

## HOW CAN WE CONTROL THE FASTEST SYSTEMS? ONE SOLUTION IS QFT

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**Abstract:** When we want to control a very fast system, like an operational amplifier, we have the problem that the controller must be faster than the system. The controller's work conditions are extreme and it is very difficult to find an optimal and implement solution. The QFT design methodology allows to look for real solutions to the problem. To demonstrate this affirmation, a study has been done about the operational amplifier (O.A.) LM12CL of National Semiconductor. The result is a robust controller that allows its operation in extreme conditions. *Copyright © 2002 IFAC*

**Keywords:** Quantitative Feedback Theory (QFT), Operational Amplifier (O.A.)

### 1. INTRODUCTION

In the field of electronic design, the operational amplifiers play an important role. The O.A. are used in a lot of applications, including works of middle power. The LM12CL is an O.A. designed for this kind of jobs; it is capable of driving loads  $\pm 25\text{v}$  at  $\pm 10\text{A}$ .

Although O.A.s are internally compensated, some operation conditions can drive them to a very submuffed or unstable systems. This situation force to control the system O.A.. Besides appear these extreme work conditions, as O.A. is a physical system, is also a not linear system (although in fact we can consider a negligible linearity), and it also has uncertainty in most of its parameters, due to manufacture conditions are not always the same.

If we analyse the problem, we can obtain three important causes for which the QFT methodology design is useful to solve our problem control in this kind of systems:

1. To work with low gains makes the operational amplifier instable. There are applications in which it is interesting that the O.A. works with unity gain (voltage follower).

2. When the O.A. drives capacitive loads, interact with the open-loop output resistance (about  $1\Omega$ ). The system acquires a new pole that reduce the

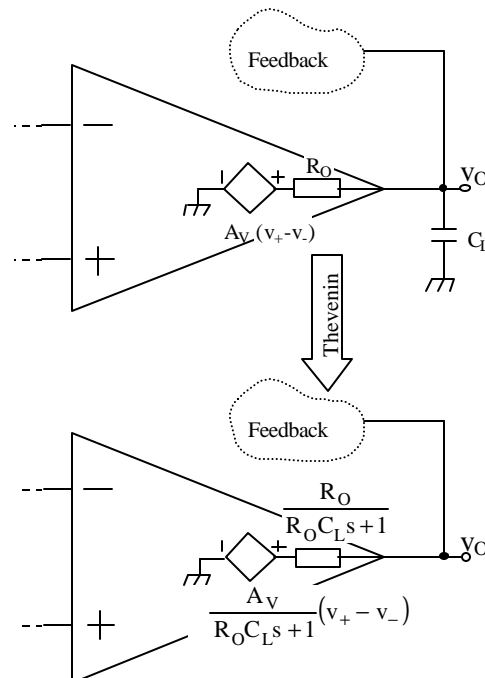


Fig. 1. Capacitive loads influence in O.A.

phase margin of the feed-back loop, ultimately causing oscillation. Figure 1 shows the effect over gain in open-loop if O.A drives capacitive loads.

3. The variation of operational amplifier parameters makes that operation conditions change significantly. It is necessary to reduce these variations in apparently equal devices.

The three above points show possible situations of instability and uncertainty interrelated, that QFT can solve suitably. Moreover QFT allows impose other kind of conditions, i.e. tracking performance specifications, that will allow to design a controller for the system, to force the response into limits indicated for designer.

## 2. QFT MODEL

QFT needs a system defined in frequency domain and with some kind of uncertainty. This is traduced in a certain number of plants, that show the worst behaviours. This plants will be forced to work into performance specifications that the designer has indicated.

For this reason the first step for controller's design is to obtain the model in frequency domain of O.A., National Semiconductor offers Bode plot of open loop, that is shown in Figure 2.

