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AN H_∞ ROBUST CONTROLLER FOR SINGLE-PHASE PWM INVERTERS

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Abstract: A robust controller for a single-phase Pulse-Width Modulation (PWM) inverter is designed. The single-phase PWM inverter generates an AC sinusoidal output from a DC link voltage. Robustness of the output waveform against load variations and minimum Total Harmonic Distortion (THD) are the main goals in this design. The importance of this design methodology results from the changes in the inverter's transfer function as affected by load variations. So it is necessary to use a robust controller such as H_∞. *Copyright © 2002 IFAC*

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1. INTRODUCTION

In many applications such as an AC voltage source or a high performance UPS, it is necessary to have a perfect sinusoidal waveform in the output, and the variation of load and other disturbances such as link DC voltage changes, nonlinear switching, or filter elements do not affect the output waveform.

As will be mentioned in Section 2 of this paper, the transfer function of an inverter changes as a result of load variations. Feed-forward open-loop control has been the common method used in this kind of inverters. In such a control, the slow feedback of the rms output value, can only regulate the amplitude of the output in small output amplitude variations and is not useful for the waveform control, also in fast variations, this method fails and the value of output THD is uncontrollable. Today, there are different methods for instantaneous control of the output voltage, which result in faster response, lower THD, disturbance reduction and lower output impedance. Some of these controllers are as follows:

1) Digital controllers with state feedback (Vukosavic, *et al*., 1990; Haneyoshi, *et al*., 1986).

2) Hysteresis controllers (Carpita, *et al*., 1987).

3) Sliding-mode controllers (Jung and Tzou, 1993; Jung and Tzou, 1996).

4) Analog controllers based on L-C current feedback (Ryan and Lorenz, 1995; Venkataramanan *et al.*, 1989).

5) Adaptive repetitive controllers (Tzou, et al.,1999).

6) High frequency controllers (Bowes *et al*., 2000).

Controllers 1) and 5) have high performance but need expensive processors to generate switches' pulse width in each cycle. 2) and 6) have a high and variable switching frequency. 3) need complex analog implementation, and 4) need wide bandwidth current sensors. The H_∞ controller has not been applied to this kind of inverters yet and is proposed here as a new control solution . It has a simple hardware implementation and good performance not only in the output waveform but also in the reduction of THD. This design methodology has been previously applied on some other applications, e.g., for control of a flexible-link manipulator (Yazdanpanah, *et al.*, 1997), and also on singularly perturbed systems (Karimi and Yazdanpanah, 2001).

The paper is organized as follows. In Section 2, the model of the inverter and the dynamic load model used in this paper will be presented, and in Section 3 the equivalent problem is discussed and stated as a standard H_∞ problem. Finally a controller is designed, for dynamic loads, using common design techniques. Section 4 is allocated to the simulation results, which show the good performance of the controller and the robustness of the closed loop system against load variations. Finally, there is a conclusion in Section 5.

2. THE DYNAMIC MODEL OF THE SINUSOIDAL INVERTER

Fig. 1 shows the simplified model of a single-phase PWM inverter. The plant discussed here consists of the half-bridge structure switches S1 and S2, the output filter LC and the RLC load. r_C and r_L are series resistances of filter elements, *C* and *L* respectively. The load is an *RLC* circuit whose L has a series resistance as seen in Fig. 2. V_{DC} is the input DC bus voltage, from which power is received and transmitted to the load with the help of the inverter. The switching of S1 and S2 are done such that the output voltage, V_{out}, has the least distortion from the reference signal, so the frequency of switching is determined by the controller design. Fig. 3 shows signal V_1 and the switching period. In the presence of switches S1 and S2, this system is nonlinear, so the state-space averaging method and small gain modeling were used to linearize the

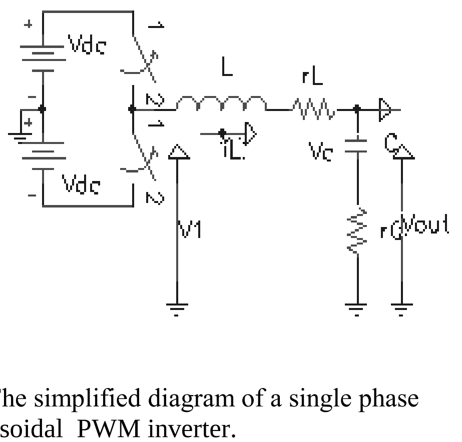

Fig. 1. The simplified diagram of a single phase sinusoidal PWM inverter.

