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DECENTRALIZED ADAPTIVE CONTROL WITH IMPROVED STEADY STATE PERFORMANCE

Boris M. Mirkin [*,1] **and Per-Olof Gutman** [*]

[*] *Faculty of Agricultural Engineering, Technion – Israel Institute of Technology, Haifa 32000, Israel,*
e-mail:bmirkin@tx.technion.ac.il, peo@tx.technion.ac.il

Abstract: In this paper we propose a new output decentralized model reference adaptive control scheme to improve the steady state performance for a case large-scale systems with unknown interconnection strengths as well as uncertainties in subsystem dynamics. Additional model reference feedforward signals are introduced in the adaptive scheme. The proposed scheme uses decentralized local output feedback with centralized model reference coordination and provides zero tracking errors. In this way the totally decentralized structure of the current information update is saved, since there is no exchange of signals between the different subsystems. The simulation results have shown the effectiveness of our proposed scheme.

Keywords: Adaptive decentralized control, large-scale systems, model coordination

1. INTRODUCTION

In recent years there has been considerable interest to study decentralized adaptive control of large-scale dynamic systems. A variety of decentralized adaptive techniques have been developed using the M-matrix test in (Ioannou and Kokotovich, 1983), (Mirkin, 1986), a high-gain approach in (Gavel and Šiljak, 1989), (Mirkin and Choi, 1991), Morse's dynamic certainty equivalence principle in (Ortega, 1996), adaptive backstepping in (Jain and Khorrami, 1997), and parameter projection together with static normalization for plants with stable dynamic interconnections in (Wen and Soh, 1999). A specific class of decentralized adaptive control is the decentralized model reference adaptive control (DMRAC). Most works focus on the state feedback case while less attention has been paid to the adaptive decentralized output feedback problem. The latter problem is of great importance from a theoretic point of view and is of great practical interest for applications.

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Unfortunately, the best that can be achieved in most known model reference adaptive decentralized control laws in the presence of parametric disturbances is the convergence of errors to some bounded residual set. The bounds of this set are unknown *a priori* and the size depends on the global bound of the strength of the unmodelled interconnections. Hence, such adaptive schemes may be unsuitable for applications, and there is a needs to develop new methods which would make it possible to avoid this disadvantage.

For the *state feedback case* (each state of subsystems x_i can be locally measured), (Mirkin, 1995) and (Mirkin, 1999) proposed a new decentralized information structure with reference model coordination, whereby coordinating information about the reference signals of the other subsystems are used in all local control laws. This structure guarantees zero residual tracking errors.

The purpose of this paper is to obtain the first solution to the more challenging problem of decentralized *output asymptotic exactly tracking* for large-scale systems with parametric uncertainties. Local control based on output measurements is a natural prerequisite for all practical control problems.

The controller will be said to be *decentralized with model coordination* if the only information available to each subsystem controller is the corresponding subsystem input and output and the states of all reference models that are assumed to be *a priori* available to all subsystems. In this way the totally decentralized structure of the current information update is saved, since there is no exchange of signals between the different subsystems.

2. SYSTEM MODEL

The class of large-scale plants with M subsystems and parametric uncertainty that we shall consider in this paper is of the form

$$\begin{aligned} \dot{x}_i(t) &= A_i x_i(t) + b_i u_i(t) + \sum_{j=1}^{M} A_{ij} x_j(t), \\ y_i(t) &= c_i^T x_i(t), \quad i = 1, 2, \ldots, M, \end{aligned} \tag{1}$$

where for the i-th subsystem $x_i \in R^{n_i}$ is the state vector $u_i(t) \in R$ is the input control vector; $y_i(t) \in R$ is the output vector, the constant matrices $A_i \in R^{n_i \times n_i}$, $b_i \in R^{n_i}$, $A_{ij} \in R^{n_i \times n_j}$, are not specified; $\sum_{j=1}^{M} n_i = n$. The following assumptions are made:

