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ALGORITHM OF PARAMETERS' IDENTIFICATION OF POLYHARMONIC FUNCTION

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Abstracts: The problem of on-line continuous-time estimation of parameters of a polyharmonic function is considered. The analytical conditions of existence of the solution are presented and the design procedure of the estimation algorithm is proposed. The presented results were obtained on the basis of the theory of adaptive systems and identification theory, mathematical theory of stability as well as approaches of the linear optimal control. *Copyright © 2002 IFAC*

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1. INTRODUCTION

In the paper the problem of on-line continuous-time estimation of the parameters (frequencies) of a polyharmonic function is considered. Given problem is a classical problem of the systems theory. A nonlinear dependency of the function on its parameters prevents using standard, well-known methods of adaptive control and identification (Fomin, 1981; Fradkov, 1999; Anderson, 1986), since a presentation of periodic function as output of dynamic system leads to the problem of a simultaneous estimation of the state variables and unknown parameters, that presents the especial difficulty in continuous-time. In the last papers dedicated to the given problem (Regalia, 1995; Hsu, et al., 1997) the estimation algorithms of the frequency of the measured sinusoidal signal $f = A\sin(\omega t + \varphi)$ were proposed. However those algorithms can not be developed for an estimation of parameters of a polyharmonic signal.
The solution of the considered problem can find the wide practical application in self-learning of robotic systems (Miroshnik et, al., 1994) and adaptive noise damping systems (Nikiforov, 1997; Pogromsky et, al., 1997). For example, the problem of overcoming the function uncertainties in control laws arises from the design of mobile robot control systems for a motion along physically detectable, but analytically unknown paths (border of physical object). Solution of this problem is based on the using of a strategy of self-learning and an algorithm of approximation of the unknown periodic functional dependences (Lyamin, et al., 1998).

In the paper analytical conditions of existence of the solution are presented for the case of on-line continuous-time estimation of parameters of a polyharmonic function. The design procedure of the estimation algorithm is proposed. Presented results were obtained on the basis of adaptive systems theory and identification (Fomin, 1981; Fradkov, 1999; Anderson, 1986), mathematical theory of stability as well as approaches of the linear optimal control (Kwakernaak, 1997; Kalman, 1964).

2. STATEMENT OF THE PROBLEM

Given polyharmonic function (signal)

$$f(t) = C_0 + \sum_{i=1}^{n} A_i \sin\omega_i t + \sum_{i=1}^{n} B_i \cos\omega_i t, \qquad (1)$$

where $C_0, A_1, \dots, A_n, B_1, \dots, B_n, \omega_1, \dots, \omega_n$ are unknown parameters.

The considered problem is to find an adaptation algorithm generating estimate of the unknown parameters on the basis of current values of the function $f(t)$ and establish conditions of the algorithm efficiency.

The solution of the above stated problem is founded on the basis of the following assumptions:

1) the number n is known;
2) the vector-function

$$F(t) = \left[f(t), f^{(1)}(t), f^{(2)}(t), \dots, f^{(2n)}(t)\right]^T \qquad (2)$$

is strongly integral nondegenerate. In other words, there exist positive numbers L, β, such that

$$\int_t^{t+L} F(s)F^T(s)ds \ge \beta I \qquad (3)$$

for any $t > 0$.

3. DYNAMIC MODELS OF THE POLYHARMONIC FUNCTION

In this section we present analysis of properties of dynamic models of the polyharmonic function (1). To get the dynamic model consider the m-dimensional vector-function $\xi_*(t) = F(t)$ and the constant vector $\theta = [\theta_1, \dots, \theta_m]^T$, which components $\theta_1, \dots, \theta_m$ satisfy the identity

$$s^m - \theta_m s^{m-1} - \dots - \theta_2 s - \theta_1 \equiv$$
$$\equiv s(s^2 + \omega_1^2) \cdot \ldots \cdot (s^2 + \omega_n^2), \qquad (4)$$

where $m = 2n+1$, s is a complex variable. Then differentiating the vector - function $\xi_*(t)$ and taking into account the relations (1), (2) and (4) we find

$$\dot{\xi}_* = \Gamma\xi_* = \Gamma_0\xi_* + b\theta^T\xi_*, \qquad (5)$$
$$f = c^T\xi_*, \qquad (6)$$

where $\Gamma_0 = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \\ 0 & 0 & 0 & \dots & 0 \end{bmatrix}$, $b = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$, $c = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}$.