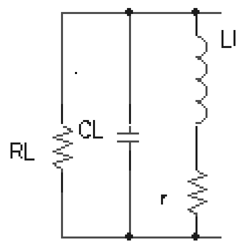

Fig. 2. RLC Load.

system. This model is valid if the carrier switching frequency $F_c = 1/T_c$ is greater than F- output frequency of the inverter. If the state vector is considered as $\mathbf{z}=[I_L \;\; \mathrm{V_c} \;\; \mathrm{I_{LL}}]^T$ the following state equation yields:

$$\dot{\mathbf{z}}=\mathbf{Az}+\mathbf{Bu}$$

$$\mathbf{A} = \begin{bmatrix} -\frac{r_L}{L} & -\frac{1}{L} & 0 \\ \frac{1}{(C+CL)} & \frac{-1}{R_L(C+C_L)} & \frac{-1}{(C+C_L)} \\ 0 & \frac{-1}{L_L} & \frac{-r}{L_L} \end{bmatrix}$$

$$\mathbf{B}= [1/L \;\; 0 \;\; 0]' \tag{1}$$

in which $\mathbf{u}=DV_{DC}$, $D = 2D_1 - 1$ and D_1 is as in Fig. 3. R_L , C_L and L_L are load elements and *C*, *L* and r_L are filter elements.

3. DESIGN OF A ROBUST CONTROLLER USING H_∞ TECHNIQUE

As seen in (1), matrix **A** varies as the load elements change, resulting in an uncertainty in **A**. If **A** is written as:

$$\mathbf{A}=\mathbf{A_0} + \Delta\mathbf{A} \tag{2}$$

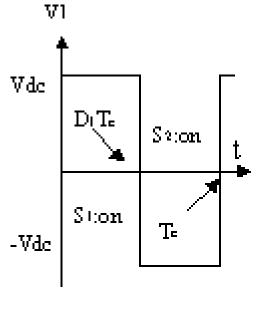

Fig. 3. The waveform of signal V_1

where $\mathbf{A_0}$ is computed by putting:

$$R_L = \infty$$
$$C_L = C_{nom}$$
$$L_L = L_{nom}$$

then $\mathbf{A_0}$ and $\mathbf{\Delta A}$ would be as follows :

$$\mathbf{A_0} = \begin{bmatrix} \frac{-r_L}{L} & \frac{-1}{L} & 0 \\ \frac{1}{(C+C_{Lnom})} & 0 & \frac{-1}{(C+C_{Lnom})} \\ 0 & \frac{1}{L_{nom}} & \frac{-r}{L_{nom}} \end{bmatrix}$$

$$\mathbf{\Delta A} = \begin{bmatrix} 0 & 0 & 0 \\ K' & \frac{-K}{R_L} & -K' \\ 0 & K'' & -rK'' \end{bmatrix}$$

$$K = \frac{1}{C+C_L}$$
$$K' = \frac{1}{C+C_L} - \frac{1}{C+C_{Lnom}}$$
$$K'' = \frac{1}{L_L} - \frac{1}{L_{nom}} \qquad (3)$$

and the model would be:

$$\dot{\mathbf{z}} = \mathbf{Az} + \mathbf{B_u u} = (\mathbf{A_0} + \mathbf{\Delta A})\mathbf{z} + \mathbf{B_u u}$$
$$= \mathbf{A_0 z} + \mathbf{B_u u} + \mathbf{B_w w} \qquad (4)$$

where

$$\mathbf{B_w} = \begin{bmatrix} 0 & 0 \\ \frac{-1}{C} & 0 \\ 0 & 1 \end{bmatrix},$$

$$\mathbf{\Delta_1} = \begin{bmatrix} -K'C & \frac{-KC}{R_L} & -K'C \\ 0 & K'' & -K''r \end{bmatrix} \qquad (5)$$

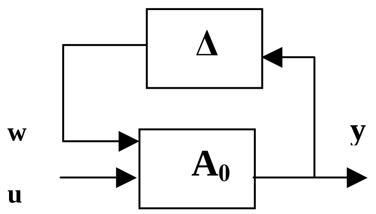

Fig. 4. Block diagram of the inverter system with uncertainty

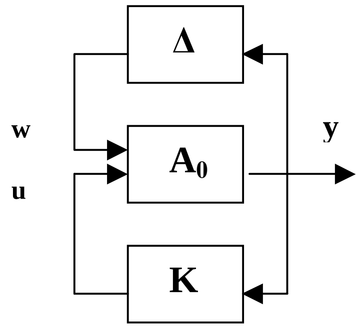

Fig. 5. Diagram of the inverter system with the controller

Consider $\boldsymbol{y}$=[$\mathbf{z}^T$ $\mathbf{u}^T]^T$, then:

$$\mathbf{w} = [\mathbf{\Delta_1}\ \mathbf{0}]\mathbf{y} = \mathbf{\Delta}\ \mathbf{y} \qquad (6)$$

The problem can be modeled as in Fig. 4. So in the present problem it is desirable to design a controller **K** such that:

1) Internal stability is guaranteed.