All subsystems are completely controllable. The signs of b_i are assumed to be positive. The matrices of subsystems A_i, b_i and matrices of interaction A_{ij} have the form

$$A_i = \begin{bmatrix} 0 & 1 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & 1 \\ -a_1^i & -a_2^i & \ldots & -a_{n_i}^i \end{bmatrix},$$

$$a_i^T = [-a_1^i, \ldots, -a_{n_i}^i], \quad b_i = [0, \ldots, b_i^*],$$

$$A_{ij} = b_i a_{ij}^{*T}, \quad a_{ij}^{*T} = [a_1^{ij}, \ldots, a_{n_j}^{ij}].$$

The composite system can be written as

$$\begin{aligned} \dot{x}(t) &= A_d x(t) + b_d u(t) + b_d A^* x(t), \\ y(t) &= c_d x(t), \end{aligned} \tag{2}$$

where $x(t) \in R^{n_1 + \ldots + n_M}, u(t) \in R^M$, $y(t) \in R^M$ are the overall state, control and output vectors, respectively, and the matrices $A_d \in R^{n \times n}$, $b_d \in R^{n \times M}$ and $c_d \in R^{n \times M}$ are block diagonal with blocks $A_1, \ldots, A_M$, $b_1, \ldots, b_M$ and $c_1, \ldots, c_M$. The subscript "d" denotes a block-diagonal matrix. The block matrix $b_d A^*$, where

$$A^* \in R^{M \times n} = \begin{bmatrix} A'^*_{11} & \ldots & A'^*_{1M} \\ \vdots & \ddots & \vdots \\ A'^*_{M1} & \ldots & A'^*_{MM} \end{bmatrix}$$

represents the interconnection pattern.

3. PROBLEM FORMULATION

Let the M reference models be given by

$$\begin{aligned} \dot{x}_{mi}(t) &= A_{mi} x_{mi}(t) + b_{mi} r_i(t), \\ y_{mi}(t) &= c_{mi}^T x_i(t), \quad i = 1, \ldots, M, \end{aligned} \tag{3}$$

where for the ith model, $x_{mi}(t) \in R^{n_i}$ is the state vector, $r_i \in R$ is the input and $y_{mi} \in R$ is the output. The matrices A_{mi}, b_{mi} are known constant matrices of appropriate dimensions.

Coordinates for each local model are chosen so that the pairs (A_{mi}, b_{mi}) are in canonical form as in (1). With this choice of coordinates, it is clear also, that there exists constant unknown vectors a_i^*, a_{ij}^*, b_i^* so that $A_{ij} = b_i a_{ij}^{*T}$, $A_i = A_{mi} + b_i a_i^{*T}$, $b_{mi} = b_i b_i^*$.

Then the *control objective* is to design decentralized controllers for system (1) and (2) such that the closed-loop system is stable and the outputs $y_i(t)$ track the outputs of the M stable local reference models y_{mi} (3) with the aim that

$$\begin{aligned} \lim_{t \to \infty} e_i(t) = \lim_{t \to \infty} (y_i(t) - y_{mi}(t)) = 0, \\ i = 1, 2, \ldots, M, \end{aligned} \tag{4}$$

i.e. we demand that the tracking errors converge to zero asymptotically with time.

4. PROPOSED DECENTRALIZED ADAPTIVE CONTROLLER

Each local controller consist of two loops, i.e. the controller for the ith subsystem is defined as the sum of two components

$$u_i(t) = u_{li}(t) + u_{gi}(t). \tag{5}$$

The local feedback controller structure The differentiator free local feedback controller structure with control component $u_{li}(t)$ is defined as in conventional MRAC schemes that have been widely analyzed in the literature of centralized and decentralized adaptive control (Narendra and Annaswamy, 1989)

$$\begin{aligned} u_{li}(t) = W_{pi}(s) u_i(t) + W_{fi}(s) y_i(t) \\ + K_{ei} e_i(t) + K_{ri} r_i(t), \end{aligned} \tag{6}$$

with state space realization

$$\begin{aligned} \dot{x}_{pi}(t) &= F_i x_{pi}(t) + g_i u_i(t), \\ \dot{x}_{fi}(t) &= F_i x_{fi}(t) + g_i y_i(t), \\ K_i(t) &= [K_{ei}, K_{pi}^T, K_{fi}^T, K_{ri}]^T, \\ \omega_i(t) &= [e_i(t), x_{pi}^T(t), x_{fi}^T(t), r_i(t)]^T, \\ u_{li}(t) &= K_i^T(t) \omega_i(t), \end{aligned} \tag{7}$$

where (F_i, g_i) is an asymptotically stable system in controllable canonical form with the elements in the last row equal to the coefficients of the characteristic polynomial, $x_{pi}(t) \in R^{n_i-1}$, $x_{fi}(t) \in R^{n_i-1}$, $F_i \in R^{(n_i-1)\times(n_i-1)}$, $g_i \in R^{n_i-1}$.