Thus, it is possible to consider the function $f(t)$ as the particular solution of the system of differential equations (5), (6). In other words, function $f(t)$ may be presented in the form

$$f(t) = c^T \exp(\Gamma t)\xi_*(0).$$

Lemma 1. Let the matrix $T = k_1 I + k_2\Gamma + k_3\Gamma^2 + \dots + k_m\Gamma^{m-1}$ be nonsingular. Then there exists transformation of coordinates such, that the model (5), (6) is equivalent to a system

$$\dot{\xi} = \Gamma\xi, \qquad (7)$$
$$f = k^T\xi, \qquad (8)$$

where $k = [k_1, k_2, \dots, k_m]^T$, $\xi = T^{-1}\xi_*$.

Proof. Let the matrix *T* be nonsingular, then differentiating vector ξ, we obtain:

$$\dot{\xi} = T^{-1}\dot{\xi}_* = T^{-1}\Gamma\xi_* = T^{-1}\Gamma T\xi.$$

Considering the equation

$$T^{-1}\Gamma T = T^{-1}\Gamma(k_1 I + k_2\Gamma + k_2\Gamma^2 + \dots + k_m\Gamma^{m-1}) =$$
$$= T^{-1}(k_1 I + k_2\Gamma + k_2\Gamma^2 + \dots + k_m\Gamma^{m-1})\Gamma = T^{-1}T\Gamma = \Gamma$$

we obtain

$$\dot{\xi} = \Gamma\xi.$$

By force of structure of the matrixes c, Γ we obtain

$$f = c^T\xi_* = c^T T\xi = k^T\xi \ \blacksquare$$

Remark. It should be noted that by appropriate choice of the coefficient k_1 we may provide nonsingularity of the matrix T. The last is satisfied under the condition $|k_1| >> |k_i|$ for $\forall i = \overline{2,m}$. Really if we present matrix T in the form

$$T = k_1 I + D,$$

where elements of the matrix $D = k_2\Gamma + k_3\Gamma^2 + \dots + k_m\Gamma^{m-1}$ are limited numbers, then under the condition $|k_1| >> |k_i|$ we have that $\det T \ne 0$.

Lemma 2. Let the vector $k = \rho P b$ and the matrix $P = P^T > 0$ be a solution of Riccati equation

$$\Gamma_0^T P + P\Gamma_0 - \rho P b b^T P = -2\alpha P - cc^T, \qquad (9)$$

where $\alpha > 0$. Then

$$\lim_{\rho\to\infty} \frac{kk^T}{\rho} = cc^T. \qquad (10)$$

Proof. Let us consider the matrix $G = \Gamma_0 + \alpha I - bk^T$. It follows from the structure of matrixes Γ_0, b, c and the asymptotic properties of the solutions of Riccati equation (9) (Kwakernaak, 1997; Kalman, 1964), that the characteristic polynomial of the matrix G under $\rho \to \infty$ asymptotically approaches the polynomial

$$a(s) = \prod_{i=1}^{m} \left(s - \omega e^{j\left(\frac{\pi}{2} + \frac{2i-1}{2m}\pi\right)} \right) =$$

$$= s^m + \alpha_{m-1}\omega s^{m-1} + \ldots + \alpha_1 \omega^{m-1} s + \alpha_0 \omega^m , \quad (11)$$

where $\omega = \sqrt[2m]{\rho}$, $\alpha_i > 0$ $(i = 0,1,\ldots,m-1)$ are the coefficients of the polynomial (11) under $\omega = 1$. From the relation (11) we find that elements k_i $(i = 1,\ldots,m)$ of the matrix k under $\rho \to \infty$ asymptotically approach the numbers $\alpha_{i-1}\rho^{\frac{m+1-i}{2m}}$ $(i = 1,\ldots,m)$ and, consequently, the limiting relation (10) is fulfilled. ■

Right now using above lemmas we will formulate and prove the main result of this section.