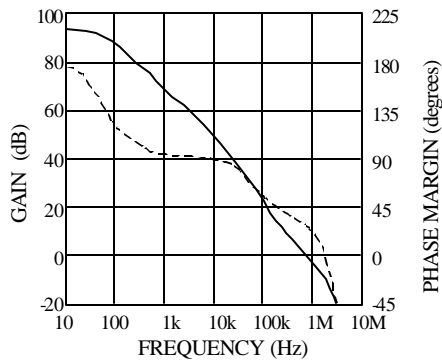


Fig. 2. LM12CL open loop response.

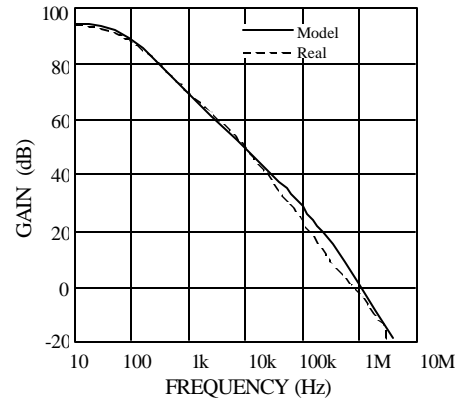
After some calculations the LM12CL is modeled by the following transfer fuction

$$G_{planta} = \frac{A_{DC}(T_1 s + 1) e^{-T_2 s}}{(T_3 s + 1)(T_4 s + 1)(T_5 s + 1)}$$

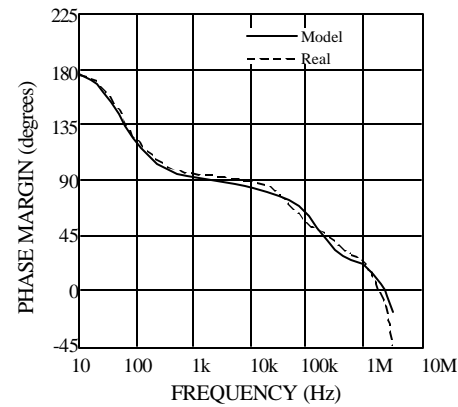
$$\begin{aligned} T_1 &= 3.062938485 \cdot 10^{-7} & T_2 &= 40 \cdot 10^{-9} \\ T_3 &= 0.002652582385 & T_4 &= 1.224268793 \cdot 10^{-6} \\ T_5 &= 1.020979495 \cdot 10^{-7} & A_{DC} &= 50000 \end{aligned}$$

This model is quite approximate, in figure 3 the great coincidence with the Bode plot of National Semiconductor is shown.

Once the O.A. is modeled, the work's conditions indicated above must be included in the transfer fuction. To this respect, according to point 1, the controller must be designed to allow works with unity gain. The point 2 indicates that the controller will allow the O.A to work with variable capacity. Finally



a)



b)

Fig. 3. Real and modeled Bode plots.

according the point 3, it will bear in mind the variability in two parameters, dc gain and output resistance.

$$20000 \leq A_{DC} \leq 90000 \quad 0.2\Omega \leq R_O \leq 1\Omega$$

Knowing this information, the transfer fuction for open loop is

$$G_{planta} = \frac{A_{DC}(T_1 s + 1) e^{-T_2 s}}{(T_3 s + 1)(T_4 s + 1)(T_5 s + 1)(R_O C_L s + 1)}$$

Calling  $A_{DC} = 10000 A$  y  $R_O C_L = 10^{-6} t$  the uncertainty in O.A. parameters can be defined as

$$2 \leq A \leq 9 \quad \text{and} \quad 0 \leq \tau \leq 1$$

The result is the following transfer fuction

$$G_{planta} = \frac{10000 A (T_1 s + 1) e^{-T_2 s}}{(T_3 s + 1)(T_4 s + 1)(T_5 s + 1)(10^{-6} \tau s + 1)}$$

### 3. CONTROLLER DESIGN

QFT offers the control solution through two freedom-degrees. Figure 4 shows this structure. The target is to calculate the  $G(s)$  and  $F(s)$  controllers.

