

## IDENTIFICATION AND PREDISTORTION FOR A CLASS OF NONLINEAR SYSTEMS

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Abstract: The identification algorithm of Wiener-Hammerstein nonlinear model is proposed in the paper. When the nonlinearity in the Wiener-Hammerstein model is approximated by polynomial or spline function, the identification algorithms can be implemented iteratively, and the compensator for the nonlinear distortion is given by using the estimated Wiener-Hammerstein model and its inverse. A numerical simulation example of the power amplifier with nonlinearity demonstrates the effectiveness of the proposed method. *Copyright ©2002 IFAC*

Keywords: Identification, nonlinear models, Wiener-Hammerstein model, nonlinearity, parameter estimation, polynomials, splines.

### 1. INTRODUCTION

The majority of real systems have nonlinearity in practice. Some of them are described by the local linear models in a restricted operating range, for example, in many classical control problems the plants are controlled to keep within an acceptable range. However, when the operating range is large, the nominal local model is not enough to describe the global dynamic behavior, so it may fail to yield satisfactory performance for the specified application purpose. For example, the power amplifier distorts the signal when working near its saturation region. As a consequence, the strong demand for nonlinear model identification arises in many engineering applications.

With the development of neural networks and optimization techniques, various nonlinear identification schemes have been proposed to handle several classes of nonlinear systems, though the practical nonlinear processes have unique properties (Sjöberg and et al., 1995; Haber and Unbehauen, 1990; Ballings, 1980). For example, there are many classical approaches dealing with the

identification problems of polynomial-based models such as Hammerstein and Wiener models. Some researchers have studied the modeling approach using Volterra series. Neural networks, support vector machine and some learning algorithms have solved a wide class of nonlinear systems. Fuzzy logic also offers an important tool to handle fuzzy nonlinear model identification, etc.

The Hammerstein model and Wiener model are probably the most widely used nonlinear dynamic modeling approaches. Though the model structural assumptions about Hammerstein and Wiener models are restrictive, they describe the actuator, sensor and high power amplifier's nonlinearity in many engineering applications. Furthermore, it is easy to compensate the nonlinearity by a predistorter that implements the nonlinearity inverse. For these reasons Hammerstein and Wiener models are popular in control and signal processing areas. The classical algorithms estimate the parameters of the nonlinear elements and the linear dynamics simultaneously (Bai, 1998), or iteratively in a bootstrap way (Vörös, 1995), or separately in a blind manner

(Sun *et al.*, 1999), and some performance quantification results are also given (Ninness and Gibson, 2000). The nonlinearity in Hammerstein and Wiener models are classically approximated by a polynomial. On the other hand, the spline function also offers good approximation performance in many application situations (Zhu, 1999).

In this paper the identification problem for the Wiener-Hammerstein model is considered. The algorithms are developed for the polynomial approximated and spline function approximated nonlinear element. With the identified Wiener-Hammerstein model and its inverse model, the compensator is given to compensate the nonlinear distortion. Numerical simulation of the compensation for the distortion of a high power amplifier in OFDM digital communication systems illustrates the effectiveness of the proposed algorithms.

## 2. PROBLEM STATEMENT

Consider the Wiener-Hammerstein model illustrated in Fig.1. The model structure consists of a static nonlinear element  $G$  and two linear dynamics  $A$ ,  $B$ . It assumes the separation between the nonlinearity and the dynamics in the Wiener-Hammerstein models. It is seen that this assumption can reduce the complexity of the identification greatly. Denote the input, output signals as  $x$  and  $y$ , respectively.  $x_1$  and  $x_2$  are the unobservable intermediate signals. The task is the estimation of the parameters of two linear dynamics  $A$ ,  $B$  and the nonlinear element  $G$  or its inverse  $H$ , where  $G(H(x)) = x$ . Then the compensator is given to compensate the distortion caused by the nonlinear element  $G$ .

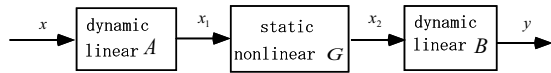


Fig. 1. Illustration of Wiener-Hammerstein model

In many applications, e.g. in communication systems, the input signal is composed by a set of frequency waves, so it is convenient to deal the problem in frequency domain. The assumptions in the identification and compensation problem are given as follows.