Theorem 1. Let the vector $k = \rho P b$ and the matrix $P = P^{\mathrm{T}} > 0$ be a solution of Riccati equation

$$\Gamma_0^T P + P\Gamma_0 - \rho P b b^T P = -2\alpha P - cc^T , \quad (12)$$

where $\rho > 0, \alpha > 0$ and $k = [k_1, k_2, ..., k_m]^T$. Then there exists a number ρ_0 such that the matrix T is nonsingular.

Proof. Using results of lemma 2, it is easy to show that

$$\lim_{\rho \to \infty} \frac{k_i}{k_1} = 0 \quad \forall i = \overline{2,m} .$$

Then there exists $\rho_0 > 0$ such that we have

$$k_1 >> k_i \quad \forall i = \overline{2,m} ,$$

under $\rho \geq \rho_0$ and, consequently, taking into account the remark, the matrix T is nonsingular. ■

4. DESIGN OF THE ESTIMATION ALGORITHM

Choose parameter ρ to guarantee the nonsingularity of the matrix

$$T = k_1 I + k_2 \Gamma + k_3 \Gamma^2 + ... + k_m \Gamma^{m-1},$$

where $k_1, k_2, \ldots, k_m$ are elements of the vector $k = \rho P b$, matrix $P = P^{\mathrm{T}} > 0$ is the solution of Riccati equation (12). Then the algorithm for estimation of functions $f(t)$ takes the form

$$\dot{\hat{\xi}} = (\Gamma_0 + b\hat{\theta}^T)\hat{\xi} + \mu b (f - \hat{f}), \quad (13)$$

$$\hat{f} = k^T \hat{\xi} , \quad (14)$$

where $\hat{\xi}$ is a current estimation of the vector $\xi(t)$ and $\hat{f}$ is a current estimation of the polyharmonic function $f(t)$, $\hat{\theta}$ is an estimation of the vector θ and the number $\mu > 1$.

For tuning the vector of the estimation $\hat{\theta}$ of unknown parameters θ we will use the algorithm of adaptation (Fomin, 1981; Fradkov, 1999)

$$\dot{\hat{\theta}} = K_a \hat{\xi} (f - \hat{f}) , \quad (15)$$

where $K_a = K_a^T > 0$ is a matrix of constant coefficients.

Using equations (7), (8) and (13) - (15) we obtain the model of deviations

$$\dot{\tilde{\xi}} = (\Gamma_0 + b\theta^T - \mu b k^T)\tilde{\xi} + b\hat{\xi}^T \tilde{\theta} , \ \tilde{f} = k^T \tilde{\xi} , \quad (16)$$

$$\dot{\tilde{\theta}} = -K_a \hat{\xi} \tilde{f} , \quad (17)$$

where $\tilde{f} = f - \hat{f}$, $\tilde{\xi} = \xi - \hat{\xi}$, $\tilde{\theta} = \theta - \hat{\theta}$.

To prove the main result of this paper it is needed to set the following fact.

Lemma 3. Let the vector-function (2) be strongly integral nondegenerate. Then the state vector $\xi(t)$ of the system (7) is strongly integral nondegenerate.

Proof. Consider the chain of equalities

$$\int_t^{t+L} F(s)F^T(s)ds = \int_t^{t+L} \xi_*(s)\xi_*^T(s)ds = \int_t^{t+L} T\xi(s)\xi^T(s)T^T ds .$$

The matrix T is nonsingular and

$$\int_t^{t+L} \xi(s)\xi^T(s)ds = T^{-1} \int_t^{t+T} F(s)F^{\mathrm{T}}(s)ds (T^{-1})^T \geq$$

$$\geq \beta\, (T^{-1})(T^{-1})^T \geq \beta\, \lambda I ,$$

where λ is minimum eigenvalue of matrix $(T^{-1})(T^{-1})^T$. ■

The conditions of asymptotic stability of solution $\tilde{\xi} = 0$, $\tilde{f} = 0$ and $\tilde{\theta} = 0$ of the systems (16), (17) are given in the following theorem.