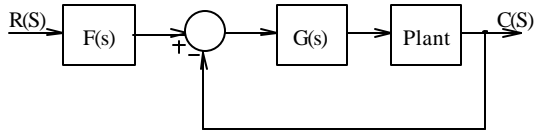


Fig. 4. Two freedom-degrees control is proposed by QFT

Firstly we calculate the parametric uncertainty obtaining 65 plants. These plants represent the worst O.A. conditions. Variability is converted in  $A$  and  $\tau$  combinations shown in figure 5.

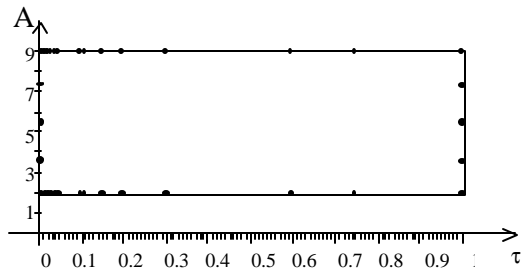


Fig. 5. Parametric uncertainty and combinations for design.

Figure 6 shows some plant's frequency responses in Black plot, for different combinations. (4 representative plants of 65).

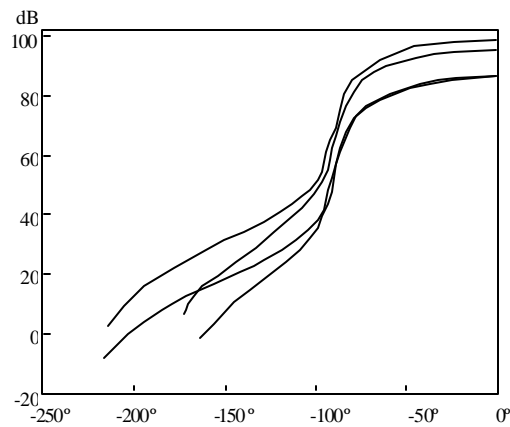


Fig 6. Black plot for some plants.

It is easy to see the great variability in response due to variable parameters O.A.

Once the plants are defined, the second step is to mark the desirable performance specifications. They can be divided in two parts: stability performance and tracking performance. The stability performance specifications are indicated with a gain

margin of 5dB and a phase margin of 50°. In QFT design, this involves the following specification

$$\left| \frac{P(s)G(s)}{1+P(s)G(s)} \right| \leq 1.2$$

[  $P(s) \equiv$  Plant ]

On the one hand, the tracking performance specifications are determined to get the loop close system response, enclosed into two limits. These limits are defined in frequency domain by two transfer functions

$$Mu(s) = \frac{3.3s^2 + 4.2 \cdot 10^7 s + 1.2 \cdot 10^{13}}{2.5 \cdot 10^{-6} s^3 + 9s^2 + 3.4 \cdot 10^7 s + 1.2 \cdot 10^{13}}$$

$$Ml(s) = \frac{-1 \cdot 10^{-7} s + 1}{7.9 \cdot 10^{-20} s^3 + 1.2 \cdot 10^{-12} s^2 + 2.6 \cdot 10^{-6} s + 1}$$

and the step response is shown in figure 7.

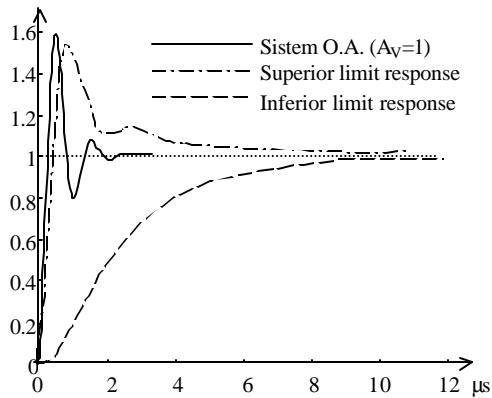


Fig. 7. Step response for limits  $Mu(s)$  y  $Ml(s)$ .

The marked specifications are translated in the QFT bounds. These bounds indicate the frequency response restriction to get the proposed objectives.

