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MINIMAX MPC FOR SYSTEMS WITH UNCERTAIN GAIN

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Abstract: Robust synthesis is one of the remaining challenges in model predictive control (MPC). One way to robustify an MPC controller is to formulate a minimax problem, i.e., optimize a worst-case performance measure. For systems modeled with an uncertain gain, there are many results available. Typically, the minimax formulations have given intractable problems, or unorthodox performance measures have been used to obtain tractable problems. In this paper, we show how the standard quadratic performance measure can be used in a computationally tractable minimax MPC controller. The controller is developed in a linear matrix inequality framework that easily allows extensions and generalizations.

Keywords: Predictive control, Robust control, Minimax

1. INTRODUCTION

Despite the tremendous amount of results and research on robust control during the last decades, model predictive control (MPC) still suffers from a lack of general and tractable results on robust synthesis. Many interesting approaches based on minimax (worst-case) optimization have been proposed, but they often come with some drawback such as computationally intractable (Lee and Cooley, 1997; Casavola *et al.*, 1999), use of non-standard performance measures (Campo and Morari, 1987; Allwright and Papavasiliou, 1991; Oliviera *et al.*, 2000) or restriction to systems with particular structure (Zheng, 1995).

In this paper, we present a framework for minimax MPC with a traditional quadratic performance measure, and the tools we use are robust linear matrix inequalities and semidefinite programming.

2. PROBLEM FORMULATION

A problem setup that has been used in many approaches to robust MPC is models with an uncertain gain. With an uncertain gain, we mean an uncertain input matrix B or output matrix C. The approach we will present in this paper can be used for both cases, but we will focus on an uncertain input gain.

$$x(k+1) = Ax(k) + B(k)u(k) \quad (1a)$$
$$y(k) = Cx(k) \quad (1b)$$

The time-varying uncertainty in $B(k)$ can be modeled in various ways. A common choice has been a polytopic model $B(k) \in \mathbf{Co}(B_1, \ldots, B_q)$

$$B(k) = \sum_{i=1}^{q} \lambda_i B_i, \quad \sum_{i=1}^{q} \lambda_i = 1,\ \lambda_j \geq 0 \quad (2)$$

In this work, we will turn our attention to a different model, a so called norm-bounded uncertainty model (Boyd *et al.*, 1994).

$$B(k) = B_0 + B_p\Delta(k)C_p, \quad \Delta(k) \in \mathbf{\Delta} \quad (3a)$$
$$\mathbf{\Delta} = \{\Delta : ||\Delta|| \leq 1\} \quad (3b)$$

Notice that a polytopic uncertainty, if necessary, can be approximated by a model of this type, see (Boyd *et al.*, 1994).

In nominal MPC, we typically use a quadratic finite horizon performance measure (Q and R for simplicity assumed positive definite)

$$J = \sum_{j=0}^{N-1} ||x(k+j+1|k)||_Q^2 + ||u(k+j|k)||_R^2 \quad (4)$$

Since this is a well established performance measure, and many results are available concerning MPC using this set-up, it is our intention to robustify this performance measure with a minimax formulation. Hence, the optimization problem we wish to solve is

$$\min_{u(\cdot|k)} \max_{\Delta^N} J \quad (5)$$

In the expression above, we introduced the total uncertainty along the future trajectory

$$\Delta^N = [\Delta(k), \ldots, \Delta(k+N-1)] \in \mathbf{\Delta}^N = \mathbf{\Delta} \times \ldots \times \mathbf{\Delta} \quad (6)$$

The paper is organized as follows. We begin in Section 3 with a review of some approaches to minimax MPC that have been proposed earlier in the literature. Section 4 introduces some central mathematical concepts. The main results are presented in Section 5 and we finally conclude the paper with a simple example.

3. REVIEW OF AVAILABLE APPROACHES

The fundamental property that is exploited in minimax MPC for systems with an uncertain input gain is that with a convex uncertainty, the maximum of a convex performance measure will occur at the border of the uncertainty model (Bertsekas, 1999). With a polytopic model of $B(k)$, the maximum is thus found at a vertex of the uncertainty model.

