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ITERATIVE LEARNING CONTROL FOR NON-UNIFORM TRAJECTORY TRACKING PROBLEMS

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Abstract: In this work Iterative Learning Control is extended to non-uniform trajectory tracking problems for certain classes of nonlinear uncertain systems. The proposed ILC scheme can learn from different motion patterns and guarantee the asymptotic convergence even if the target trajectory varies at every iteration. The concept of Composite Energy Function (CEF) is adopted to facilitate the convergence property analysis, which shows that the uniform convergence of the tracking error is achieved.

Keywords: Iterative Learning Control, Non-uniform Trajectory, Composite Energy Function, Nonlinear Uncertain Systems

1. INTRODUCTION

Iterative learning control (ILC) is one kind of control methodology effectively dealing with repeated tracking control problems or periodic disturbance rejection problems (Arimoto *et al.*, 1984; D. H. Hwang and Oh, 1991; Moore, 1993; Kurek and Zaremba, 1993; Xu, 1997; Wang, 2000; Park and Bien, 2000).

Most ILC schemes developed hitherto are only valid where the target trajectory is uniform in all iterations. If any change occurs in target trajectory due to the variation of control objectives or task specifications, no matter how small it might be, the control system has to start learning process from the very beginning and the previously learned control input profiles can no longer be used. Tracking control tasks can be classified into uniform and non-uniform ones, from practical point of view we often face non-uniform trajectory tracking tasks in which the target trajectory may vary from iteration to iteration. Obviously non-uniform learning control is much more challenging.

Can a control system learn from non-uniform tasks? Intuitively, the control information of a particular task should contain the system information such as input-output relationship or input-to-state relationships, etc. It is possible for a control system to learn from the execution of distinct trajectories and improve the tracking performance gradually. Direct Learning Control (Xu and Zhu, 1999) and Recursive Direct Learning Control schemes (Xu *et al.*, 2001) have been developed recently to make use of previously obtained control information to generate control input for a new trajectory. A difficulty encountered in further expansion is the requirement for perfect preceding control information and the open-loop control nature. Note that an ILC can always start learning from scratch, but limited to a simple task with a uniform trajectory. Can we merge the two kinds of learning control methods such that the control system can learn from scratch with distinct tasks?

In this paper we present an ILC scheme that can partially fulfill the challenging objective. As far as the matching condition is satisfied and system unknowns to be learned are confined to time varying parametric types and trajectory irrelevant, a Lyapunov function can be constructed together with a learning mechanism. The Lyapunov function ensures the asymptotic convergence of the system nominal part in iteration domain. The learning mechanism estimates the parametric uncertainties in a pointwise manner over iterations. To analyze the convergence of the proposed ILC, the concept of Composite Energy Function (CEF) is further applied. The CEF is composed of a standard Lyapunov function and a L^2 norm of parametric learning errors. Through rigorous proof it is shown that the proposed ILC scheme guarantees the uniform convergence of tracking error for distinct trajectories. Therefore, ILC approaches handling same kinds of systems with identical target trajectory can be viewed as the specific cases of the proposed one.

The paper is organized as follow. Section 2 formulates the nonlinear dynamic system and the non-uniform tracking tasks. Section 3 presents the configuration of the ILC scheme with convergence analysis based on CEF. Section 4 applies the developed learning control approach to a nonlinear system and gives the simulation results.

