Section 2.2, a natural question is: Under what condition this model is controllable and observable? To answer this problem, we first assume controllability and observability of the continuous-time model Σ_c in (1). Then we will give a sufficient condition for model Σ_l to preserve the two properties. See the following Theorem.

Theorem 3. In the discretization process from Σ_c in (1) to Σ_l in Proposition 1, assume the frame period T is non-pathological (Chen and Francis (1995)). Then

(1) $(\underline{A}, \underline{B})$ is controllable if (A, B) is controllable;
(2) $(\underline{C}, \underline{A})$ is observable if (C, A) is observable.

Note that this sufficient condition also guarantees that the uniformly sampled system with period

T is controllable and observable. A question may arise: Do non-uniformly sampled systems have any advantage over uniformly sampled systems in preserving controllability and observability? The answer is positive. Looking at controllability, for example, model Σ_c is controllable with $A = \begin{bmatrix} 0 & -\pi \\ \pi & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$. If the frame period T is taken to be 3 sec, and $m = 3$. It can be verified that this T is pathological. Under the uniform updating pattern, i.e., $t_1 = 0$, $t_2 = 1$, and $t_3 = 2$, the lifted pair $(\underline{A}, \underline{B})$ is clearly uncontrollable by Proposition 1. However, if we keep $T = 3$ and $m = 3$ and use a non-uniform updating pattern, say, $t_1 = 0$, $t_2 = 0.8$, and $t_3 = 1.2$, then $(\underline{A}, \underline{B})$ turns out to be controllable.

3. GPC ALGORITHM FOR NON-UNIFORMLY SAMPLED SYSTEMS

In this section, we study the GPC design problem for the non-uniformly sampled systems discussed in the preceding section. We will see that conventional GPC algorithms fail to provide causal control laws; thus we propose a new GPC solution, taking into account the causality constraint in (7).

In view of the lifted model in (4), the GPC design is to minimize a cost function of the form

$$J_k = \sum_{i=1}^{n_y} [w_{k+i} - \hat{\underline{y}}_{k+i|k}]^T [w_{k+i} - \hat{\underline{y}}_{k+i|k}] + \lambda \sum_{i=1}^{n_u} \Delta \underline{u}_{k+i-1}^T \Delta \underline{u}_{k+i-1}, \quad (8)$$

by computing the incremental control moves $\Delta \underline{u}_{k+i}$ for $i = 0, 1, \cdots, n_u - 1$, subject to the condition that $\Delta \underline{u}_{k+i} = 0$ for $i = n_u, n_u + 1, \cdots, n_y$. Here $\Delta = 1 - q^{-1}$, and q^{-1} is the backward shift operator. The vector sequence w_{k+i} is the output tracking reference; $\hat{\underline{y}}_{k+i|k}$ is the i-step ahead prediction of the future lifted output at present time k. The minimum and maximum prediction horizons are 1 and n_y, respectively; n_u is the control horizon. The weighting for the error signal between w and $\hat{\underline{y}}$ is an identity matrix, and for the lifted incremental control signal is a constant diagonal matrix λI. For simplicity, in the following, we assume $n_y = n_u = n$.

To avoid estimating the state vector, we will adopt the input-output GPC design as that by Rossiter (1993). Including an integrator $1/\Delta$ to the lifted model in (4), we can obtain a transfer function representation from $\Delta \underline{u}$ to $\underline{y}$:

$$\underline{y}_k = \frac{N(q^{-1})}{d(q^{-1})} \Delta \underline{u}_k, \quad (9)$$

where $d(q^{-1})$ is the common denominator of the form

$$d(q^{-1}) = 1 + d_1 q^{-1} + \cdots + d_l q^{-l} \quad (10)$$

(assuming the order of the system involved is l); $N(q^{-1})$ is a $p \times m$ matrix polynomial of the form

$$N(q^{-1}) = \begin{bmatrix} N_{11}(q^{-1}) & \cdots & N_{1m}(q^{-1}) \\ \vdots & \vdots & \vdots \\ N_{p1}(q^{-1}) & \cdots & N_{pm}(q^{-1}) \end{bmatrix} \quad (11)$$

with each element being an l-th order polynomial:

$$N_{ij}(q^{-1}) = N_{ij}^0 + N_{ij}^1 q^{-1} + \cdots + N_{ij}^l q^{-l}. \quad (12)$$

In the following, we will omit q^{-1} in the polynomials, if no confusion will arise.

