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FUNDAMENTAL PROPERTIES OF RESET CONTROL SYSTEMS [1,2]

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Abstract: Reset controllers are linear controllers that reset some of their states to zero when their input is zero. We are interested in their feedback connection with linear plants, and in this paper we establish fundamental closed-loop properties. This paper considers more general reset structures than previously considered, allowing for higher-order controllers and partial-state resetting. It gives a testable necessary and sufficient condition for quadratic stability and links it to uniform bounded-input bounded-output state stability. Unlike previous related research, which includes the study of impulsive differential equations, our stability results require no assumptions on the evolution of reset times.

Keywords: reset actions, stability analysis, nonlinear control systems

1. INTRODUCTION

In this paper we study the control system depicted in Figure 1 which consists of a *reset controller* R connected in feedback with a plant transfer function $P(s)$. [3] A reset controller is a linear time-invariant system whose states, or subset of states, *reset* to zero when the controller input e is zero. Motivation for reset control comes from two sources. First, from the limitations of linear feedback control systems imposed by Bode's gain-phase relationship. Second, from the favorable sinusoidal describing function of reset controllers which promise relief from Bode's constraint. Indeed, a reset integrator, also referred to as a Clegg integrator, has a describing function similar to the frequency response of a linear integrator but with only 38.1° phase lag; see [1]. The purpose of this paper is to report on some fundamental properties of these reset control systems and complements the papers [2] – [5] which show, either through theory, simulation or experiment, the potential benefit of reset control.

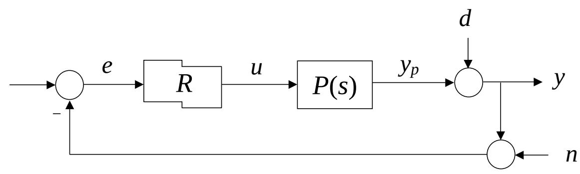

Fig. 1. Block diagram of a reset control system.

Before we discuss previous research, we first give a simple illustration of reset control. Consider the feedback system in Figure 1 with plant

$$P(s) = \frac{s+1}{s(s+0.2)}.$$

We take the reset controller to be a first-order filter $\frac{1}{s+1}$ whose state x_r resets (to zero) whenever

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[3] They are not to be confused with integral action in process control, which is also referred to as reset control.

the loop error is zero; i.e., $e(t) = 0$.[4] We can describe this reset controller by the impulsive differential equation:

$$\begin{aligned} \dot{x}_r(t) &= -x_r(t) + e(t), \quad & e(t) \neq 0; \\ x_r(t^+) &= x_r(t), & e(t) = 0; \\ u(t) &= x_r(t). \end{aligned}$$

If this first-order filter is not allowed to reset, then, the resulting linear closed-loop system responds to a unit step reference signal $r(t)$ as shown in the top plot of Figure 2. The response, when the filter does reset, is shown in the middle plot, while the last plot shows the reset controller's output u. The introduction of reset decreases the overshoot and settling time without sacrificing rise time.

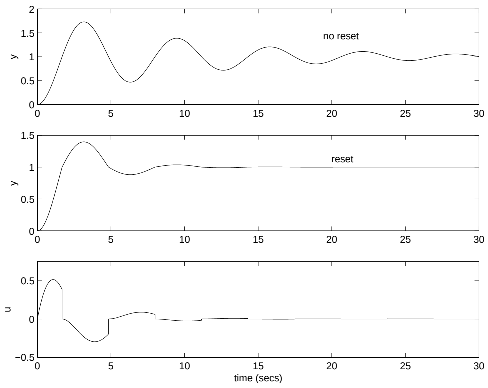

Fig. 2. Step response y of the linear control system (top), reset control system (middle) and reset control action u (bottom).