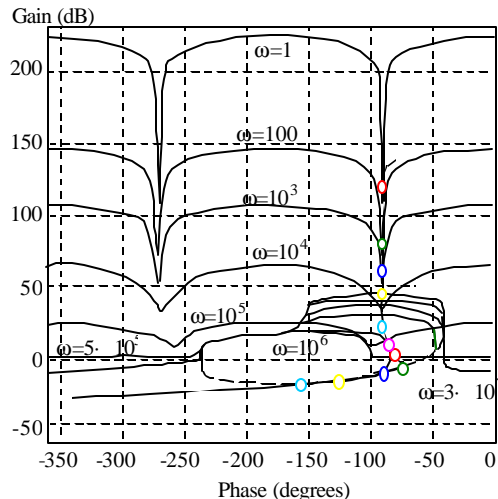


Fig. 8. Loop-Shaping for controller design.

The stability and tracking bounds obtained are intersected to get the most restrictive specifications. Once this operation is carried out, we go to the design.

It is important to look for an easy controller to get a implemently system. Figure 8 shows the obtained solution, that involves the following controller

$$G = \frac{89.03s^2 + 6.767 \cdot 10^7 s + 2.555 \cdot 10^{10}}{s^2 + 4.923 \cdot 10^8 s + 2.135 \cdot 10^6}$$

The controller is of a low order and the specifications are almost perfectly fitted, it would just test to calculate the pre-filter. The design itself is similar to the controller,  $G(s)$ , and it's obtained as a result

$$F(s) = \frac{2.582 \cdot 10^6}{s + 2.582 \cdot 10^6}$$

The 65 plants simulation behaviour facing the unitary step is practically found as a whole in the fixed specifications. The answers of the further fixed specification plants are shown in figure 9, and as it can be seen, the discordance among them is perfectly acceptable.

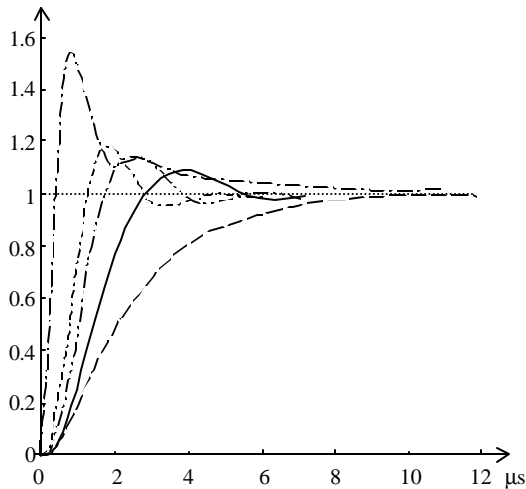


Fig. 9. Worst temporal responses for the controlled system.

#### 4. SEARCH OF SOLUTIONS

Once the controller is designed, our next goal is the system implementation. However two problems appear:

1. The controller has a pole in 87 MHz.
2. The controller has a gain of low frequency superior to 10000.

Which physical system is able to offer a pole in 87 MHz and gain of 10000, with the rest poles further away so as not to produce a frequencial charge effect?

This solution is not possible. Given the O.A. bandwidth, a controller with a huge bandwidth is necessary, what produces the necessity of no-using

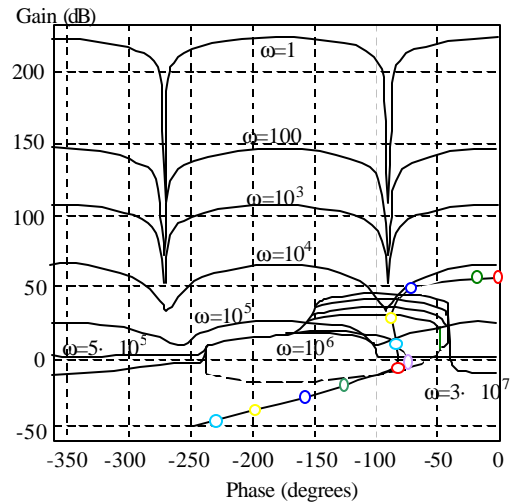


Fig. 10. Passive controller Loop-shaping..

active devices (since the bandwidth of them is not so big). This means the only real solution can be obtained by means of passive elements, what at the time will force, without any doubt to relax the fixed specifications. In order to look for a controller which could be implemented with passive components, and wich reaches the specifications in its maxims as well (always bearing the stability conditions), we have obtained

$$G(s) = 0.0243 \frac{2.602 \cdot 10^{-6} s + 1}{3.2691 \cdot 10^{-7} s + 1}$$

The frequencial response located upon Nichols's chart and the bounds founds are shown in figure 10.