- (A.1) The separation between the linear dynamics and nonlinear element holds in the nonlinear system. This is the most important assumption in Hammerstein and Wiener models.
- (A.2) The system is stable, and the signals are bounded.
- (A.3) The nonlinear element  $G$  is a smooth continuous function, and can be approximated by

$$x_2 = G(x_1) = g_0 + g_1 x_1 + g_2 x_1^2 + \dots \quad (1)$$

or other static approximation such as spline function, which will be discussed later.

- (A.4) The linear dynamics  $A$  and  $B$  are described by rational transfer function model

$$A(s) = \frac{P_A(s)}{Q_A(s)}, \quad B(s) = \frac{P_B(s)}{Q_B(s)} \quad (2)$$

where the orders are known as *a priori*. When the model operates in the discrete-time framework, they can also be given by the corresponding discrete-time models.

- (A.5) In order to ensure the unique parameterization, some extra constrained conditions are necessary. Here it assumed that  $g_1 = 1$  and the constant coefficient of  $P_B$  is 1.
- (A.6) It assumed that the inverse models of the nonlinear element and linear dynamics exist to give the distortion compensator.

## 3. IDENTIFICATION ALGORITHM

The identification algorithms are considered for two situations where the static nonlinearity is approximated by classical polynomial, and spline function respectively. Without loss of the generality, it is assumed that the transfer function model of the linear dynamics is given in the continuous time form and the input signal consists of a finite number of periodic frequency waves.

### 3.1 Case of Polynomial Approximation

Assume that the nonlinear element  $G$  and its inverse  $H$  are approximated by polynomials. Their polynomial degrees are  $L_g$  and  $L_h$  respectively. Denote the estimates in the  $i$ -th iteration as  $\hat{A}^{(i)}(s)$ ,  $\hat{B}^{(i)}(s)$ ,  $\hat{G}^{(i)}$  and  $\hat{H}^{(i)}$  respectively. For the sake of simplicity, the affection of observation noise and other disturbances are not discussed in this paper. The identification algorithm is implemented iteratively. The initial estimate of  $A$  is given by  $\hat{A}^{(0)} = 1$ . The  $(i + 1)$ -th iteration is performed as follows.

#### 3.1.1. Estimation of $\hat{B}^{(i+1)}(j\omega)$ and $\hat{G}^{(i+1)}(x)$

Denote the Fourier transformation  $\mathcal{F}(x(t))$  as  $\mathcal{X}(j\omega)$ . It can be estimated through DFT at a discrete set of frequencies. Since the frequency of the output signal in nonlinear system might be infinite, the sampling frequency and DFT size should be large to grasp the main frequency characters. Then

$$\hat{\mathcal{X}}_1(j\omega) = \hat{A}^{(i)}(j\omega)\mathcal{X}(j\omega) \quad (3)$$

where  $\mathcal{X}_1(j\omega)$  is the Fourier transformation of  $x_1(t)$ . Then the frequency characteristics of the output signal  $y(t)$  can be approximated by

$$\mathcal{Y}(j\omega) = \hat{B}^{(i+1)}(j\omega) \left( \hat{g}_0^{(i+1)} + \hat{g}_1^{(i+1)} \hat{\mathcal{X}}_1(j\omega) \right)$$

$$+\hat{g}_2^{(i+1)}\hat{\mathcal{X}}_1^{(2)}(j\omega)+\dots+\hat{g}_{L_g}^{(i+1)}\hat{\mathcal{X}}_1^{(L_g)}(j\omega)\Big)(4)$$

at the discrete set of frequencies, where  $\mathcal{X}_1^{(l)}(j\omega) = \mathcal{F}(x_1^l(t))$ . Furthermore (4) leads to

$$\begin{aligned} & \left(1+j\hat{q}_{b,1}^{(i+1)}\omega-\hat{q}_{b,2}^{(i+1)}\omega^2-j\hat{q}_{b,3}^{(i+1)}\omega^3+\dots\right)\mathcal{Y}(j\omega) \\ &= \left(\hat{p}_{b,0}^{(i+1)}+j\hat{p}_{b,1}^{(i+1)}\omega-\hat{p}_{b,2}^{(i+1)}\omega^2-j\hat{p}_{b,3}^{(i+1)}\omega^3+\dots\right) \cdot \\ & \left(1+\hat{g}_1^{(i+1)}\hat{\mathcal{X}}_1^{(1)}(j\omega)+\hat{g}_2^{(i+1)}\hat{\mathcal{X}}_1^{(2)}(j\omega)+\dots+\hat{g}_{L_g}^{(i+1)}\hat{\mathcal{X}}_1^{(L_g)}(j\omega)\right) \end{aligned} \quad (5)$$