2) The property 1) is achieved for all uncertainties of the kind $\|\mathbf{\Delta}\|_\infty < 1/\gamma$ $(\gamma>0)$

The last problem is a standard H_∞ problem and can be solved using the classic methods, such as solving the Ricatti equation, or using a Hamiltonian system (Green and Limebeer, 1995). In (Abedor, *et al.*, 1995), it is proved that this problem is equivalent to the following:

For the system in Fig. 6, design **K** in such a way that:

1) **K** stabilizes $\mathbf{A_0}$ internally,

2) $\|T_{yw}\|_\infty < \gamma$ (T_{yw} is the transfer function from **w** to the **y**)

So if this problem is solved, the main problem is solved too. The equivalence of these two problems means that the uncertainty in the model can be considered as an external disturbance with bounded energy. If the optimum γ - the smallest $\gamma > 0$ for which a solution exists-is showed with γ_r, then the controller **K** exists if $\|\mathbf{\Delta}\|_\infty < 1/\gamma_r$ holds.

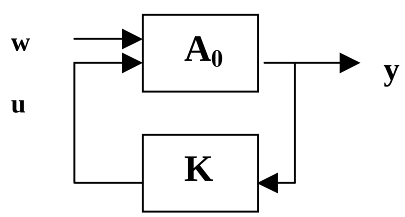

Fig. 6. Diagram for the inverter equivalent system

Assume $\|\mathbf{\Delta}\|_\infty = \gamma_s$ then the inequality $\gamma_s \gamma_r < 1$ results in the internal stability according to the small gain theorem, and $\|\mathbf{\Delta}\|_\infty < 1/\gamma$ determines the acceptable range of load variations.

If parameter values are considered as in Table 1, then $\gamma_r = 20.35$, so the value of $\|\mathbf{\Delta}\|_\infty$ should be always smaller than $\frac{1}{20.35} \approx 0.049$. With $\|\mathbf{\Delta}\|_\infty$ as the greatest eigenvalue of $\mathbf{\Delta}^T \mathbf{\Delta}$, its value for different load variations is calculated and it was seen by try and error methods that if the inequality $\gamma_s \gamma_r < 1$ is to be met , the load parameters should be chosen as:

$$\begin{gathered} R_L > 4\ \Omega \\ r > 1\ \Omega \\ 9\ \text{uF} < C_L < 11\ \text{uF} \\ L_L = 40\ \text{mH} \end{gathered} \tag{7}$$

This results in :

$$\|\mathbf{\Delta}\|_\infty = \gamma_s = 0.0496 \tag{8}$$

The related **K** is too large for optimum case and not practical, so a suboptimal solution is used with $\gamma=30$ and the obtained **K** is:

$$\mathbf{K} = [-2.8427 \quad 0.8724 \quad 3.0155] \tag{9}$$

Table 1: Parameter values

Parameter	Value
L_{nom}	40 mH
C_{nom}	11 uF
r	1.9 Ω
r_L	.3 Ω
r_c	0 Ω
L	2 mH
C	6 uF
V_{DC}	300 V
$I_L(0)$	5 A
$I_{LL}(0)$	3 A
$V_o(0)$	200 V

4. SIMULATIONS

With the help of Simulink^{TM} the mentioned system with the designed **K** is simulated. Fig. 7. shows the output voltage in the closed-loop case for different load variations with sinusoidal input reference as:

$$V_{in} = .8 \sin(100\pi t) \tag{10}$$

As seen, the variations of the output signal with the load variations at the time 25 ms, are negligible, except in fig. 7.d. for which the load variation is out of the allowed range. Fig. 8. shows the output voltage without using the controller, for different values of load. As seen, the output voltage is a function of the load variations. Fig. 9 shows the output voltage using the controller, against load variation. The robustness of the output is obvious., except in case c in which the load variation is out of the allowed range.

5. CONCLUSIONS

In this paper, a H_∞ robust controller was designed for a single-phase PWM inverter to have a perfect sinusoidal waveform in the output. To do this, first the uncertainty in transfer function was modeled, and then the H_∞ controller was designed and applied to the plant. As simulations show, the output voltage has good robustness against dynamic load changes.