The present paper, which reports on results from [8], provides a summary of fundamental properties of reset control. It considers more general reset structures than previously considered, allowing for higher-order controllers and partial-state resetting. The paper shows the previously mentioned H_β-condition to be necessary and sufficient for *quadratic stability* and links it to uniform bounded-input bounded-output state (UBIBS) stability. It also identifies a non-trivial class of reset control systems that is quadratically stable. Another contribution of this work is the removal of all assumptions on reset times. Such assumptions were required in previous analysis and also appear in the study of impulsive differential equations (IDEs); e.g., see [9] and [10].

The paper is organized as follows: In Section 2 we set up the reset control problem by expressing the dynamics of reset in terms of impulsive differential equations. Section 3 presents our main results where we state Lyapunov-based conditions for closed-loop stability, give a necessary and sufficient condition for quadratic stability and show that quadratic stability implies UBIBS stability. In Section 4 we identify a non-trivial class of reset control systems that are always quadratically stable, and, as a result, are input-output stable. For brevity, proofs of results are omitted and are available in the more complete version of this paper [11].

[4] In the literature, this simple reset controller is referred to as a *first-order reset element* (FORE).

2. SETUP

The reset control system considered in this paper is shown in Figure 1 where the reset controller R is described by the IDE

$$\begin{aligned} \dot{x}_r(t) &= A_r x_r(t) + B_r e(t), \quad & e(t) \neq 0; \\ x_r(t^+) &= A_\varrho x_r(t), & e(t) = 0; \\ u(t) &= C_r x_r(t) & (1) \end{aligned}$$

and where $x_r(t)$ is the reset controllers state, $u(t)$ its output, $A_r \in \mathbb{R}^{n_r \times n_r}$, $B_r \in \mathbb{R}^{n_r \times 1}$ and $C_r \in \mathbb{R}^{1 \times n_r}$. The matrix $A_\varrho \in \mathbb{R}^{n_r \times n_r}$ selects the states to be reset. Without loss of generality we assume the block diagonal form $A_\varrho = \begin{bmatrix} I_{n_{\bar{\varrho}}} & 0 \\ 0 & 0_{n_\varrho} \end{bmatrix}$ where n_ϱ (of the n_r controller states) are reset. Examples of reset controllers include the Clegg Integrator (CI):

$$A_r = 0; \quad B_r = 1; \quad C_r = 1; \quad A_\varrho = 0$$

and the First-Order Reset Element (FORE):

$$A_r = -b; \quad B_r = 1; \quad C_r = 1; \quad A_\varrho = 0. \qquad (2)$$

In both of these cases, $n_\varrho = n_r = 1$. We assume that plant $P(s)$ in Figure 1 accounts for any linear pre-compensation. In fact, the design of reset control systems as developed in [3] involves the synthesis of both linear compensator $C(s)$ and reset controller R. Typically, the linear compensator is used to stabilize and shape the loop to satisfy classical Bode specifications at high and low frequencies. The reset controller is then designed to meet overshoot constraints. We assume $P(s)$ strictly proper and adopt the realization:

$$\begin{aligned} \dot{x}_p(t) &= A_p x_p(t) + B_p u(t); \\ y_p(t) &= C_p x_p(t) \end{aligned}$$

where $A_p \in \mathbb{R}^{n_p \times n_p}$, $B_p \in \mathbb{R}^{n_p \times 1}$ and $C_p \in \mathbb{R}^{1 \times n_p}$. The closed-loop reset control system can then be described by the IDE

$$\begin{aligned} \dot{x}(t) &= A_{c\ell} x(t) + B_{c\ell} w(t), \quad & x(t) \notin \mathcal{M}(t); \\ x(t^+) &= A_R x(t), & x(t) \in \mathcal{M}(t); \\ y(t) &= C_{c\ell} x(t) + d(t); & (3) \\ e(t) &= w(t) - C_{c\ell} x(t) \end{aligned}$$

where

$$x \triangleq [\, x_p' \; x_r' \,]' ,$$
$$A_{c\ell} \triangleq \begin{bmatrix} A_p & B_p C_r \\ -B_r C_p & A_r \end{bmatrix}, \quad B_{c\ell} \triangleq \begin{bmatrix} 0 \\ B_r \end{bmatrix},$$
$$A_R \triangleq \begin{bmatrix} I & 0 \\ 0 & A_\varrho \end{bmatrix},$$
$$C_{c\ell} \triangleq [\, C_p \; 0 \,]$$

and $w(t) \triangleq r(t) - n(t) - d(t)$. The so-called *reset surface* $\mathcal{M}(t)$ defines those states triggering reset and is formally defined by

$$\mathcal{M}(t) = \left\{ \xi \in \mathbb{R}^{n_p+n_r} \colon \; e(t) = 0, (I - A_R)\xi \neq 0 \right\}. \tag{4}$$