2. PROBLEM FORMULATION

Consider a higher order MIMO nonlinear dynamical system described by

$$\begin{aligned} \dot{\mathbf{x}}_j &= \mathbf{x}_{j+1}, \quad j = 1, \cdots, m-1 \\ \dot{\mathbf{x}}_m &= \mathbf{f}(\mathbf{x}, t) + B(\mathbf{x}, t)[\mathbf{u}(t) + \mathbf{d}_1(\mathbf{x}, t)] \end{aligned} \tag{1}$$

where $\mathbf{x}_j \in R^n$, $j = 1, \cdots, m$; $\mathbf{x} \triangleq [\mathbf{x}_1^T, \mathbf{x}_2^T, \cdots, \mathbf{x}_m^T]^T \subseteq R^{nm}$ is the physically measurable state vector of the system; $\mathbf{u} \in R^n$ is the control input vector of the system; $B(\mathbf{x}, t) : R^{nm} \times R_+ \to R^{n\times n}$ is a known function with full rank; $\mathbf{f}(\mathbf{x}, t) : R^n \times R_+ \to R^n$ is a known mapping; $\mathbf{d}_1(\mathbf{x}, t) \in R^n$ is system uncertainties. This dynamic system is repeatable over a finite time interval $[0, T]$.

Since the desired state trajectories are different from iteration to iteration, for the i-th iteration, the desired trajectory for $\mathbf{x}_{1,i}$ is denoted as $\mathbf{x}_{1d,i}$ which is defined on the $[0, T]$. $\mathbf{x}_{1d,i}$ is differential with respect to t up to the mth order and all its higher-order derivatives

$$\mathbf{x}_{1d,i}^{(j)} \triangleq \mathbf{x}_{(j+1)d,i}, \quad j = 0, \cdots, m$$

are available over $t \in [0, T]$.

For the mth order dynamic system (1), an extended tracking error is defined at the ith iteration as

$$\boldsymbol{\sigma}_i(t) = \sum_{j=1}^{m} c_j \mathbf{e}_{j,i}(t), \quad c_m = 1$$

where $\mathbf{e}_{j,i}(t) \triangleq \mathbf{x}_{j,i}(t) - \mathbf{x}_{jd,i}(t)$ and c_j $(j = 1, \cdots, m)$ are coefficients of a Hurwitz polynomial.

Taking derivative of $\boldsymbol{\sigma}_i(t)$ with respect to time t yields

$$\begin{aligned} \dot{\boldsymbol{\sigma}}_i(t) = &\sum_{j=1}^{m-1} c_j \mathbf{x}_{j+1,i} - \sum_{j=1}^{m} c_j \dot{\mathbf{x}}_{(j+1)d,i} + \mathbf{f}_i \\ &+ B_i(\mathbf{u}_i + \mathbf{d}_{1,i}) \end{aligned} \tag{2}$$

where $\mathbf{f}_i = \mathbf{f}(\mathbf{x}_i, t)$, $B_i = B(\mathbf{x}_i, t)$, $\mathbf{u}_i = \mathbf{u}_i(t)$ and $\mathbf{d}_{1,i} = \mathbf{d}_1(\mathbf{x}_i, t)$.

In this paper, we assume that the system extended error dynamics satisfy the following assumptions.

Assumption 1. For each desired trajectory $\mathbf{x}_{d,i}$, there exist a C^1 Lyapunov function $V_i : R^n \to R_+$ and functions $\gamma_{1,i}$, $\gamma_{2,i}$ and $\gamma_{3,i}$, where $\gamma_{1,i}$, $\gamma_{2,i}$ belong to class-**KR** and $\gamma_{3,i}$ belongs to class-**K**, such that for a vector $\boldsymbol{\zeta}_i \in R^n$

$$\begin{aligned} &0 \le \gamma_{1,i}(||\boldsymbol{\zeta}_i||) \le V_i(\boldsymbol{\zeta}_i, t) \le \gamma_{2,i}(||\boldsymbol{\zeta}_i||) \\ &\frac{\partial V_i^T}{\partial t} + \frac{\partial V_i^T}{\partial \boldsymbol{\zeta}} \mathbf{g}_i(\boldsymbol{\zeta}_i, t) \le -\gamma_{3,i}(\|\boldsymbol{\zeta}_i\|). \end{aligned} \tag{3}$$

Note that the zero state of the system extended error described by

$$\dot{\boldsymbol{\zeta}}_i = \mathbf{g}_i(\boldsymbol{\zeta}_i, t) \tag{4}$$

is uniformly asymptotically stable.