3.1 *Conventional GPC design*

First, following the work by Rossiter (1993), we review how the GPC solution is derived for model (9), without considering the causality constraint.

Rewrite (9) as follows:

$$\underline{y}_k = \sum_{j=0}^{l} N_j \Delta \underline{u}_{k-j} - \sum_{j=1}^{l} D_j \underline{y}_{k-j}. \quad (13)$$

Here, based on (11) and (10), we have

$$N_i = \begin{bmatrix} N_{11}^i & \cdots & N_{1m}^i \\ \vdots & \cdots & \vdots \\ N_{p1}^i & \cdots & N_{pm}^i \end{bmatrix}, \; D_i = \begin{bmatrix} d_i & & \\ & \ddots & \\ & & d_i \end{bmatrix}_{p \times p}.$$

Then for the MIMO system in (13), we can write out the i-step ahead prediction of future output at current time k, namely, $\hat{\underline{y}}_{k+i|k}$. Letting i vary from 1 to n; minimizing J_k with respect to the future control sequence $\Delta U = \begin{bmatrix} \Delta \underline{u}_k^T & \cdots & \Delta \underline{u}_{k+n-1}^T \end{bmatrix}^T$, we can obtain the optimal solution ΔU. While in the frame period $[kT, (k+1)T)$, only the first element, i.e., $\Delta \underline{u}_k$, will be implemented:

$$\Delta \underline{u}_k = \mathbf{K}(W - P_2 \Delta U_p - P_3 Y_p). \quad (14)$$

Here we have defined
$\mathbf{K} = \begin{bmatrix} I_{m \times m} & 0 & \cdots & 0 \end{bmatrix} (P_1^T P_1 + \lambda I)^{-1} P_1^T$,
$W = \begin{bmatrix} w_{k+1}^T & \cdots & w_{k+n}^T \end{bmatrix}^T$,
$\Delta U_p = \begin{bmatrix} \Delta \underline{u}_{k-1}^T & \cdots & \Delta \underline{u}_{k-l+1}^T \end{bmatrix}^T$,
$Y_p = \begin{bmatrix} \underline{y}_k^T & \cdots & \underline{y}_{k-l+1}^T \end{bmatrix}^T$,
and P_1, P_2, P_3 are big matrices in terms of N_i, D_i or linear combinations of them.

Since the first element of Y_p is $\underline{y}_k$, the direct feedthrough term from $\underline{y}_k$ to $\Delta \underline{u}_k$, say, $\underline{D}_c$, is thus the first p columns of matrix $\mathbf{K}P_3$. Clearly, this $\underline{D}_c$ is a general $m \times p$ matrix. There is no guarantee that it has the block lower triangular structure as shown in (7). So such a conventional GPC solution is not implementable in real time.

3.2 *Proposed GPC design*

To obtain causal GPC control law, the idea we adopt is as follows: During every frame period T, construct a chain of new lifted signals and models, corresponding to the control moves; then apply the conventional GPC design to every one of them.

In our proposed algorithm, during the k-th frame period $[kT, (k+1)T)$, each control move will be calculated separately; at each time instant $t = kT + t_i$, we will construct a corresponding lifted output $\underline{y}_k^i$, consisting of the most recent p measurements (some elements in $\underline{y}_k^i$ are in fact measured in the last frame period), where all the elements are listed in their order of occurrence. It is relatively easy to derive a model from $\Delta \underline{u}_k$ to $\underline{y}_k^i$ (denoted by M^i). Thus, in one frame period T, a chain of m models are obtained.