Theorem 2. Let the vector-function (2) be strongly integral nondegenerate, then there exist numbers $\rho > 0$ and $\mu > 1$, such that solution $\tilde{\xi} = 0$, $\tilde{f} = 0$ and $\tilde{\theta} = 0$ of the systems (16), (17) are uniformly asymptotically stable on the whole.

Proof. Let us consider the Lyapunov function

$$V = \tilde{\xi}^T P \tilde{\xi} + \tilde{\theta}^T (K_a \rho)^{-1} \tilde{\theta} . \quad (18)$$

Differentiating (18) by force of the equations (16)-(17), we obtain

$$\dot V = \tilde\xi^T(\Gamma_c^T P + P\Gamma_c)\tilde\xi + \\ + \tilde\xi^T P b \hat\xi^T \tilde\theta + \tilde\theta^T \hat\xi b^T P \tilde\xi - \tilde\theta^T \hat\xi b^T P \tilde\xi - \\ - \tilde\xi^T P b \hat\xi^T \tilde\theta \ , \quad (19)$$

where $\Gamma_c = \Gamma_0 + b(\theta - \mu k)^T$. Taking into account the equations (12), (19) we find that

$$\dot V = \tilde\xi^T(\Gamma_0^T P + P\Gamma_0 - 2\mu\rho P b b^T P + P b\theta^T + \theta b^T P)\tilde\xi \le \\ \le \tilde\xi^T(-2\alpha P - (\mu+\varepsilon)\rho P b b^T P - \\ - c c^T + P b \theta^T + \theta b^T P)\tilde\xi \ , \quad (20)$$

where $\varepsilon = \mu - 1$. Since

$$\tilde\xi^T\left(\theta b^T P + P b \theta^T\right)\tilde\xi \le \tilde\xi^T\left(\rho\mu P b b^T P + \frac{1}{\rho\mu}\theta\theta^T\right)\tilde\xi \ ,$$

then the inequality (20) takes the form

$$\dot V \le \tilde\xi^T\Big(-2\alpha P - (\mu+\varepsilon)\rho P b b^T P + \rho\mu P b b^T P + \\ + \frac{1}{\rho\mu}\theta\theta^T\Big)\tilde\xi = \tilde\xi^T\left(-2\alpha P + \frac{1}{\rho\mu}\theta\theta^T\right)\tilde\xi - \frac{\varepsilon}{\rho}\tilde f^T \tilde f \ . \quad (21)$$

From above inequality it follows that there exist numbers $\rho > 0$ and $\mu > 1$, such that

$$\dot V \le -\delta \tilde\xi^T \tilde\xi - \frac{\varepsilon}{\rho}\tilde f^T \tilde f \ , \quad (22)$$

where $\delta > 0$. Thereby the solution $\tilde\xi = 0$, $\tilde f = 0$ and $\tilde\theta = 0$ of the systems (16), (17) are Lyapunov stable. Integrating inequality (22), it can be seen that

$$\tilde\theta \in L^\infty, \ \tilde f \in L^2 \cap L^\infty \ \text{and} \ \tilde\xi \in L^2 \cap L^\infty . \quad (23)$$

Since

$$\dot{\tilde\xi} = (\Gamma_0 + b\theta^T - \mu b k^T)\tilde\xi + b(\xi^T - \tilde\xi^T)\tilde\theta \in L^\infty \quad (24)$$

then the function $\tilde\xi(t)$ uniformly continuous and, consequently, according to Barbalat theorem (Popov, 1970)

$$\lim_{t\to\infty}\tilde\xi(t) = 0 \ , \ \lim_{t\to\infty}\tilde f(t) = 0 \ . \quad (25)$$

Then from (23)-(25), we find that $\lim_{t\to\infty}\xi^T\tilde\theta = 0$ and since by force of the lemma 3 the vector-function $\xi(t)$ is strongly integral nondegenerate, we have $\lim_{t\to\infty}(\theta - \hat\theta) = 0$ (Fomin, 1981; Fradkov, 1999).■

Substituting the found values $\hat\theta$ to the equation (4) we discover the unknown quantitative parameters ω_i.