Then,  $\hat{B}^{(i+1)}(j\omega)$ ,  $\hat{G}^{(i+1)}(x)$  can be estimated from (5). Though the parameters in the right side of (5) is nonlinear because the parameter products of  $\hat{P}_B$  and  $\hat{G}$  appear, it is possible calculate them through a generalized model where the identical terms are over-parameterized in a linear manner followed by the separating operation from the oversized parameters (Bai, 1998). In the broadband situation, the frequency weighting should also be considered for the affect of high frequencies. Notice that the over-parameterized products of  $\hat{P}_B$  and  $\hat{G}$  can also be arranged in the multiplication of two polynomials such as

$$\begin{aligned} & (\hat{p}_{b,0}+\hat{p}_{b,1}z+\dots)(1+\hat{g}_1z+\dots) \\ &= \alpha_1+\alpha_2z+\alpha_3z^2+\dots \\ & \dots \\ & (\hat{p}_{b,0}+\hat{p}_{b,1}z+\dots)(\hat{g}_{L_g}+\dots+\hat{g}_1z^{L_g-1}+z^{L_g}) \\ &= \beta_1+\beta_2z+\beta_3z^2+\dots \end{aligned} \quad (6)$$

Then  $\hat{P}_B$  and  $\hat{G}$  can also be separated from these multiplications.

When the linear dynamics can be modeled by an AR model, (5) reduces to

$$\begin{aligned} & \left(1+j\hat{q}_{b,1}^{(i+1)}\omega-\hat{q}_{b,2}^{(i+1)}\omega^2-j\hat{q}_{b,3}^{(i+1)}\omega^3+\dots\right)\mathcal{Y}(j\omega) \\ &= \hat{p}_{b,0}^{(i+1)}\left(1+\hat{g}_1^{(i+1)}\hat{\mathcal{X}}_1^{(1)}(j\omega)+\hat{g}_2^{(i+1)}\hat{\mathcal{X}}_1^{(2)}(j\omega)+\dots+\hat{g}_{L_g}^{(i+1)}\hat{\mathcal{X}}_1^{(L_g)}(j\omega)\right) \end{aligned} \quad (7)$$

then the computation reduces to simple linear parameterization problem.

### 3.1.2. Estimation of $\hat{A}^{(i+1)}(j\omega)$

Denote Fourier transformation of  $x_2(t)$  as  $\mathcal{X}_2(j\omega)$ . At the given discrete set frequencies corresponding to DFT,  $\mathcal{X}_2(j\omega)$  is estimated by

$$\hat{\mathcal{X}}_2(j\omega) = \frac{\mathcal{Y}(j\omega)}{\hat{B}^{(i+1)}(j\omega)} \quad (8)$$

Then  $\hat{A}^{(i+1)}(j\omega)$  satisfies

$$\hat{A}^{(i+1)}(j\omega)\mathcal{X}(j\omega) = \hat{h}_0^{(i)} + \hat{h}_1^{(i)}\mathcal{X}_2^{(1)}(j\omega)$$

$$+\hat{h}_2^{(i)}\mathcal{X}_2^{(2)}(j\omega)+\dots+\hat{h}_{L_h}^{(i)}\hat{\mathcal{X}}_2^{(L_h)}(j\omega) \quad (9)$$

where  $\hat{\mathcal{X}}_2^{(l)}(j\omega) = \mathcal{F}(\hat{x}_2^l(t))$ , and the estimated signal  $x_2(t)$  is given by  $\mathcal{F}^{-1}(\hat{\mathcal{X}}_2(j\omega))$ ,  $\mathcal{F}^{-1}$  denotes the inverse Fourier transformation. Then the parameters of  $\hat{A}^{(i+1)}(j\omega)$  are estimated from

$$\begin{aligned} & \left(\hat{p}_{a,0}^{(i+1)}+j\hat{p}_{a,1}^{(i+1)}\omega-\hat{p}_{a,2}^{(i+1)}\omega^2-j\hat{p}_{a,3}^{(i+1)}\omega^3+\dots\right)\mathcal{X}(j\omega) \\ &= \left(1+j\hat{q}_{a,1}^{(i+1)}\omega-\hat{q}_{a,2}^{(i+1)}\omega^2-j\hat{q}_{a,3}^{(i+1)}\omega^3+\dots\right) \cdot \\ & \left(1+\hat{h}_1^{(i)}\hat{\mathcal{X}}_2^{(1)}(j\omega)+\hat{h}_2^{(i)}\hat{\mathcal{X}}_2^{(2)}(j\omega)+\dots+\hat{h}_{L_h}^{(i)}\hat{\mathcal{X}}_2^{(L_h)}(j\omega)\right) \end{aligned} \quad (10)$$