Following the results of (Narendra and Annaswamy, 1989) it can be shown that a constant control parameter vector

$$K_i^* = [K_{ei}^*, K_{pi}^{*T}, K_{fi}^{*T}, K_{ri}^*]^T$$

exists such that if $K_i(t) = K_i^*$, the transfer function of the isolated subsystem ($A_{ij} = 0$) (1) together with the local controller matches that of the reference model (3) exactly.

The reference model based feedforward coordinated local controller structure The reference model based feedforward coordinated control component $u_{gi}(t)$ is structured as a linear combination of the states of *all reference models* as follows

$$\dot{z}_{mij}(t) = F_{di} z_{mij}(t) + g_{di} x_{mj}(t), \tag{8}$$
$$u_{gi}(t) = -\sum_{j=1}^{M} K_{ij}^T(t) x_{mj}(t) + \sum_{j=1}^{M} K_{zij}^T(t) z_{mij}(t),$$

where $K_{ij}(t)$, $K_{zij}(t)$ are the block time-varying adaptation gain vectors, the block vectors $z_{mij}(t) = [z_{mij}^{1T}(t), \ldots, z_{mij}^{n_jT}(t)]^T$ have components from equations

$$\dot{z}_{mij}^1(t) = F_i z_{mij}^1(t) + g_i x_{mj}^1(t),$$
$$\vdots$$
$$\dot{z}_{mij}^{n_j}(t) = F_i z_{mij}^{n_j}(t) + g_i x_{mj}^{n_j}(t) \tag{9}$$

and $F_{di} = \text{block-diag}(F_i)$, $g_{di} = \text{block-diag}(g_i)$.

The difference between the control structure (8) and the DMRAC scheme in (Mirkin, 1999) is the use also of the feedforward dynamic term $\sum_{j=1}^{M} K_{zij}^T z_{mij}$ in addition to the static term $\sum_{j=1}^{M} K_{ij}^T x_{mj}$.

The main difference from standard DMRAC schemes used in decentralized adaptive control is defined by the global component $u_{gi}(t)$. *This is the main contribution of our approach.* We assume that every local controller uses *the reference trajectories of all subsystems*. Such a control law makes it possible to achieve *the zero tracking error* even though the coefficients are unknown in the interconnection matrices.

The proposed structure with reference model coordination for decentralized model reference adaptive control uses totally decentralized output feedback but centralized model reference feedforward and provides zero tracking errors.

In this way the totally decentralized structure of the current information update is saved.

5. ERROR EQUATION

With the controller in (5) and the parameter errors $\Delta K_i(t) = K_i(t) - K_i^*$ the closed-loop interconnected system becomes

$$\dot{\hat{x}}_i = \hat{A}_i \hat{x}_i + \hat{b}_i[\Delta K_i^T \omega_i + K_{ri}^* r_i - K_{ei}^* y_{mi}]$$
$$+ \bar{b}_i[u_{gi} + \sum_{j=1}^{M} a_{ij}^{*T} x_j],$$
$$y_i = \hat{c}_i^T \hat{x}_i, \tag{10}$$

where $\hat{c}_{mi} = [c_i^T\ 0\ 0]^T$, $\hat{x}_i(t) = [x_i^T, x_{pi}^T, x_{fi}^T]^T$ and

$$\hat{A}_i = \begin{bmatrix} A_i + b_i K_{ei}^* c_i^T & b_i K_{pi}^{*T} & b_i K_{fi}^{*T} \\ g_i K_{ei}^* c_i^T & F_i + g_i K_{pi}^{*T} & g_i K_{fi}^{*T} \\ g_i c_i^T & 0 & F_i \end{bmatrix},$$