If $x(t_*) \in \mathcal{M}(t_*)$, then $x(t_*)$ is called a *reset state* and t_* a *reset time*. From (4) they satisfy

$$x(t_*) \in \mathcal{M}(t_*) \quad \Rightarrow \quad x(t_*^+) \notin \mathcal{M}(t_*^+).$$

Thus, we can collect these times in the ordered set

$$\mathcal{T}(x_0) \triangleq \{ t_i \in \mathbb{R}_+ \colon \; t_i < t_{i+1}; \\ x(t_i) \in \mathcal{M}(t_i), \; i \in \tilde{\mathbb{N}} \subseteq \mathbb{N} \}$$

which emphasizes that reset times depend on initial conditions as well as the exogenous signal $w(t)$. Finally, the *reset intervals* τ_i are defined as

$$\tau_1 \triangleq t_1;$$
$$\tau_{i+1} \triangleq t_{i+1} - t_i, \quad i \in \tilde{\mathbb{N}} \subseteq \mathbb{N}.$$

In absence of resetting; i.e., when $A_R = I$, the resulting closed-loop system $C_{c\ell}(sI - A_{c\ell})^{-1} B_{c\ell}$ is called the *base-linear system*. We define the loop transfer function as

$$L(s) = P(s) R_{b\ell}(s)$$

where $R_{b\ell}(s) = C_r(sI - A_r)^{-1} B_r$ is the transfer function associated with reset controller R in the absence of resetting. For example, for CI, $R_{b\ell}(s) = \frac{1}{s}$ and for FORE, $R_{b\ell}(s) = \frac{1}{s+b}$. Associated with the base-linear system are its sensitivity function $S(s) = \frac{1}{1+L(s)}$ and the complementary sensitivity $T(s) = \frac{L(s)}{1+L(s)}$.

3. STABILITY

In this section we establish internal stability of (3) by giving a necessary and sufficient condition (called the "H_β-condition") for the existence of a quadratic Lyapunov function (quadratic stability). The H_β-condition is a strict positive real (SPR) constraint on the base-linear system and amounts to a requirement over and above base-linear stability. This is significant in light of examples demonstrating that reset can destabilize a stable base-linear system; e.g., see [2]. Moreover, the H_β-condition appears to have non-trivial application. In Section 4 we prove that a large class of reset control systems satisfy the H_β-condition. We will wrap-up this section by showing that quadratic stability implies UBIBS stability of (3).

3.1 *Preliminaries*

Consider the unforced version of (3) described by the autonomous IDE

$$\dot{x}(t) = A_{c\ell} x(t), \quad x(t) \notin \mathcal{M}, \quad x(0) = x_0;$$
$$x(t^+) = A_R x(t), \quad x(t) \in \mathcal{M} \tag{5}$$

where

$$\mathcal{M} = \left\{ \xi \in \mathbb{R}^{n_p+n_r} \colon \; C_{c\ell}\xi = 0, (I - A_R)\xi \neq 0 \right\}.$$

Our first theorem gives a general Lyapunov-like stability condition similar to those in [9] and [10].