### 3.1.3. Estimation of Inverse $\hat{H}^{(i+1)}(x)$

The frequency characteristics estimation  $\mathcal{X}_1(j\omega)$  and  $\mathcal{X}_2(j\omega)$  of  $x_1(t)$  and  $x_2(t)$  can be updated using the estimates of  $\hat{A}^{(i+1)}(j\omega)$  and  $\hat{B}^{(i+1)}(j\omega)$ , then in the nonlinearity inverse model,

$$\begin{aligned} \hat{\mathcal{X}}_1(j\omega) &= \hat{h}_0^{(i+1)} + \hat{h}_1^{(i+1)}\hat{\mathcal{X}}_2^{(1)}(j\omega) \\ &+ \hat{h}_2^{(i+1)}\hat{\mathcal{X}}_2^{(2)}(j\omega) + \dots + \hat{h}_{L_h}^{(i+1)}\hat{\mathcal{X}}_2^{(L_h)}(j\omega) \end{aligned} \quad (11)$$

thus the estimate of  $\hat{H}^{(i+1)}(x)$ , which can be utilized to compensate the nonlinear distortion directly, is obtained.

Let  $i = i + 1$  then return to 3.1.1 for the next iteration.

## 3.2 Case of Spline Function Approximation

In the polynomial approximation, high polynomial degree is sometimes necessary to ensure high approximation accuracy, which leads to that the number of parameters significantly increases so the estimation will suffer from more computation complexity. It can be resolved by using the low order piecewise polynomials, e.g. the spline functions (Zhu, 1999). As illustrated in Fig.2, let the nonlinearity inverse  $H$  within the range of  $[\xi_i, \xi_{i+1}]$ , which is denoted as  $H_i$ , be approximated by a piecewise polynomial

$$H_i(y) = \sum_{l=0}^m h_{i,l}y^l \quad (12)$$

where  $\xi_i$  is the knot,  $m$  is the degree of the spline function. The similar structure can also be utilized to approximate the nonlinearity  $G$ .

The number of knots is  $I$ , then the parameter number of the spline function is  $(m+1)(I+1)$ . Assume that  $H_i$  and its derivation are continuous in  $[\xi_i, \xi_{i+1}]$ . Furthermore, at knot  $\xi_i$ , the following constrained condition

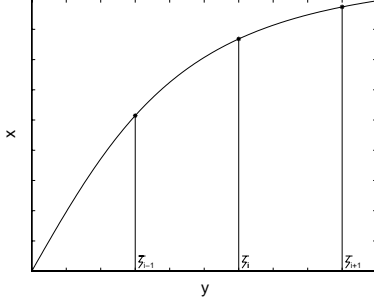


Fig. 2. Spline function and its knots

$$\left. \frac{\partial^l H_i(y)}{\partial^l y} \right|_{y=\xi_i} = \left. \frac{\partial^l H_{i-1}(y)}{\partial^l y} \right|_{y=\xi_i} \quad (13)$$

is often imposed on the smooth piecewise polynomial  $H_i$ , where  $l = 0, \dots, m-1$ . It means that the values of  $m$ -th degree spline function and its  $1, \dots, (m-1)$ -th derivation are continuous at every knot  $\xi_i$ . Then there are  $mI$  constrained conditions for (12), and the number of the independent parameters of the spline function reduces to  $(m+1)(I+1) - mI = m+I+1$ .

Now consider the estimation of the nonlinearity  $G$  and its inverse  $H$ .

### 3.2.1. Estimation of $H_0$

Following (12), the piecewise polynomial function satisfies the following relation

$$x_1(t) = H_0(x_2(t)) = \sum_{l=0}^m h_{0,l} x_2^l(t), \quad (14)$$

for  $x_2(t) < \xi_1$

then  $h_{0,l}$  can be calculated using classical algorithms such as least squares method.