$$\hat{b}_i = [b_i^T, g_i^T, 0]^T, \ \bar{b}_i = [b_i^T, 0, 0]^T.$$

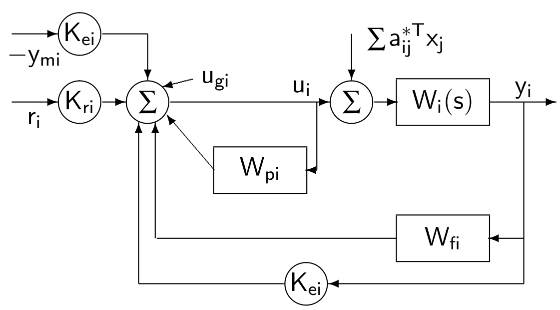

Fig. 1. Block diagram of closed loop system

As follows from Fig. 1, the interconnection terms $\sum_{j=1}^{M} a_{ij}^{*T} x_j$ that enter at the input to the plant are not available signals for input to the precompensator W_{pi}. Therefore, like in (Gavel and Šiljak, 1989), we shall reflect the signals $\sum_{j=1}^{M} a_{ij}^{*T} x_j$ to the input of the closed-loop system using standard transfer function manipulations.

In doing this, we have introduced new subsystems into the analysis, whose transfer functions are $\hat{W}_{pi}^{-1}(s) = 1 - W_{pi}(s)$ with inputs $\sum_{j=1}^{M} a_{ij}^{*T} x_j$ and outputs y_{xi}

$$y_{xi} = (1 - W_{pi}(s))[\sum_{j=1}^{M} a_{ij}^{*T} x_j]$$
$$= \sum_{j=1}^{M} a_{ij}^{*T} x_j - \sum_{j=1}^{M} W_{pi}(s)[a_{ij}^{*T} x_j].$$

Since a_{ij}^{*T} is a constant, y_{xi} can be rewritten as

$$y_{xi} = \sum_{j=1}^{M} a_{ij}^{*T} x_j - \sum_{j=1}^{M} a_{ij}^{*T} W_{pi} I_{n_j}[x_j], \tag{11}$$

where I_{n_j} is identity matrix of order $n_j \times n_j$.

The state space realization of (11) is

$$\dot{z}_{xij}(t) = F_{di} z_{xij}(t) + g_{di} x_j(t), \;\; z_{xij}(t_0) = 0,$$
$$y_{xi} = \sum_{j=1}^{M} a_{ij}^{*T} x_j - \sum_{j=1}^{M} \hat{a}_{ij}^{*T} z_{xij}, \tag{12}$$

where

$$z_{xij}(t) = [z_{xij}^{1T}(t), \ldots, z_{xij}^{n_j T}(t)]^T$$
$$K_{pid} = \text{ block-diag}(K_{pi}^{*T}), \;\; \hat{a}_{ij}^* = K_{pid}^T a_{ij}^*$$

and $F_{di}, \; g_{di}$ from (9).

With this modification the closed-loop systems from (10) are now described by

$$\begin{aligned} \dot{\hat{x}}_i &= \hat{A}_i \hat{x}_i + \hat{b}_i [\Delta K_i^T \omega_i + K_{ri}^* r_i - K_{ei}^* y_{mi} \\ &\quad + u_{gi} + \sum_{j=1}^{M} a_{ij}^{*T} L^T \hat{x}_j - \sum_{j=1}^{M} \hat{a}_{ij}^{*T} z_{xij}], \\ y_i &= \hat{c}_i^T \hat{x}_i, \end{aligned} \tag{13}$$

where $L = [I_{n_j \times n_j}, \; 0_{n_j \times n_{j-1}}, \; 0_{n_j \times n_{j-1}}]^T$