5. CONCLUSION AND SIMULATION RESULTS

To illustrate the possibilities of the algorithm presented above we consider the problem of estimation of the function

$$f = C_0 + A\sin\omega t \ , \quad (26)$$

and its parameters $C_0 = A = \omega = 1$.
For given functions the models (13), (14) take the forms

$$\dot{\hat\xi}_1 = \hat\xi_2 \ , \quad (27)$$

$$\dot{\hat\xi}_2 = \hat\xi_3 \ , \quad (28)$$

$$\dot{\hat\xi}_3 = \hat\xi_1\hat\theta_1 + \hat\xi_2\hat\theta_2 + \hat\xi_3\hat\theta_3 + \mu(f - \hat f) \ , \quad (29)$$

$$\hat f = k_1\hat\xi_1 + k_2\hat\xi_2 + k_3\hat\xi_3 \ , \quad (30)$$

where $\hat\theta_1, \hat\theta_2, \hat\theta_3$ are the estimations of parameters $\theta_1 = 0, \theta_2 = -\omega^2, \theta_3 = 0$ and supplied by the algorithm (15)

$$\dot{\hat\theta}_i = k_{ai}\hat\xi_i(f - \hat f) \ , \ i = \overline{1,3} \ . \quad (31)$$

Choose the coefficients $k_1 = 154$, $k_2 = 66$, $k_3 = 12$ (for $\alpha = 1$ and $\rho = 10^4$ see equation (12)) to guarantee the nonsingularity of the matrix T, the number $\mu = 1{,}1$, and the coefficients of the adaptation $k_{a1} = k_{a2} = k_{a3} = 50$. Results of the simulation are illustrated on time-diagrams of the estimations $\hat\theta_1, \hat\theta_2, \hat\theta_3$ (fig. 1) and the error $\tilde f$ (fig. 2). The results demonstrate convergence of $\tilde f$ to zero and the estimates $\hat\theta_1, \hat\theta_2, \hat\theta_3$ of parameters $\theta_1, \theta_2, \theta_3$.

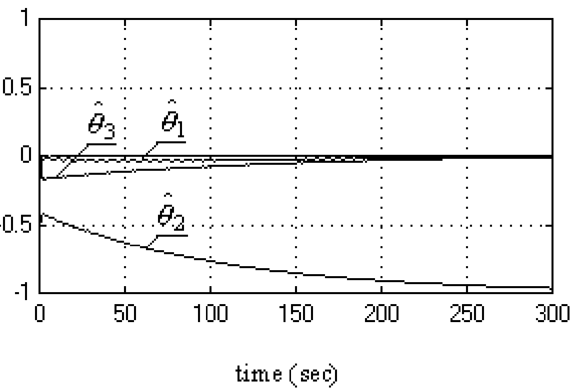

Fig. 1. Processes in a system identification: estimations of parameters.

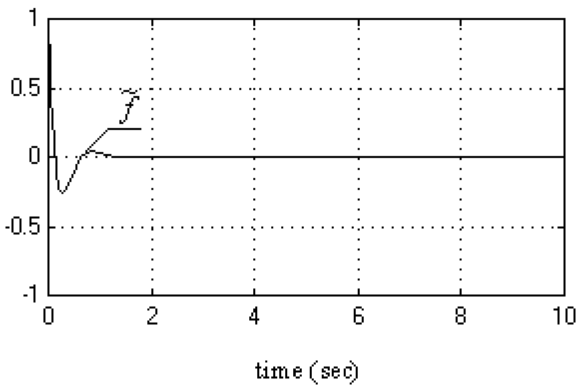

Fig. 2. Processes in system identification: signal of the error $\tilde{f}$.

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