### 3.2.2. Expressing $H_i$ by $H_{i-1}$

Substituting the estimated parameters of  $H_{i-1}$  into (13) yields that

$$\begin{aligned} \sum_{l=0}^m h_{i-1,l}(\xi_i)^l &= \sum_{l=0}^m h_{i,l}(\xi_i)^l \\ \sum_{l=1}^m l h_{i-1,l}(\xi_i)^{l-1} &= \sum_{l=1}^m l h_{i,l}(\xi_i)^{l-1} \\ &\dots \\ \sum_{l=m-1}^m l(l-1)\dots(l-m+2) h_{i-1,l}(\xi_i)^{l-m+1} \\ &= \sum_{l=m-1}^m l(l-1)\dots(l-m+2) h_{i,l}(\xi_i)^{l-m+1} \end{aligned} \quad (15)$$

Then  $h_{i,0}, \dots, h_{i,m-1}$  can be expressed by  $h_{i-1,0}, \dots, h_{i-1,m}$  and  $h_{i,m}$ .

### 3.2.3. Estimation of $H_i$

Substituting the expressions of  $h_{i,0}, \dots, h_{i,m-1}$  in (15) into  $H_i$  yields that there only  $h_{i,m}$  remains unknown in the expression of  $H_i$ , so it can be easily estimated from the observation data in the range of  $[\xi_i, \xi_{i+1}]$ . Moreover,  $h_{i,0}, \dots, h_{i,m-1}$  can be calculated from the estimates of  $h_{i-1,0}, \dots, h_{i-1,m}$  and  $h_{i,m}$  using (15).

Let  $i = i+1$ . Return 3.2.2 to estimate the parameters of  $H_{i+1}, \dots, H_I$ .

The polynomial degree  $m$  and the spline knots may be different, however, the estimation procedures of the nonlinearity  $G$  are almost the same as that of  $H$ . Furthermore, using the relations given in 3.2.2, there only  $m+I+1$  unknown parameters remain in the approximation of  $x_1(t) \sim x_2(t)$ . Substituting it into (4) or (9), the estimation can also be implemented in frequency domain easily.

### 3.3 Compensation of Nonlinear Distortion

The compensator of the nonlinear distortion can be designed by using the estimated Wiener-Hammerstein model. For example, in some signal processing applications, the output signal  $y(t)$  is required to close to the desired signal  $s(t)$ . Predistortion is the most popular method to compensate the distortion. It can be implemented as in Fig.3. First calculate the signals  $z_1(t)$  and  $z_2(t)$  using

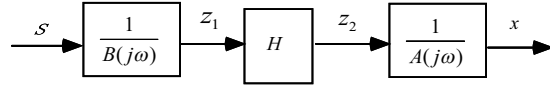


Fig. 3. Compensator of the Wiener-Hammerstein nonlinear model

the estimates of the Wiener-Hammerstein model as follows.

$$\begin{aligned} \mathcal{S}(j\omega) &= \mathcal{F}(s(t)) \\ \mathcal{Z}_1(j\omega) &= \frac{1}{\hat{B}^{(i+1)}(j\omega)} \mathcal{S}(j\omega) \\ z_1(t) &= \mathcal{F}^{-1}\{\mathcal{Z}_1(j\omega)\} \\ z_2(t) &= \hat{h}_0^{(i+1)} + \hat{h}_1^{(i+1)} z_1(t) + \hat{h}_2^{(i+1)} z_1^2(t) \\ &\quad + \dots + \hat{h}_{L_h}^{(i+1)} z_1^{L_h}(t) \end{aligned} \quad (16)$$

then the input signal of the Wiener-Hammerstein model  $x(t)$  is given by

$$\begin{aligned} \mathcal{X}(j\omega) &= \frac{\mathcal{F}(z_2(t))}{\hat{A}^{(i+1)}(j\omega)} \\ x(t) &= \mathcal{F}^{-1}(\mathcal{X}(j\omega)) \end{aligned} \quad (17)$$

When Assumption (A.6) holds, the inverse of the Wiener-Hammerstein model exist and stable, the bounded signal  $x(t)$  is the input signal to the power amplifier such that  $y(t) \approx s(t)$  holds.