For $K_i = K_i^*$ the triplet $(\hat{A}_i, \hat{b}_i, \hat{c}_i^T)$ is a non-minimal representation of the reference model (3)

$$\begin{aligned} \dot{\hat{x}}_{mi}(t) &= \hat{A}_i(K_i^*) \hat{x}_{mi} + \hat{b}_i r_i, \\ y_{mi}(t) &= \hat{c}_{mi}^T \hat{x}_{mi}(t), \end{aligned} \tag{14}$$

where $\hat{x}_{mi} = [x_{mi}^T, x_{mpi}^T, x_{mfi}^T]^T \in R^{3n_i - 2}$. The equations for the state error $\hat{e}_i(t) = \hat{x}_i(t) - \hat{x}_{mi}(t) \in R^{3n_i-2}$ can be expressed as

$$\begin{aligned} \dot{\hat{e}}_i &= \hat{A}_i \hat{e}_i + \hat{b}_i [\Delta K_i^T \omega_i - K_{ei}^* \hat{c}_{mi}^T \hat{x}_{mi} \\ &\quad + u_{gi} + \sum_{j=1}^{M} a_{ij}^{*T} L^T \hat{x}_j - \sum_{j=1}^{M} \hat{a}_{ij}^{*T} z_{xij}], \\ \dot{z}_{xij}(t) &= F_{di} z_{xij}(t) + g_{di} L^T \hat{x}_j(t), \\ e_i &= y_i - y_{mi} = \hat{c}_{mi}^T \hat{e}_i. \end{aligned} \tag{15}$$

Now we introduce $z_{eij}(t) + z_{mij}(t) = z_{xij}(t)$. Then from (15) we can write

$$\begin{aligned} \dot{\hat{e}}_i &= \hat{A}_i \hat{e}_i + \hat{b}_i \sum_{j=1}^{M} a_{ij}^{*T} L^T \hat{e}_j - \hat{b}_i \sum_{j=1}^{M} \hat{a}_{ij}^{*T} z_{eij} \\ &\quad + \hat{b}_i [\Delta K_i^T \omega_i + u_{gi} \\ &\quad + \sum_{j=1}^{M} a_{ij}^{**T} L^T \hat{x}_{mj} - \sum_{j=1}^{M} \hat{a}_{ij}^{*T} z_{mij}], \\ \dot{z}_{eij}(t) &= F_{di} z_{eij}(t) + g_{di} L^T \hat{e}_j(t), \\ \dot{z}_{mij}(t) &= F_{di} z_{mij}(t) + g_{di} x_{mj}(t), \\ e_i &= y_i - y_{mi} = \hat{c}_{mi}^T \hat{e}_i, \end{aligned} \tag{16}$$

where

$$a_{ij}^{**} = \begin{cases} a_{ij}^*, & \text{if } i \neq j, \\ a_{ii}^* - K_{ei}^* c_{mi}, & \text{if } i = j. \end{cases}$$

Using the control component u_{gi} as given by (8), the error equation (16) can be written as

$$\begin{aligned} \dot{\hat{e}}_i &= \hat{A}_i \hat{e}_i + \hat{b}_i \sum_{j=1}^{M} a_{ij}^{*T} L^T \hat{e}_j - \hat{b}_i \sum_{j=1}^{M} \hat{a}_{ij}^{*T} z_{eij} \\ &\quad + \hat{b}_i \Delta K_i^T(t) \omega_i + \hat{b}_i \sum_{j=1}^{M} \Delta K_{ij}^T(t) x_{mj} \\ &\quad + \hat{b}_i \sum_{j=1}^{M} \Delta K_{zij}^T(t) z_{mij}, \\ \dot{z}_{eij}(t) &= F_{di} z_{eij}(t) + g_{di} L^T \hat{e}_j(t), \\ \dot{z}_{mij}(t) &= F_{di} z_{mij}(t) + g_{di} x_{mj}(t), \\ e_i &= y_i - y_{mi} = \hat{c}_{mi}^T \hat{e}_i, \end{aligned} \tag{17}$$

where $\Delta K_{ij}(t) = a_{ij}^{**} - K_{ij}(t)$, $\Delta K_{zij}(t) = K_{zij}(t) - \hat{a}_{ij}^*$.