Theorem 1. Let $V(x) : \mathbb{R}^n \to \mathbb{R}$ be a continuously-differentiable, positive-definite, radially-unbounded function such that

$$\dot{V}(x) \triangleq \left[\frac{\partial V}{\partial x} \right]' A_{c\ell} x < 0, \qquad x \neq 0; \tag{6}$$
$$\Delta V(x) \triangleq V(A_R x) - V(x) \leq 0, \quad x \in \mathcal{M}. \tag{7}$$

Then,

(*i*) there exists a left-continuous function $x(t)$ satisfying (5) for all $t \geq 0$,
(*ii*) the equilibrium point $x = 0$ is globally uniformly asymptotically stable. ◇

In the next subsection we specialize to quadratic Lyapunov function $V(x) = x'Px$. This leads to a tight, easily tested stability condition.

3.2 *Quadratic stability*

We state one of our main results which gives a necessary and sufficient condition, called the H_β-condition, for (5) to possess a quadratic Lyapunov function.

Definition 2. The reset control system (5) is said to satisfy the H_β-condition if there exists a $\beta \in \mathbb{R}^{n_\varrho}$ such that

$$H_\beta(s) \triangleq \left[\; \beta C_p \quad 0_{n_{\bar{\varrho}}} \quad I_{n_\varrho} \; \right] (sI - A_{c\ell})^{-1} \begin{bmatrix} 0 \\ 0'_{n_{\bar{\varrho}}} \\ I_{n_\varrho} \end{bmatrix} \tag{8}$$

is strictly positive real [5] where I_{n_ϱ} denotes an identity matrix of size $n_\varrho \times n_\varrho$ and $0_{n_{\bar{\varrho}}}$ denotes a matrix of zeros of size $n_\varrho \times n_{\bar{\varrho}}$. ◁

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[5] A square transfer function matrix $X(s)$ is called *strictly positive real* if *i)* $X(s-\varepsilon)$'s elements are analytic in $Re[s] > 0$ and *ii)* $X^\dagger(s-\varepsilon) + X(s-\varepsilon) \geq 0$ for $Re[s] > 0$ for some $\varepsilon > 0$. Here $\dagger$ denotes the complex conjugate transpose.

We now specialize to quadratic Lyapunov functions.

Definition 3. The reset control system (5) is said to be *quadratically stable* if there exists a positive-definite symmetric matrix P such that $V(x) = x'Px$ satisfies conditions (6) and (7). $\triangleleft$

We now state our quadratic stability result.

Theorem 4. The reset control system (5) is quadratically stable if and only if it satisfies the H_β-condition. $\diamond$

Theorem 4 gives an easily testable condition for quadratic stability. We illustrate with a simple example.

Example 5. Consider the unforced reset control system (5) with $P(s) = \frac{1}{s}$ and $R_{b\ell}(s) = \frac{1}{s+1}$. To check the H_β-condition we use (8) and form the stable transfer function

$$H_\beta(s) = \frac{s+\beta}{s^2+s+1}.$$

Given any $\beta \in (0,1)$, it is easy to show that the real part of $H_\beta(j\omega)$ is positive for all $\omega > 0$. Hence, the system is quadratically stable from Theorem 4. A quadratic Lyapunov function verifying (6) and (7) is

$$V(x) = x' \begin{bmatrix} 0.7282 & -0.3953 & 0.1128 \\ -0.3953 & 0.8691 & 0 \\ 0.1128 & 0 & 1 \end{bmatrix} x.$$

$\triangleright$

Remark 6. In this remark we address the prevalence of quadratically-stable reset control systems.