#### 4. NUMERICAL EXAMPLES

In the simulation example the predistortion problem of a high power amplifier (HPA) is considered in the Orthogonal Frequency Division Multiplexing (OFDM) communication system. The desired source signal  $s(t)$  in the base band is given by

$$s(t) = \sum_{k=0}^{N-1} c_k e^{j2\pi f_k t} \quad (18)$$

where  $c_k = a_k + jb_k$  is the 64 QAM information symbol,  $a_k, b_k \in \{\pm 1, \pm 3, \pm 5, \pm 7\}$ ,  $f_k$  is the normalized carrier frequency, the carrier number  $N = 1405$ . The typical spectrum of  $s(t)$  is illustrated in Fig.4.

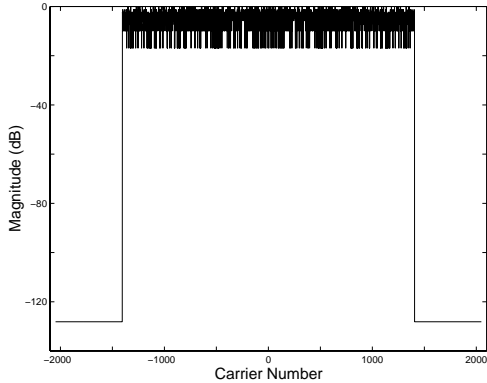


Fig. 4. Normalized spectrum of source signal  $s(t)$

The modulated signal is amplified by the radio frequency high power amplifier, which has saturation nonlinearity and can be modeled by the Wiener-Hammerstein model. Since the modulated signal  $s(t)$  is complex signal in the OFDM system, usually the power amplifier has both the amplitude (AM/AM) and phase (AM/PM) nonlinearities, as shown in Fig.5. The true nonlinear model in the simulation is given by

$$G(x) = g(x)e^{j[\angle x + \phi(x)]} \quad (19)$$

where

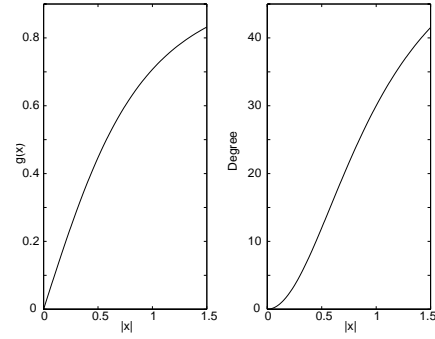
$$g(x) = \frac{\gamma_1 |x|}{(1 + \alpha_1 |x|^{2p})^{1/2p}}, \quad \phi(x) = \frac{\pi}{3} \frac{\gamma_2 |x|^2}{1 + \alpha_2 |x|^2}$$

and  $\alpha_1, \alpha_2, \gamma_1$  and  $\gamma_2$  are real numbers. Then the coefficients of the polynomial approximation and spline function approximation are handled as complex numbers.

Assume that the distortion caused by linear dynamics  $A$  and  $B$  are modeled by two low-pass AR model as follows.

$$A(j\omega) = \frac{4.43807294714819}{1 + 0.00111846510451j\omega} \quad (20)$$

$$B(j\omega) = \frac{1.16002598448587}{1 + 0.00074369688893j\omega}$$



(a) Amplitude distortion  $g(x)$  (b) Phase distortion  $\phi(x)$

Fig. 5. The amplitude and phase nonlinearities

where  $\omega$  is the normalized frequency. In the simulation,  $\alpha_1 = \alpha_2 = 1.0$ ,  $\gamma_1 = \gamma_2 = 1.0$ , and  $p = 1$ . Let the DFT size be 2048. The spectrum of signal  $y(t)$  with distortion is illustrated in Fig.6. Due to the amplitude, phase nonlinearities and the low pass linear dynamics, the high order cross-modulated products, which are the severe interference to the recovered signal as shown in Fig.7, occur at the carrier band, as well as the adjacent channel interference occurs at the outside of the carrier band. Then the predistortion is necessary for the high power amplifier (Nojima, 1999; Saleh and Salz, 1983).