The composite system error can be written as

$$\begin{aligned} \dot{e}(t) &= A_{md} e(t) + \hat{b}_d A^* e(t) - \hat{b}_d A_z^* z_e(t) \\ &\quad + \hat{b}_d \Delta K_d(t) \omega(t) + \hat{b}_d \Delta K_m(t) x_m(t) \\ &\quad + \hat{b}_d \Delta K_z(t) z_m(t), \\ \dot{z}_e(t) &= \hat{F}_d z_e(t) + \hat{g}_d e(t), \\ \dot{z}_m(t) &= \hat{F}_d z_m(t) + \hat{g}_d x_m(t), \\ e &= y - y_m = \hat{c}_{md}^T \hat{e} \end{aligned} \tag{18}$$

where $z_e = [z_{e11}^T \ldots z_{e1M}^T | \ldots | z_{eM1}^T \ldots z_{eMM}^T]^T$, the block vectors e, ω, x_m, z_m have as components $\hat{e}_i, \omega_i, x_{mi}, z_{mij}$, respectively, and the matrices A_{md}^*, A^*, A_z^*, $\Delta K_d(t)$, $\Delta K_m(t)$, $\Delta K_z(t)$ are block-matrices with parameter components from (17), respectively.

6. STABILITY

We denote the solutions of the system (18) by $e(\Delta K_d, \Delta K_m(t), \Delta K_z(t))(t)$ and prove the following theorem

Theorem Consider the closed-loop system consisting of a plant described by (1) and (2), controllers with control law given by (5). Then all the signals in the system are bounded and the tracking errors $e_i(t) \to 0$ as $t \to \infty$ $(i = 1, \ldots, M)$, if we choose the local adaptive laws as

$$\begin{aligned} \Delta \dot{K}_i &= -\Gamma_{1i} e_i \omega_i \\ \Delta \dot{K}_{mij} &= -\Gamma_{2i} e_i x_{mj} \\ \Delta \dot{K}_{zij} &= -\Gamma_{3i} e_i z_{mij}. \end{aligned} \tag{19}$$

where $\Gamma_{1i} = \Gamma_{1i}' > 0$, $\;\; \Gamma_{2i} = \Gamma_{2i}' > 0$ are constant matrices.

Proof: Define the function V as

$$V = \sum_{i=1}^{M} V_i, \;\; (V_i = \sum_{k=1}^{5} V_{ki}) \tag{20}$$

where

$$V_{1i} = \hat{e}_i^T \hat{P}_i \hat{e}_i,$$

$$V_{2i} = (\Delta K_i - \bar{K}_{1i})^T \Gamma_{1i}^{-1} (\Delta K_i - \bar{K}_{1i}),$$
$$V_{3i} = \sum_{j=1}^{M} \Delta K_{ij}^T \Gamma_{2i}^{-1} \Delta K_{ij},$$
$$V_{4i} = \sum_{j=1}^{M} \Delta K_{zij}^T \Gamma_{3i}^{-1} \Delta K_{zij},$$
$$V_{5i} = \sum_{j=1}^{M} z_{eij}^T S_i z_{eij}, \qquad (21)$$

with $\Gamma_{ki}^T = \Gamma_{ki} > 0$ and $\bar{K}_{1i} = -r_0 \hat{b}_i^T \hat{P}_i$. Since W_{mi} is a strictly positive real (SPR) transfer function and F_{di} is a stable matrix $\hat{P}_i$, S_i satisfies the equations (bearing in mind the Kalman-Yakubovich lemma)

$$\hat{A}_i^T \hat{P}_i + \hat{P}_i \hat{A}_i = -\hat{Q}_i,$$
$$\hat{P}_i \hat{b}_i = \hat{c}_i,$$
$$F_{di}^T S_i + S_i F_{di} = -Q_{zi}, \qquad (22)$$

where both $\hat{Q}_i$ and Q_{zi} are positive definite matrices suitable dimensions.

Taking the time derivative of V_{1i} with respect to (17), we get

$$\dot{V}_i = -\hat{e}_i^T \hat{Q}_i \hat{e}_i - \hat{e}_i^T r_0 \hat{P}_i \hat{b}_i \hat{b}_i^T \hat{P}_i \hat{e}_i - \sum_{j=1}^{M} z_{eij}^T Q_{zi} z_{eij}$$
$$-2 \sum_{j=1}^{M} z_{eij}^T E_{ij} \hat{e}_i + 2 \sum_{j=1}^{M} z_{eij}^T D_{ij} \hat{e}_j, \qquad (23)$$

where $E_{ij} = \hat{a}_{ij}^* \hat{b}_i^T \hat{P}_i$ and $D_{ij} = S_i g_{di} L$.