Then we can solve the standard GPC problem using each model M^i and a similar cost function as in (8) to get the optimal $\Delta \underline{u}_k^i$, while implementing $\Delta u^i(kT+t_i)$ only. As time goes (i increases from 1 to m), the actual lifted incremental control signal implemented in the k-th frame period is

$$\Delta \underline{u}_k^\star = \begin{bmatrix} \Delta u^1(kT+t_1) \\ \Delta u^2(kT+t_2) \\ \vdots \\ \Delta u^m(kT+t_m) \end{bmatrix}. \qquad (15)$$

It is clear from the discussions that the proposed GPC control law is causal because of the way the chain of new lifted output signals and new models are constructed, and therefore can be implemented in real time. Furthermore, closed-loop expression for the optimal and causal control in (15) can be derived.

Consider the frame interval $[kT, (k+1)T)$, based on (14), $\Delta u^i(kT+t_i)$ in (15) has the following form

$$\Delta u^i(kT+t_i) = K_1^i W + K_2^i \Delta U_p + K_3^i Y_p^i, \qquad (16)$$

where $Y_p^i = \left[\underline{y}_k^{i\,T} \cdots \underline{y}_{k-l+1}^{i\quad T} \right]^T$, K_1^i, K_2^i, and K_3^i are $1 \times pn$, $1 \times m(l-1)$, and $1 \times pl$ matrices, respectively, all depending on the i-th model M^i.

Assume that the future reference signal is a constant w along the horizon, then equation (16) can be written as

$$\Delta u^i(kT+t_i) = F^i w + G^i \Delta \underline{u}_k^\star + H^i \underline{y}_k^i,$$

where F^i, G^i, and H^i all are matrices in terms of K_1^i, K_2^i and K_3^i, respectively.

Let i vary from 1 to m, and note that

$$\underline{y}_k^i = O^i \underline{y}_k, \qquad (17)$$

$$O^i = \left[\begin{array}{c|c} 0 & diag\{q^{-1}, \cdots, q^{-1}\}_{x \times x} \\ \hline I_{y \times y} & 0 \end{array} \right], \qquad (18)$$

where $y = n_0 + n_1 + \cdots + n_{i-1}$ with $n_0 = 1$, $x = p - y$; and we have assumed that $t_i^1 = t_i$, i.e., the output signal is available right at the time instant $kT + t_i$, when the control input is updated. Thus we obtain the closed-loop expression for the optimal control in (15):

$$\Delta \underline{u}_k^\star = \Psi^{-1} \cdot \left[\Theta w + \Xi \underline{y}_k \right], \qquad (19)$$

where Ψ, Θ, and Ξ are matrices in terms of G^i, F^i and $H^i O^i$, respectively. The result is summarized in the Theorem below.

Theorem 4. Assume that the future reference signal is a constant w along the horizon. The optimal lifted control law is given by its closed-loop form in (19), where the term from $\underline{y}_k$ to $\Delta \underline{u}_k^\star$, $\Psi^{-1}\Xi$, always satisfies the causality structure in (7).

4. EXAMPLE

In this section, we illustrate our proposed GPC algorithm by an example. For a SISO continuous-time model $G(s)$ with a minimal realization

$$A = \begin{bmatrix} -0.075 & -0.0003 \\ 1.000 & 0 \end{bmatrix}, \ B = \begin{bmatrix} 1 \\ 0 \end{bmatrix},$$
$$C = \begin{bmatrix} 0.0039 & 0.0028 \end{bmatrix}, \ D = 0,$$

we assume that the control signal is updated every 8 sec, while the output signal is sampled every 12 sec; thus the frame period is $T = 24$ sec. Such a multirate system is a special case of the non-uniformly sampled systems discussed in this paper, so all the results in Sections 2 and 3 are directly applicable.