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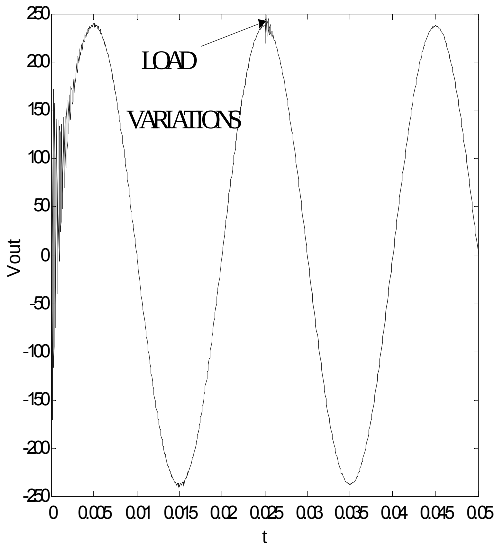
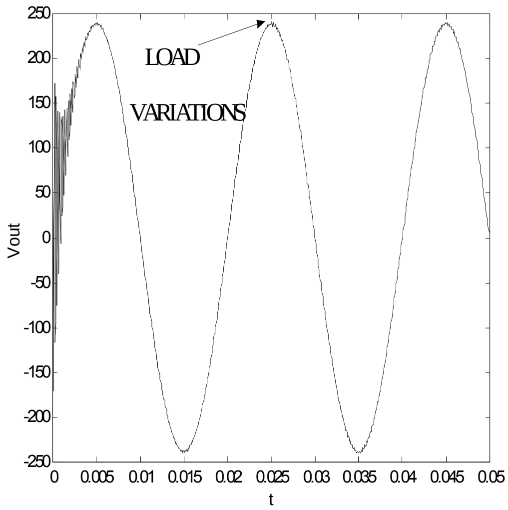
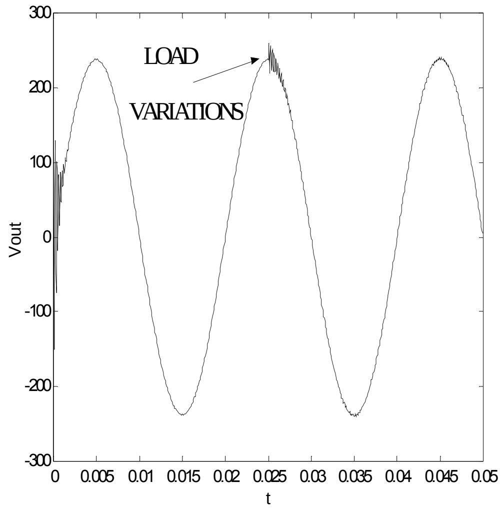


7.a

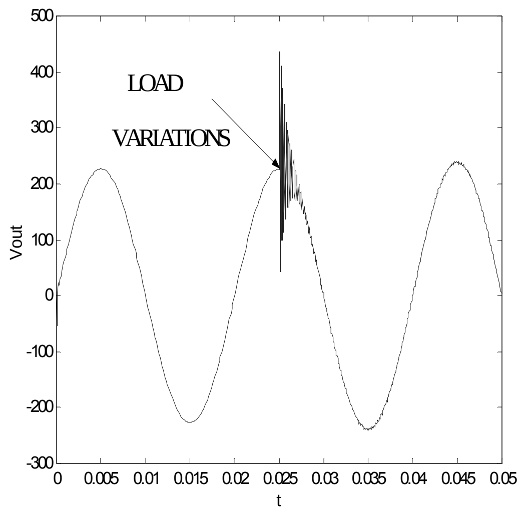
Fig. 7. Output voltage in the presence of controller for closed loop system and change of variable:
a. R_L=100 Ω ,C_L=9 uF to R_L=20 Ω ,C_L=11uF.
b. R_L=100 Ω ,C_L=9 uF to R_L=1000 Ω ,C_L=11uF.
c. R_L=∞ ,C_L=9 uF to R_L=20 Ω ,C_L=11uF.
d. R_L=100 Ω ,C_L=9 uF to R_L=2 Ω ,C_L=14uF.