Work on minimax MPC can be traced back to (Campo and Morari, 1987). The uncertainty model was an uncertain FIR model,

$$y(k+1) = \sum_{i=0}^{n} g_i u(k-i) \quad (7)$$

where each impulse coefficient is subjected to a polytopic (time-invariant) uncertainty. By straightforward manipulations, this can be converted to a system with a polytopic uncertainty in the $B(k)$ matrix. The performance measure was chosen as the largest deviation (over a finite horizon) of the output $y(k+j|k)$ from some reference $r(k+j|k)$. Loosely speaking, this yields the problem (with Δ meaning the polytopic uncertainty)

$$\min_{u(\cdot|k)} \max_{\Delta} \max_{j} ||y(k+j|k) - r(k+j|k)||_\infty \quad (8)$$

It was shown that this can be written as a linear programming (LP) problem. Unfortunately, the optimization problem had exponential complexity in the number of uncertain variables.

The complexity was improved in (Allwright and Papavasiliou, 1991) where an equivalent LP problem with polynomial complexity was derived. It was also noted that the formulation with $|| \cdot ||_\infty$ could be extended to $|| \cdot ||_1$. Furthermore, the approach was extended to time-varying uncertainties.

Similar work on minimization of the worst-case deviation along a predicted trajectory, given a polytopic model the gain, can be found in, e.g., (Zheng, 1995) and (Oliviera *et al.*, 2000).

A quadratic minimax performance measure, such as (5) which will be addressed in this work, has not been studied to the same extent, at least not in the sense of efficient formulations.

Since the quadratic performance measure is convex, a polytopic uncertainty in the $B(k)$ matrix can be taken care of by just enumerating all the vertices of the uncertainty model along the future trajectory and solve a quadratic program for every possible combination. However, this has to be considered an intractable result since this will lead to problems with exponential complexity. If there are q uncertain parameters in $B(k)$, there will be 2^{Nq} vertices of the uncertainty realization along the trajectory. Schemes based on straightforward enumeration can be found in, e.g., (Lee and Cooley, 1997) and (Casavola *et al.*, 1999).

4. MATHEMATICAL PRELIMINARIES

The results in this paper are based on linear matrix inequalities, LMIs.

Definition 1. (LMI, (Boyd *et al.*, 1994)). An LMI is an inequality, in the free scalar variables x_i, that for some fixed symmetric matrices F_i can be written

$$F(x) = F_0 + x_1F_1 + x_2F_2 + \ldots + x_nF_n \succeq 0$$

LMIs are used in semidefinite programming.

Definition 2. (SDP, (Boyd *et al.*, 1994)). An SDP is an optimization problem that can be written

$$\begin{aligned} &\min_{x} \; c^T x \\ &\text{subject to } \; F(x) \succeq 0 \end{aligned}$$

An SDP is a convex optimization problem that can be solved with high efficiency using solvers based on, e.g., interior-point methods.

The following lemma will be used repeatedly

Lemma 1. (Schur complement, (Zhang, 1999)). If $W \succ 0$, then for any $X \succeq 0$

$$X - ZW^{-1}Z^T \succeq 0 \Leftrightarrow \begin{bmatrix} X & Z \\ Z^T & W \end{bmatrix} \succeq 0$$

The importance of this lemma is that it allows us to rewrite certain nonlinear matrix inequalities into linear matrix inequalities (LMIs). The lemma is a slight variation of the standard Schur complement which involves strict inequalities.

5. MAIN RESULT

In this section, we show how to transform our original minimax problem to a semidefinite programming problem. The calculations will be done in a vector formulation, so we begin by defining the stacked state predictions and the future control sequence that we are trying to find

$$X = \begin{bmatrix} x(k+1|k) \\ x(k+2|k) \\ \vdots \\ x(k+N|k) \end{bmatrix}, \; U = \begin{bmatrix} u(k|k) \\ u(k+1|k) \\ \vdots \\ u(k+N-1|k) \end{bmatrix} \tag{9}$$