Then the time derivative of V from (20) can be written in the compact form

$$\dot{V} = e^T[-\hat{Q}_d - r_0 \hat{P}_d \hat{b}_d \hat{b}_d^T \hat{P}_d + A^* \hat{b}_d^T \hat{P}_d \qquad (24)$$
$$+ \hat{P}_d \hat{b}_d A^{*T}]e + 2 z^T \tilde{E} e - z^T \hat{Q}_{zd} z - e^T \hat{Q}_d e,$$

where

$$2\hat{Q}_d = \text{blockdiag}[\hat{Q}_i],\ \hat{P}_d = \text{blockdiag}[\hat{P}_i],$$

$\hat{b}_d = \text{blockdiag}[\hat{b}_i]$, $\tilde{E} = \hat{E} - \hat{D}$, $\hat{Q}_{zd} = \text{blockdiag}[\hat{Q}_{zi}]$, where $\hat{D}$ and $\hat{E}$ are block diagonal matrices with elements E_{ij} and D_{ij}, respectively.

Setting $\hat{Q}_{zd} = q_z I$ ($q_z \in R^1 > 0$) after completing the squares in (24) and dropping negative terms, we obtain

$$\dot{V} \le -[r_0 \lambda_{min}(\hat{Q}_d) - \lambda_{max}(A^{*T} A^*)] \|e\|^2$$
$$-[q_z \lambda_{min}(\hat{Q}_{zd}) - \lambda_{max}(\tilde{E}^T \tilde{E})] \|e\|^2, \qquad (25)$$

where $\lambda_{min}(.)$ and $\lambda_{max}(.)$ are the minimum and maximum eigenvalues. By selecting sufficiently large finite values r_0^* and q_z^* so that

$$r_0^* > \lambda_{max}(A^{*T} A^*) \lambda_{min}^{-1}(\hat{Q}_{0d}),$$
$$q_z^* > \lambda_{max}(\tilde{E}^T \tilde{E}) \lambda_{min}^{-1}(\hat{Q}_{zd}) \qquad (26)$$

we get $\dot{V} \le 0$.

Further using standard arguments from the Lyapunov theory (Narendra and Annaswamy, 1989), we conclude that the solutions $e(\cdot)(t)$ are bounded and $e_i(t) \to 0$ as $t \to \infty$ and the proof is complete.

Remark The adaptive controller developed in this paper can be extended to the case when the relative degree of the plant exceeds unity. In our structure for local adaptation, we can use Monopoli's augmented error concept (Narendra and Annaswamy, 1989) or, for example, parameter projection together with static normalization (Wen and Soh, 1999).

7. SIMULATION RESULTS

In this simulation, Gavel and Šiljak's example (Gavel and Šiljak, 1989) of a fourth order system is used to demonstrate the effectiveness of the proposed scheme.

$$\dot{x}_1(t) = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} x_1(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_1(t) + \begin{bmatrix} 0 & 0 \\ 2 & 0 \end{bmatrix} x_2(t),$$
$$y_1(t) = [1.0\ \ 0.1] x_1(t),$$
$$\dot{x}_2(t) = \begin{bmatrix} 0 & 1 \\ 3 & 0 \end{bmatrix} x_2(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_2(t) + \begin{bmatrix} 0 & 0 \\ -2 & 0 \end{bmatrix} x_1(t),$$
$$y_2(t) = [1.0\ \ 0.1] x_2(t). \qquad (27)$$

In this case we have two second order subsystems and it is required to design MRDAC u_1 and u_2 such that the outputs $y_1(t)$, $y_2(t)$ track the corresponding outputs $y_{m1}(t)$, $y_{m2}(t)$ of the reference models

$$\dot{x}_{mi}(t) = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix} x_{mi}(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r_i(t),$$
$$y_{mi}(t) = [0.5\ 0.5] x_{mi}(t), \qquad (28)$$

with the reference inputs $r_i(t) = \sin(t)$.

Simulation results are shown in Figures 2 – 5.