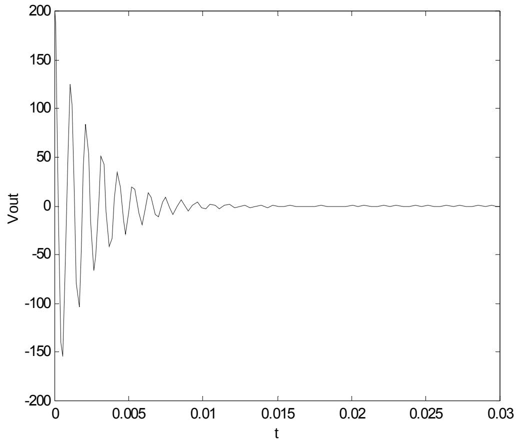
7.b



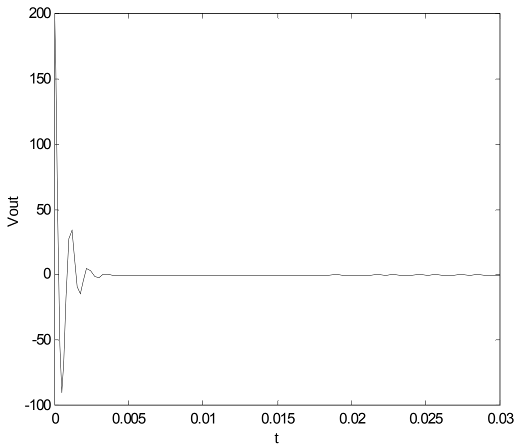
7.c



7.d



8.a.



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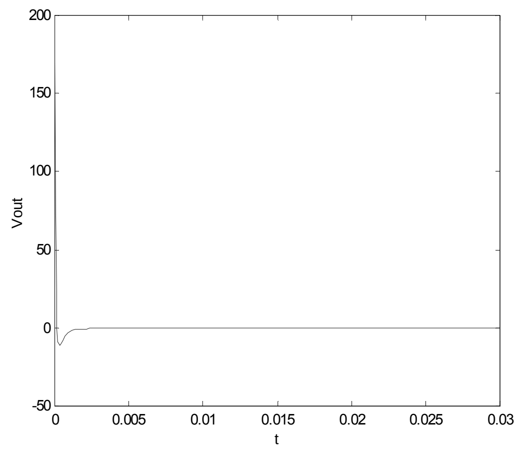
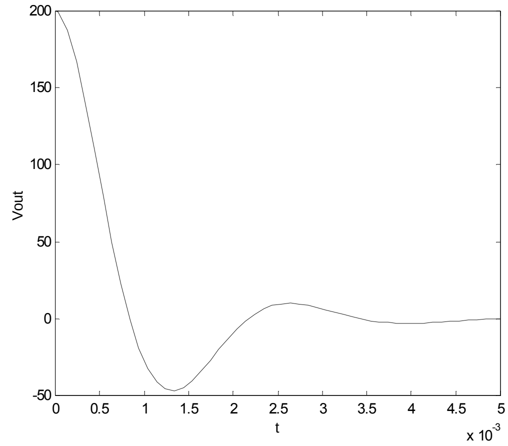


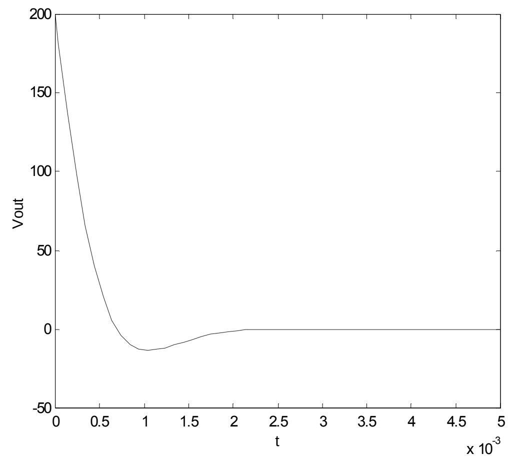
8.c.

Fig. 8. Output voltage without controller for different loads:

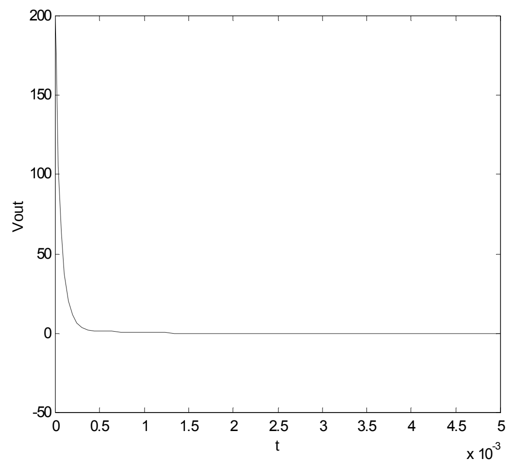
a. R_L=100 Ω, C_L=9 uF
b. R_L=20 Ω, C_L=11 uF
c. R_L=4 Ω, C_L=9 uF



9.a.



9.b.



9.c.

Fig. 9. Output voltage in the presence of the controller for different loads:

a. R_L=100 Ω, C_L=9 uF
b. R_L=20 Ω, C_L=11 uF
c. R_L=4 Ω, C_L=9 uF