1. Some stable reset control systems are not quadratically stable. For example, in [12], it was shown that a Clegg integrator exponentially stabilizes the plant
$$P(s) = \frac{(3+\alpha)s+1}{s^2+3s-\alpha}$$
for $\alpha \in (-6.1, 1.1)$. However, computation shows the H_β-condition is only satisfied for the smaller range $\alpha \in (-3, 0)$. Thus, there is a difference between the classes of reset control systems that are stable and quadratically stable. It is interesting to note that the quadratically stable systems in this example exactly coincide to those $P(s)$'s that are both minimum phase and stable.
2. In spite of the previous example, it does appear that the class of quadratically stable systems is rich. For example:
   (*a*) Consider those reset control systems whose base-linear transfer functions have the classic second-order form:
   $$T(s) = \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$
   where $\zeta, \omega_n > 0$. Also, assume the reset controller is a FORE (with pole $s = -b$). In Section 5, we will show this class of reset control systems to be quadratically stable for all $b > 0$. This is encouraging given the ubiquity of control systems having this type of complementary sensitivity functions which arise when integral action is required and one loop-shapes a stable, minimum-phase plant. This class also covers the example in [5] which demonstrates that reset control satisfies specifications unachievable by any linear stabilizing compensator.
   (*b*) The experimental demonstrations of reset control in [3] and [7] were verified to be quadratically stable, and, their associated loop transfer functions were nontrivial. For example, in [3], the transfer function was 14^{th}-order and had a pair of complex right-half plane zeros.
3. The class of quadratically stable reset control systems require their base-linear systems to be stable – this follows immediately from the H_β-condition. This begs the question: Do there exist stable reset control systems with unstable base-linear system? Said another way, can mere application of reset stabilize a linear, unstable feedback loop? The answer is "yes" and the example comes from [12] where
$$P(s) = \frac{3.1s+1}{s^2+0.3322s-0.1}.$$
This plant cannot be stabilized by a linear integrator, but it can be stabilized by a Clegg integrator. This stability is not deducible from the H_β-condition but from the techniques in [12]. $\circ$

Remark 7. As mentioned, Horowitz and his coworkers (for example, see [6]) incorporated FOREs into control system design by advocating a two-step process in which a linear controller was first designed followed by selection of the FORE's pole. In [6], specific guidelines were provided which explicitly link the design of the FORE to the linear compensation. However, considerations of closed-loop stability were not addressed. The question, then, is whether quadratic stability can be incorporated into this design scheme. Assuming FORE as the reset element, one possibility comes from the following expression for $H_\beta(s)$:

$$H_\beta(s) = \left(\frac{1}{s+b} - \beta\right) S(s) + \beta.$$

Design of the linear compensator insures base-linear stability and consequently the stability of

$H_\beta(s)$. Therefore, for quadratic stability it suffices to guarantee

$$\mathrm{Re}\left\{\left(\frac{1}{j\omega+b}-\beta\right)S(j\omega)\right\} > -\beta$$

for all $\omega > 0$. For fixed β and b it seems possible to express the above as a useful constraint on the linear loop-shape. This would allow one to bring quadratic stability directly into the design process. ○

The H_β-condition is useful in establishing other properties of reset control systems. In the next subsection we will address one such property, UBIBS stability.

3.3 *UBIBS stability*

In [7], a bounded-input bounded-output (BIBO) stability result was given for a special class of reset control systems that utilize FOREs. It was assumed that the reset intervals were lower-bounded. We generalize this result to a larger class of reset control systems, extend it to UBIBS stability and remove the assumption on reset times. We now consider the forced reset control system described by the IDE in (3) and recall the definition of UBIBS stability. In the following $\|\cdot\|$ denotes the usual Euclidean vector norm and $\|\cdot\|_\infty$ the signal norm: $\|x\|_\infty \triangleq \sup_t \|x(t)\|$.

Definition 8. The reset control system (3) is said to be *uniformly bounded-input bounded-state* (UBIBS) *stable*, if, for each $\eta > 0$, there exists $\mu > 0$ such that for each initial condition x_0 and each bounded input $w(t)$:

$$\|x_0\| < \eta, \;\; \|w\|_\infty < \eta \quad \Rightarrow \|x(t, x_0, w)\| < \mu$$

for all $t \geq 0$. ◁

For UBIBS stability, we require the following assumptions on the reset controller and reset times.

Assumption 9. The state matrix A_r in (1) satisfies

$$A_r = \begin{bmatrix} A_{r_{11}} & A_{r_{12}} \\ 0 & A_{r_{22}} \end{bmatrix}.$$

That is, $\dot{x}_\varrho$ does not explicitly depend on the non-resetting states $x_{\bar{\varrho}}$. ○

Remark 10. Assumption 9 holds if all states of R are reset. Examples include CI and FORE. ○

We now state our result on UBIBS stability.