The linear system and the uncertainty model $B(k) = B_0 + B_p\Delta(k)C_p$ allows us to write (for short notation $\Delta_j = \Delta(k+j)$)

$$X = Hx(k) + SU + \sum_{j=0}^{N-1} V_j\Delta_j W_j U \tag{10}$$

where

$$V_0 = \begin{bmatrix} B_p \\ AB_p \\ \vdots \\ A^{N-1}B_p \end{bmatrix}, \; V_1 = \begin{bmatrix} 0 \\ B_p \\ \vdots \\ A^{N-2}B_p \end{bmatrix}, \; \ldots$$

$$W_0 = \begin{bmatrix} C_p & 0 \; 0 \; \ldots \; 0 \end{bmatrix}, \; W_1 = \begin{bmatrix} 0 \; C_p \; 0 \; \ldots \; 0 \end{bmatrix}, \; \ldots$$

The matrices H and S are defined in the standard way to account for the nominal part

$$H = \begin{bmatrix} A \\ A^2 \\ \vdots \\ A^N \end{bmatrix}, S = \begin{bmatrix} B_0 & 0 & \ldots & 0 \\ AB_0 & B_0 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ A^{N-1}B_0 & A^{N-2}B_0 & \ldots & B_0 \end{bmatrix}$$

For notational convenience, we redefine the weight matrices

$$Q := \text{diag}(Q, \ldots, Q) \text{ and } R := \text{diag}(R, \ldots, R)$$

This makes it possible to write the minimax problem as

$$\begin{aligned} &\min_{t,U} \; t \\ &\text{subject to } \max_{\Delta^N} X^TQX + U^TRU \leq t \end{aligned} \tag{11}$$

For the sake of a more compact notation, we define

$$\vartheta_i = \sum_{j=i}^{N-1} V_j\Delta_j W_j U \tag{12}$$

$$\bar{X} = Hx(k) + SU \tag{13}$$

In other words, we split the state predictions to one nominal part $\bar{X}$, and one uncertain contribution ϑ_0. The constraint in the optimization problem (11) can be rewritten with a Schur complement, and we obtain

$$\begin{bmatrix} t & \bar{X}^T + \vartheta_0^T & U^T \\ \bar{X} + \vartheta_0 & Q^{-1} & 0 \\ U & 0 & R^{-1} \end{bmatrix} \succeq 0 \tag{14}$$

At this point, we would like to eliminate the uncertainties, and our tool to do this is the following theorem (Ghaoui and Lebret, 1997)

Theorem 1. (Robust LMI). Robust satisfaction of the uncertain LMI

$$F + L\Delta R + R^T\Delta^T L^T \succeq 0 \quad \forall \Delta \in \mathbf{\Delta}$$

is equivalent to the LMI

$$\begin{aligned} \begin{bmatrix} F & L \\ L^T & 0 \end{bmatrix} &\succeq \begin{bmatrix} R & 0 \\ 0 & I \end{bmatrix}^T \begin{bmatrix} \tau I & 0 \\ 0 & -\tau I \end{bmatrix} \begin{bmatrix} R & 0 \\ 0 & I \end{bmatrix} \\ \tau &\geq 0 \end{aligned}$$

We write our constraint in a form suitable for the above theorem by pulling out one uncertainty $\vartheta_0 = \vartheta_1 + V_0\Delta_0 W_0 U$

$$\begin{bmatrix} t & \bar{X}^T + \vartheta_1^T & U^T \\ \bar{X} + \vartheta_1 & Q^{-1} & 0 \\ U & 0 & R^{-1} \end{bmatrix} + \begin{bmatrix} U^TW_0^T \\ 0 \\ 0 \end{bmatrix} \Delta_0^T \begin{bmatrix} 0 \; V_0^T \; 0 \end{bmatrix} + \begin{bmatrix} 0 \\ V_0 \\ 0 \end{bmatrix} \Delta_0 \begin{bmatrix} W_0U \; 0 \; 0 \end{bmatrix} \succeq 0$$

Clearly, this uncertain LMI has the structure addressed in the theorem, so we obtain

$$\begin{bmatrix} t & \bar{X}^T + \vartheta_1^T & U^T & U^TW_0^T \\ \bar{X} + \vartheta_1 & Q^{-1} & 0 & 0 \\ U & 0 & R^{-1} & 0 \\ W_0U & 0 & 0 & 0 \end{bmatrix} \succeq \begin{bmatrix} 0 & 0 \\ V_0 & 0 \\ 0 & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \tau_0 I & 0 \\ 0 & -\tau_0 I \end{bmatrix} \begin{bmatrix} 0 & V_0^T & 0 & 0 \\ 0 & 0 & 0 & I \end{bmatrix} \tag{15}$$