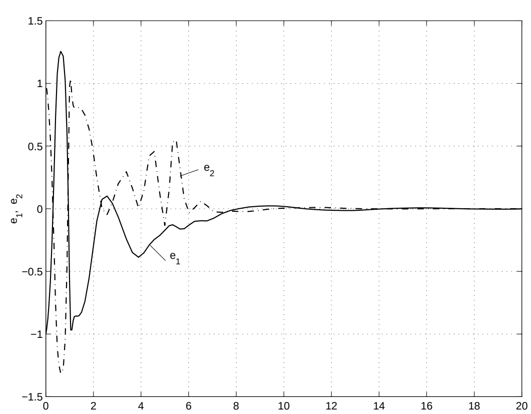

Fig. 2. Tracking errors responses

In Fig. 2 we show the time responses of the tracking errors e_1, e_2. Controller adaptive gain responses are shown in Figures 3 – 5. As seen in the figures, the output errors tend to zero in all cases, and the performance of the proposed algorithms is considerably

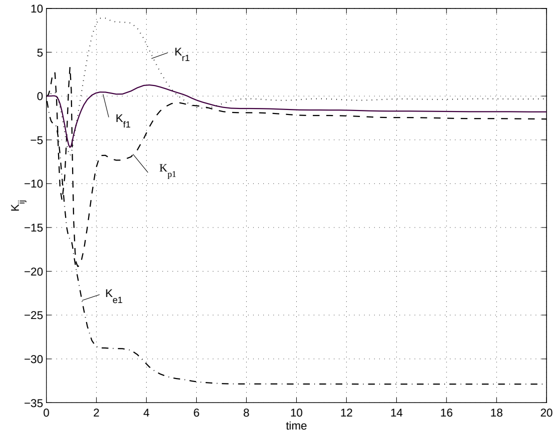

Fig. 3. First local controller gain responses

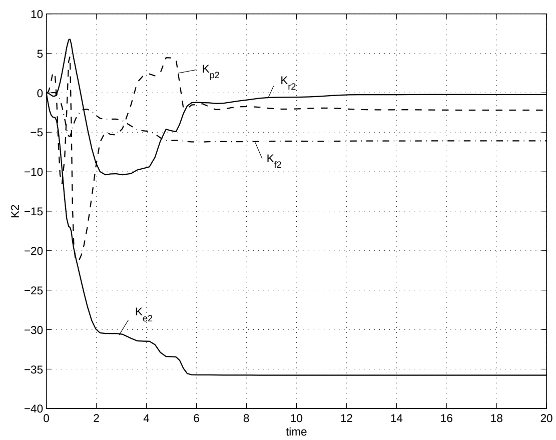

Fig. 4. Second local controller gain responses

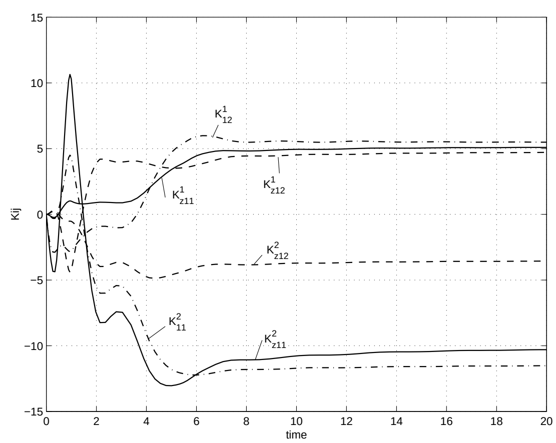

Fig. 5. Feedforward coordinated controller gain responses

better than that with the standard adaptive laws (Gavel and Šiljak, 1989).

8. CONCLUSION

In this paper, we have developed coordinated decentralized output adaptive controllers for a class of large-scale systems with unknown interconnected strengths. We presented a modified DMRAC scheme which requires an exchange of signals between the different reference models, but does not involve the exchange of output signals between the different subsystems. Our scheme can be classified as *a decentralized adaptive control scheme with model coordination.* It can not only guarantee closed-loop stability but also *asymptotic zero tracking errors* when uncertainties are present in the subsystems and interconnections. Since the reference model signals can be exchanged between the subsystems off-line before the operation of the system, this scheme is *feasible.* The local control laws are the same as those found in the literature. The proposed control structure can be viewed as an upgrade of existing schemes.Simulation results show the effectiveness of our proposed scheme.

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