First, by applying Proposition 1, it is easy to check that the lifted pair $(\underline{A}, \underline{B})$ is controllable and $(\underline{C}, \underline{A})$ is observable. This is also true by Theorem 3, because $T = 24$ sec is non-pathological.

Next we will design a multirate GPC controller for the uniformly sampled system. Notice here that in one frame period $T = 24$ sec, the control input is updated 3 times, and according to the results in Section 3.2, a chain of 3 lifted models should be defined. However, in this example (and many industrial processes), the output is sampled at a slower rate: The first sample is taken at $t = kT$, and the next is not available until time

instant $t = kT + 12$. Thus the first two incremental control moves, $\Delta u(kT)$ and $\Delta u(kT+8)$, can be computed together in one time; so only two lifted models: M^1 and M^2 need to be used. And
$\underline{y}_k^1 = \left[\, y(kT-12)\ \ y(kT)\, \right]^T$,
$\underline{y}_k^2 = \left[\, y(kT)\ \ y(kT+12)\, \right]^T$,
$\Delta \underline{u}_k^\star = \left[\, \Delta u^1(kT)\ \ \Delta u^1(kT+8)\ \ \Delta u^2(kT+16)\, \right]^T$.

The purpose of the GPC design is to minimize the cost function in (8), where the tuning parameters are n_y, n_u and λ. If we choose $n_y = 6$, $n_u = 5$, $\lambda = 0.1$, the tracking performance of the closed loop with the multirate GPC controller can be simulated, see the solid lines in Figure 3. The simulation time is 800 sec, and the setpoint

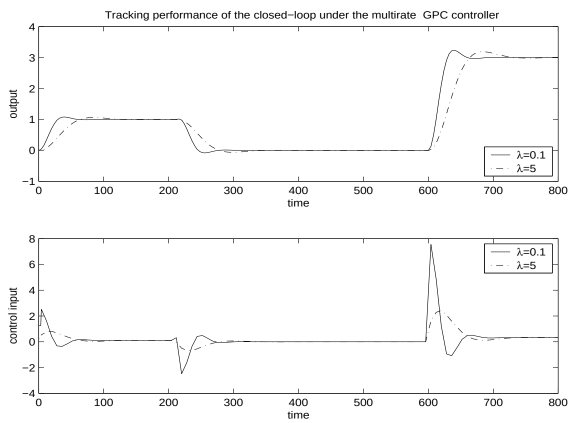

Fig. 3. Tracking performance of the closed-loop with the multirate GPC controller

changes from 0 to 1 at $t = 0$, from 1 to 0 at $t = 203$, and from 0 to 3 at $t = 600$.

Tuning the parameters n_y, n_u and λ can affect tracking performance and control effort. For example, increasing λ – the weighting factor on the control signal – from 0.1 to 5 will result in decrease in the maximum value of the control input, but the price paid is that the tracking becomes more sluggish, see the dash-dot lines in Figure 3.

Finally, with the tuning parameters $n_y = 6$, $n_u = 5$, and $\lambda = 0.1$, we can compute the closed-loop form of the multirate GPC controller. According to (16) to (19), the direct feedthrough term from $\underline{y}_k$ to $\Delta \underline{u}_k^\star$, $\Psi^{-1}\Xi$, is computed to be

$$\Psi^{-1}\Xi = \begin{bmatrix} 1.1321 & 0 \\ 0.0369 & 0 \\ -1.5729 & 1.4937 \end{bmatrix}.$$

This matrix is block lower triangular, satisfying the causality constraint in this case.

5. CONCLUSIONS

In this paper, we studied non-uniformly sampled systems, which are characterized by non-uniform but periodic updating and sampling patterns. Lifted and LTI model were derived, and a sufficient condition was given for the lifted models to inherit controllability and observability from continuous time.

We also dealt with the GPC problems for non-uniformly sampled systems. Starting from the conventional GPC design for MIMO systems, we proposed an algorithm which can handle the so-called causality constraint on lifted controllers in the GPC design.

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