Simplification yields

$$\begin{bmatrix} t & \bar{X}^T+\vartheta_1^T & U^T & U^TW_0^T \\ \bar{X}+\vartheta_1 & Q^{-1}-\tau_0V_0V_0^T & 0 & 0 \\ U & 0 & R^{-1} & 0 \\ W_0U & 0 & 0 & \tau_0 I \end{bmatrix} \succeq 0$$

The LMI above is still uncertain, due to the remaining term ϑ_1. However, the structure is the same as the original LMI, so we can apply Theorem 1 recursively until all uncertainties have been eliminated. The result will be a large LMI

$$\begin{bmatrix} t & \bar{X}^T & U^T & U^TW_0^T & U^TW_1^T & \dots \\ \bar{X} & Q^{-1}-\sum_{j=0}^{N-1}\tau_jV_jV_j^T & 0 & 0 & 0 & 0 \\ U & 0 & R^{-1} & 0 & 0 & 0 \\ W_0U & 0 & 0 & \tau_0 I & 0 & 0 \\ W_1U & 0 & 0 & 0 & \tau_1 I & 0 \\ \vdots & 0 & 0 & 0 & 0 & \ddots \end{bmatrix} \succeq 0$$

Working with such a large LMI might be inconvenient, but Schur complements can be used to write it as a system of smaller LMIs, and we obtain our optimization problem, which also is our main result

$$\begin{aligned} \min_{t,\tau,U}\ & t_x+t_u+\sum_{j=0}^{N-1}t_j \\ \text{subject to}\ & \begin{bmatrix} t_x & \bar{X}^T \\ \bar{X} & Q^{-1}-\sum_{j=0}^{N-1}\tau_jV_jV_j^T \end{bmatrix} \succeq 0 \\ & \begin{bmatrix} t_u & U^T \\ U & R^{-1} \end{bmatrix} \succeq 0 \\ & \begin{bmatrix} t_j & U^TW_j^T \\ W_jU & \tau_j I \end{bmatrix} \succeq 0 \end{aligned}$$

5.1 *Connection to nominal MPC*

The resulting LMI can be analyzed to some extent. A Schur complement on the large LMI yields the equivalent constraint

$$\begin{aligned} &\bar{X}^T(Q^{-1}-\sum_{j=0}^{N-1}\tau_jV_jV_j^T)^{-1}\bar{X} \\ &\qquad +U^T(R+\sum_{j=0}^{N-1}\frac{1}{\tau_j}W_j^TW_j)U \le t \end{aligned} \quad (16)$$

Recall that when we solve a nominal MPC problem, we have the constraint

$$\bar{X}^TQ\bar{X}+U^TRU \le t \quad (17)$$

Hence, the difference is, to begin with, the additional $\sum_{j=0}^{N-1}\frac{1}{\tau_j}W_j^TW_j$ on the control weight. By recalling the definition of the matrices W_j, we see that the matrices $W_j^TW_j$ are block diagonal with zeros in the diagonal blocks except at the jth block. Hence, the extra weight on $u(k+j|k)$ will be proportional to τ_j^{-1}. The modified state weight is a bit harder to interpret. However, intuitively, if we rewrite the state cost using the matrix inversion lemma and then neglect higher order terms we obtain

$$(Q^{-1}-\sum_{j=0}^{N-1}\tau_jV_jV_j^T)^{-1} \approx Q+\sum_{j=0}^{N-1}\tau_jQV_jV_j^TQ$$

We see that the state weight also will be increased, i.e. the robustification is not done by only increasing the control weight, and the adjustment of the state weight depends on both the original state weight and the uncertainty model (of course, this can be seen directly in (16), but the expression above makes it easier to see in what directions we are changing the state weight).

6. EXTENSIONS

For the proposed framework to be interesting, it is important that standard extensions to nominal MPC can be applied also in our minimax framework. Indeed, this is the case as we will show here.

6.1 *Control constraints*

To begin with, we note that any linear constraint on U, such as amplitude or rate constraints, can be incorporated unaffected as standard linear constraints, since these constraints are unrelated to the uncertainty.