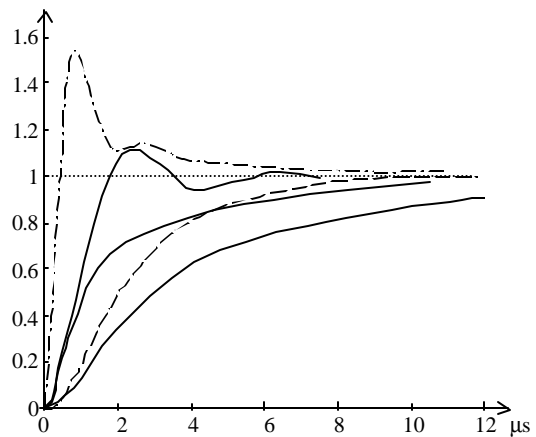


Figure 11. Worst response for the system with passive controller.

It is observed that the tracking specifications give up working in many frequencies, but those of stability do still work. In this situation we can avoid the pre-filter calculation. Figure 11 shows the step response for most extreme plants.

Obviously the system works much more slowly, but watching the step responses is checked that they don't differ that much from the imposed limit responses.

### 5. ELECTRONIC IMPLEMENTATION

The block diagram to implement is the one shown in figure 4, with the particularity that in the end, the pre-filter is not necessary. The main problem is to know how to carry out the difference junction. Electronically, a difference junction is built by means of a differential amplifier, but for that it is necessary to use an O.A.; which will produce a frequencial charge due to the poles its gain owns. This situation does to look for other solution.

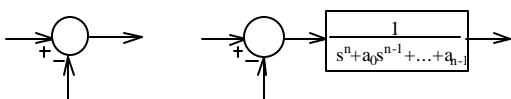


Fig. 12. Difference junction ideal and real model.

The problem answer could result in using the O.A. in itself as a difference junction. The O.A. is a differential amplifier and, in order to use it as a difference junction, it would be enough just to transform the block diagram of figure 4 into the one of figure 13.

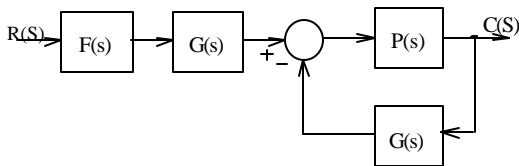
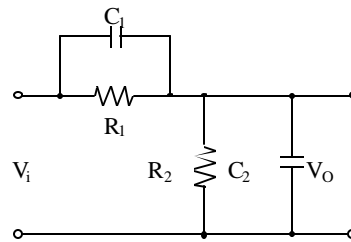


Fig. 13. New structure of QFT block diagram control.

When the pre-filter is designed, the improvement behavior is poor. For this reason the pre-filter can be eliminated.

With this last consideration we introduce the controller. The electronic circuit that would implement the transfer function is shown in figure 14. The total system without pre-filter is showed in figure 15.



$$\frac{V_o}{V_i}(s) = \frac{R_2}{R_1 + R_2} \frac{R_1 C_1 s + 1}{\frac{R_1 R_2}{R_1 + R_2} (C_2 + C_1) s + 1}$$

Fig. 14. Controlador.  $C_1=2.3979\text{nF}$ ,  $R_1=1085.11\Omega$ ,  $R_2=27.0248\Omega$ ,  $C_2=10\text{nF}$