Theorem 11. If Assumption 9 holds and (5) is quadratically stable, then, the reset control system (3) is UBIBS stable. ◇

Example 12. As an illustration of Theorem 11, consider the reset control system with $P(s) = \frac{s+1}{s(s+0.2)}$ and $R_{b\ell}(s) = \frac{1}{s+1}$. To establish its boundedness to a step input $r(t)$ we invoke Theorem 11. Since the reset controller is a FORE and therefore its only state is reset, Assumption 9 holds. Using (8) we form

$$H_\beta(s) = \frac{s^2 + (0.2+\beta)s + \beta}{(s+1)(s^2+0.2s+1)}.$$

Clearly, $H_\beta(s)$ is stable, and for $\beta = 0.25$ the real part of $H_\beta(j\omega)$ is positive for all $\omega \geq 0$. Hence, the H_β-condition is satisfied and the step response is bounded. ▷

In the next section we consider an important class of second-order base-linear systems that satisfy the H_β-condition. As a result, the associated reset control systems enjoy the previously discussed stability and performance properties.

4. A CLASS OF SECOND-ORDER BASE-LINEAR SYSTEMS

In this section we follow-up on Remark 6.2a and show there exists a rich class of reset control systems that are quadratically stable. To begin, consider the feedback system in Figure 3 where the reset controller is a FORE (with pole b). Assume the linear loop has transfer function

$$P(s) = \frac{(s+b)\omega_n^2}{s(s+2\zeta\omega_n)}$$

resulting in a base-linear system with complementary sensitivity function

$$T(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}.$$

This transfer function has classical second-order form and is often encountered in feedback control systems where integral control is used and response is second-order dominant. This setup is

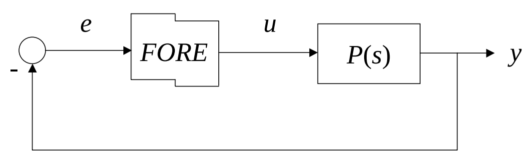

Fig. 3. Block diagram of a reset control system with FORE.

described by the IDE in (5) with data:

$$A_{c\ell} = \begin{bmatrix} -2\zeta\omega_n & 1 & 1 \\ 0 & 0 & b \\ -\omega_n^2 & 0 & -b \end{bmatrix}, \quad B_{c\ell} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix};$$

$$A_R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix};$$

$$C_{c\ell} = \begin{bmatrix} \omega_n^2 & 0 & 0 \end{bmatrix}. \tag{9}$$

Our next result shows this class of reset control system to be quadratically stable.

Theorem 13. The reset control system described in (5) and (9) is quadratically stable for all positive b, ω_n and ζ. Consequently, this reset control system is also UBIBS stable. ◇

Remark 14.

1. Examples 5 and 12, as well as the reset control system in [5], have the second-order base-linear structures considered here. As a result, their quadratic stability is ensured by Theorem 13.
2. A useful extension to Theorem 13 would consider $P(s)$ that are second-order dominant; i.e.,

$$P(s) = \frac{(s+b)\omega_n^2}{s(s+2\zeta\omega_n)(s/p+1)}$$

where $\frac{1}{s/p+1}$ models a high-frequency parasitic. The question is whether quadratic stability (or equivalently, the H_β condition) is robust in the presence of this dynamic. ○

5. CONCLUSION

This paper shows that quadratic stability plays an important role in reset control systems, similar to that in linear feedback. For linear systems, quadratic stability is tested via a Lyapunov equation. For reset control systems, it is deduced from a constrained Lyapunov equation, or equivalently, from an SPR condition – the H_β-condition. All stable linear systems are quadratically stable, but not so for reset control; see Remark 6.1. Nevertheless, the H_β-condition has been valuable in establishing stability for some high-order experimental systems and is always satisfied for the important class of reset control systems described in Section 5. A possible topic for further research is to explore the use of non-quadratic Lyapunov functions. One step in this direction has been taken in [8] where passivity formalism has been applied.

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