6.2 *Linear state constraints*

A typical situation in MPC is constraints on states and outputs. Also these can be dealt with in a robust manner. Let us assume for simplicity that we can write the constraints as the element-wise constraint $MX \le 1$. Inserting the definition of X yields

$$M(\bar{X}+\sum_{j=0}^{N-1}V_j\Delta W_jU) \le 1 \quad (18)$$

Define unit vectors e_i to extract each row

$$e_i^TM(\bar{X}+\sum_{j=0}^{N-1}V_j\Delta_jW_jU) \le 1 \quad (19)$$

Worst-case disturbances are found with the following lemma

Lemma 2.

$$\max_{||\Delta||\le 1} x^T\Delta y = ||x||||y|| \quad (20)$$

Proof: Follows from Schwarz inequality $|x^T y| \leq ||x|| ||y||$. Equality when x and y are parallel, i.e. when Δ is chosen so that x and Δy are parallel.

If we apply this to our uncertain predictions, a worst-case constraint is obtained

$$e_i^T M\bar{X} + \sum_{j=0}^{N-1} ||e_i^T MV_j|| ||W_j U|| \leq 1 \quad (21)$$

Introduce N bounds γ_j (so called second order cone constraints)

$$||W_j U|| \leq \gamma_j \quad (22)$$

or equivalently

$$\begin{bmatrix} \gamma_j & U^T W_j^T \\ W_j U & \gamma_j I \end{bmatrix} \succeq 0 \quad (23)$$

With these bounds, we obtain the linear constraints

$$e_i^T M\bar{X} + \sum_{j=0}^{N-1} ||e_i^T MV_j||\gamma_j \leq 1 \quad (24)$$

Hence, the original linear constraints are taken care of by introducing N second order cone constraints and N new variables. Notice that no matter how many constraints there are, the number of second order cone constraints and new variables will always be N.

6.3 *State estimation and disturbances*

The results in this work can be incorporated into the framework for minimax MPC for systems with state estimation errors and bounded external disturbances developed in (Löfberg, 2001*a*).

7. EXAMPLE

We study a sampled double integrator. In order to design a robust MPC controller, we create an uncertainty model which basically models an uncertain gain.

$$A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$
$$B \in \textbf{Co}\left(\begin{bmatrix} 1.50 \\ 0.55 \end{bmatrix}, \begin{bmatrix} 0.50 \\ 0.55 \end{bmatrix}, \begin{bmatrix} 1.50 \\ 0.45 \end{bmatrix}, \begin{bmatrix} 0.50 \\ 0.45 \end{bmatrix}\right)$$
$$C = \begin{bmatrix} 0 & 1 \end{bmatrix}$$

Our goal is to control the output $y(k) = x_2(k)$, under the control constraint $|u(k)| \leq 1$. A natural tuning is thus to only put weight on $x_2(k)$ in the performance measure. The chosen tuning variables were

$$Q = \begin{bmatrix} 0.01 & 0 \\ 0 & 1 \end{bmatrix}, \; R = 0.01, \; N = 5 \quad (25)$$

The polytopic model on $B(k)$ has to be converted to a norm-bounded model. This is done using the approach described in (Boyd *et al.*, 1994) and results in

$$B_0 = \begin{bmatrix} 1 \\ 0.50 \end{bmatrix}, \; B_p = \begin{bmatrix} 0.52 & 0 \\ 0 & 0.16 \end{bmatrix}, \; C_p = \begin{bmatrix} 0.707 \\ 0.707 \end{bmatrix}$$

As a first experiment, we test a "bad" uncertainty realization (found by trial). The following uncertainty realization was used

$$B(k) = \begin{cases} \begin{bmatrix} 0.5 \\ 0.45 \end{bmatrix} & u(k) > 0 \\ \begin{bmatrix} 1.5 \\ 0.55 \end{bmatrix} & u(k) \leq 0 \end{cases} \quad (26)$$

Implementation and solution of the optimization problems were done using (Vandenberghe and Boyd, 1998) and (Löfberg, 2001*b*).