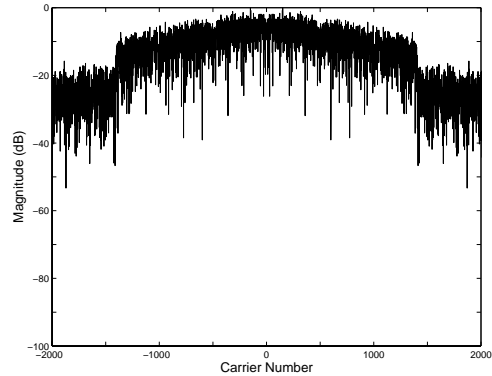


Fig. 6. The spectrum with distortion

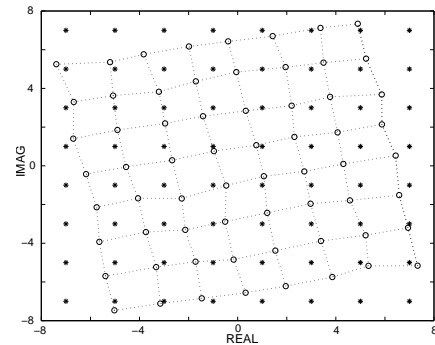


Fig. 7. The distortion of 64QAM signal obtained from the amplifier. “\*”: True source signal; “o”: Recovered signal with distortion

Since the source input signal  $s(t)$  consists of a series of harmonics frequency elements, the iden-

tification and predistortion are very easy to be implemented in frequency domain. The predistortion is performed for two cases where  $G(x)$  is approximated by ordinary polynomial and spline function. In the polynomial approximation case, the nonlinearity of  $G$  and  $H$  are approximated by

$$G(x(t)) = g_0x(t) + g_1|x(t)|^2x(t) + \dots + g_{L_g}|x(t)|^{2L_g}x(t) \quad (21)$$

$$H(y(t)) = h_0y(t) + h_1|y(t)|^2y(t) + \dots + h_{L_h}|y(t)|^{2L_h}y(t) \quad (22)$$

respectively following the nonlinear model in (19), where the coefficients  $g_l$ ,  $h_l$  and the signals  $x(t)$ ,  $y(t)$  are complex numbers. In the simulation  $L_g = 4$  and  $L_h = 5$ . The predistortion result is given in Fig.8. With 8 iterations, the distortion decreases more than 50dB. And the recovered information symbol from signal  $y(t)$  is very close to the true ones as shown in Fig.9.

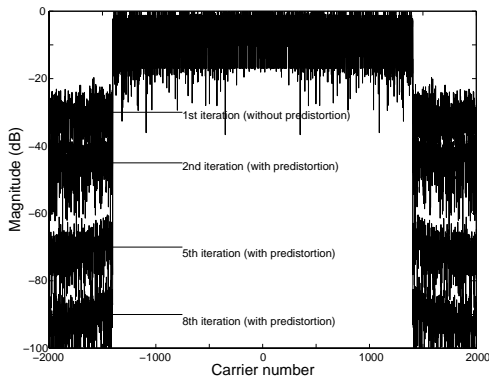


Fig. 8. Predistortion result

In the spline function case, the function degree both the nonlinearity  $G$  and inverse  $H$  is chosen as  $m = 1$ , i.e.

$$\begin{aligned} H_i(y(t)) &= h_{i,0}y(t) + h_{i,1}|y(t)|^2y(t) \\ G_i(x(t)) &= g_{i,0}x(t) + g_{i,1}|x(t)|^2x(t) \end{aligned} \quad (23)$$

The 4 knots are chosen as  $\xi_i = 0.25, 0.49, 0.64, 0.81$ . The predistortion result is almost the same as that in Fig.8 and Fig.9, but the computation is less than 2/3 of that of the ordinary polynomial case.

## 5. CONCLUSIONS

The identification and nonlinearity compensation of a class of nonlinear systems that are modeled by Wiener-Hammerstein models are investigated in this paper. By approximating the nonlinearity in traditional polynomial or spline function, the nonlinearity is estimated iteratively, and the predistortion of the nonlinearity is also performed using the inverse model. The numerical simulation illustrates that the proposed algorithms have good performance for the nonlinearity predistortion of

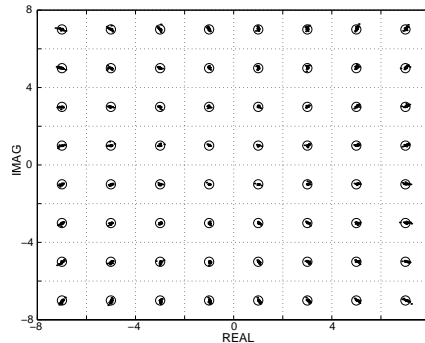


Fig. 9. The recovered information symbol with predistortion. “o”: True; Dots: Recoverd

the high power amplifier in OFDM communication systems. The next endeavor is to implement the identification and predistortion algorithm in the practical system to achieve high communication performance.

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