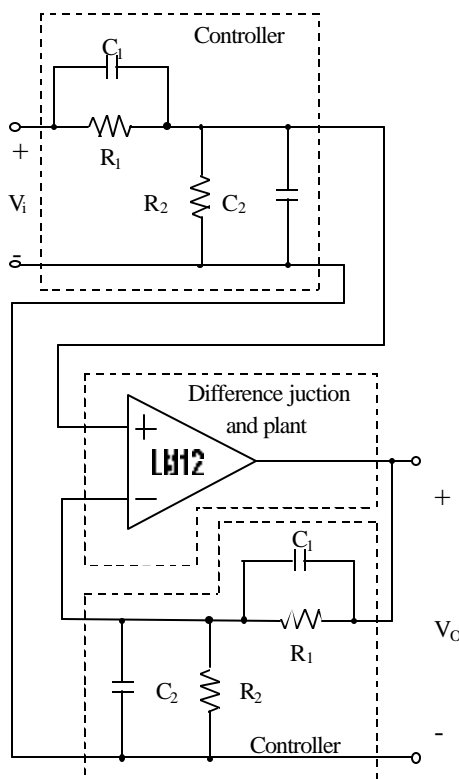


Fig. 15. Total system with controller.

In reference to write it above, it is possible change the product  $F(s) \cdot G(s)$  for a simple voltage divisor implemented with the  $R_1$  and  $R_2$  resistors ( $C_1$  y  $C_2$  can be eliminated from this block diagram part).

### 6. CONCLUSIONS

The paper has shown the possibility of controlling very fast systems, which have uncertainty by the way of QFT.

The active devices that implemented a controller, which it is used to control a fast system, must have got a frequencial response faster than the system. In other situation frequencial effect charge is produced. It is obvious, but if faster active devices do not exist, the unique solution is to use passive elements to implement the controller. It means that we must relax design specifications and work's conditions.

Finally, we have to indicate that QFT has allowed to go from the ideal, optimal and unattainable solution, to a real and implementely solution, offering the brigde between the pure theoretical design and a real engineering design.

#### REFERENCES

- Borghesani, C., Yossi Chait, Oded Yaniv. (1994) Quantitative feedback theory toolbox. The Math Works Inc.
- D'azzo, J.J., Houpis, C. H. (1995) Quantitative Feedback Theory (QFT) Tecnique. In: Linear Control System Analysis and Design. 4<sup>th</sup> Ed. , McGraw Hill.
- Ewing, R. L., Houpis, C. H., Rasmussen, S. (1997). Robust operational amplifier performance design achieved with Quantitative Feedback Theory. 3<sup>rd</sup> International Symposium of Quantitative Feedback Theory an other Frequency Domain Methods and Applications. Glasgow, Scotland.
- García-Sanz, M. (1999) Técnicas de control robusto para procesos multivariables. UPN
- Graeme, Jerald G. (1991). Feedback plots define Op Amp AC performance.. Burr-Brown. Application bulletin
- Kuo, B. C. (1996). Sistemas de control automático. 7<sup>a</sup> Edición. Prentice Hall.
- Malik, Norbert R. (1996). Circuitos electrónicos. Análisis, simulación y diseño.. Prentice Hall.
- National Semiconductor (1972). Predicting Op Amp slew rate limited response.
- National Semiconductor (1974). The monolithic operational amplifier: a tutorial study.. (IEEE Journal of solid-state circuits, vol. SC-9, N°6). Apendix A A.
- National Semiconductor (1999). LM12CL 80w operational amplifier.
- National Semiconductor (1999). LM7171. Very high speed, high output current, voltage feedback amplifier.
- Ogata, K.(1998). Ingeniería de control moderna.. 3<sup>a</sup> Edicion. Prentice Hall.
- Widlar, Robert J. (1968). Monolithic Op Amp-The universal linear component. National Semiconductor.
- Widlar, Robert J. (1986). A 150w IC Op Amp simplifies design of power circuits. National Semiconductor. Application Note 446.
- Widlar, Robert J. (1998) A monolithic power Op Amp. National Semiconductor. Application note 446B.
- Zalotas, A.C., Halikias, G. D. (1999). Optimal design of PID controllers using QFT method. IEE proc.- Control Theory appl., vol. 146, N° 6.