The closed loop performance from the initial condition $x(0) = \begin{bmatrix} 0 & 5 \end{bmatrix}^T$ with this tuning and uncertainty realization can be seen in Figure 1. The nominal MPC controller has very poor performance, while the robust minimax MPC controller gives pretty good performance. Of course, the aggressive tuning is doomed

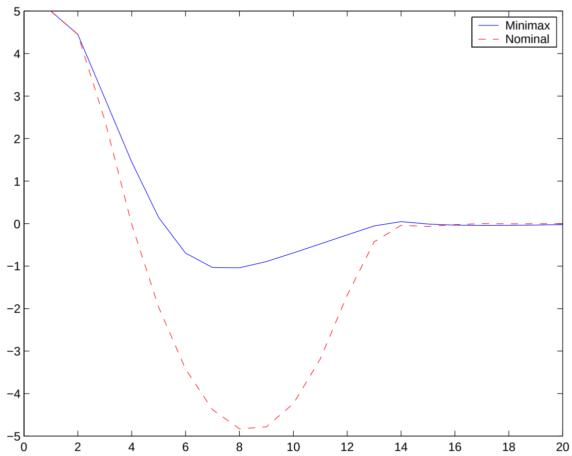

Fig. 1. Aggressive tuning $R = 0.01$ and bad uncertainty.

to give a nominal controller with poor robustness. A natural solution is to detune the controller, so we chose $R = 1$ instead and perform the same experiment. Surprisingly, this did actually not improve performance of the nominal controller that much. The minimax controller gives pretty much the same response as before, see Figure 2.

By tuning the nominal controller carefully, it is possible to obtain better performance. However, the idea with robust control is that this should not be necessary. The tuning variables should reflect the actual performance criteria, and the robustness should be built-in. Given a new uncertainty model, it should not be necessary to re-tune the controller.

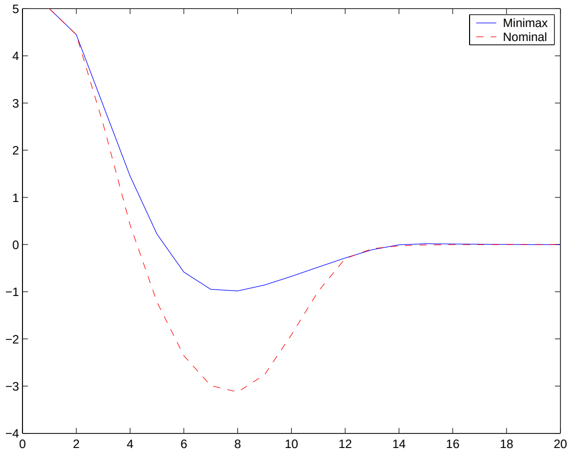

Fig. 2. Detuned controllers $R = 1$ and bad uncertainty.

So what is the price we have to pay? Conservativeness is the main problem with robust controllers, i.e., the performance in the "non-worst-case" situation can detoriate. However, for this example, this is actually not a major problem. In Figure 3 we simulate the system with a random uncertainty using both the nominal and the minimax controller. We see that the minimax controller gives a slightly sluggish behavior compared to the nominal controller. However, the minimax controller does not seem to be overly conservative.

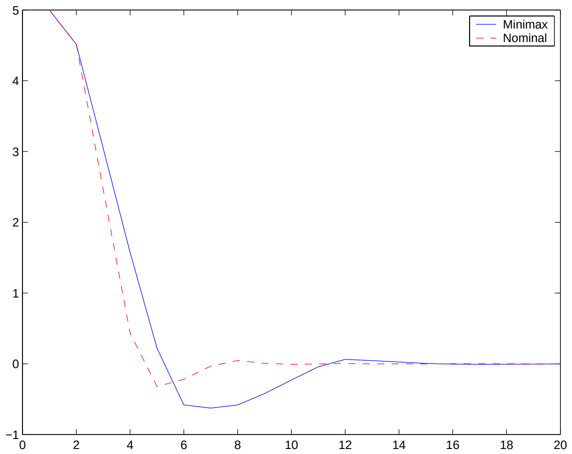

Fig. 3. Aggressive tuning $R = 0.01$ and random uncertainty.

8. CONCLUSIONS

Minimax MPC is not applicable to all systems. The example which we studied is of course chosen to point out the possible benefits of a minimax controller. However, the purpose of this paper has not been to advocate the use of minimax controllers, but to show that the problem at least can be solved efficiently and incorporated into the existing MPC framework.

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