Assumption 2. The deterministic dynamic system (1) will repeat itself under the initial resetting condition, i.e., $\mathbf{e}_{j,i}(0) \triangleq \mathbf{x}_{j,i}(0) - \mathbf{x}_{jd,i}(0) = 0$, $\forall i \in N_+ = \{1, 2, \cdots\}$.

According to Assumption 1, the extended error dynamics (2) can be rewritten as

$$\dot{\boldsymbol{\sigma}}_i(t) = \mathbf{g}_i(\boldsymbol{\sigma}_i, t) + B_i[\mathbf{u}_i + \mathbf{d}_i - B_i^{-1}\mathbf{g}_i(\boldsymbol{\sigma}_i, t)] \tag{5}$$

where $\mathbf{d}_i \triangleq \mathbf{d}_{1,i} + B_i^{-1}[\mathbf{f}_i + \sum_{j=1}^{m-1} c_j \mathbf{x}_{j+1,i} - \sum_{j=1}^{m} c_j \dot{\mathbf{x}}_{(j+1)d,i}]$ are the system uncertainties satisfying following assumption.

Assumption 3. The system uncertainties $\mathbf{d}_i$ can be represented as

$$\mathbf{d}_i = \Theta(t)\boldsymbol{\xi}(\mathbf{x}_i, \mathbf{x}_{d,i}, t), \quad \Theta \in R^{n\times n_1} \quad \boldsymbol{\xi} \in R^{n_1} \tag{6}$$

where n_1 is an appropriate number of dimension. $\Theta(t)$ is an unknown continuous time varying parameter matrix and $\boldsymbol{\xi}$ is a known vector function.

Remark 1. Although the second term in $\mathbf{d}_i$, $B_i^{-1} \cdot[\mathbf{f}_i + \sum_{j=1}^{m-1} c_j \mathbf{x}_{j+1,i} - \sum_{j=1}^{m} c_j \dot{\mathbf{x}}_{(j+1)d,i}]$, is known, it is treated by learning control. In this way the learning capability can be maximized.

The control objective is to track the non-uniform trajectories by determining a sequence of control input $\mathbf{u}_i \in R^n$, such that

$$\forall t \in [0, T] \quad \lim_{i\to\infty} \|\mathbf{e}_{1,i}(t)\|_s = 0 \tag{7}$$

where $\mathbf{e}_{1,i}(t) \triangleq \mathbf{x}_{1,i} - \mathbf{x}_{1d,i}$ is the tracking error of $\mathbf{x}_{1,i}$ at the i-th learning iteration.

Remark 2. It is assumed in the system that the input distribution matrix B is invertible. This is indispensable in non-uniform tracking problem. It is well known that a system is completely function-space controllable if and only if rank $B = n$ (Banks *et al.*, 1975). Since the target trajectory $\mathbf{x}_d$ is no longer a specific curve but may span the whole $\mathcal{R}^n$, n independent manipulating variables are needed.

3. ILC CONFIGURATION AND CONVERGENCE ANALYSIS

The proposed learning control law at the ith learning iteration is

$$\mathbf{u}_i = -\hat{\Theta}_i(t)\boldsymbol{\xi}_i + B_i^{-1}\mathbf{g}_i(\boldsymbol{\sigma}_i, t). \tag{8}$$

where $\boldsymbol{\xi}_i = \boldsymbol{\xi}(\mathbf{x}_i, \mathbf{x}_{d,i}, t)$. Here $\hat{\Theta}$ is to learn Θ and updated pointwisely over $[0, T]$ as

$$\hat{\Theta}_i(t) = proj(\hat{\Theta}_{i-1}(t)) + \beta\boldsymbol{\alpha}_i(\mathbf{x}_i, t)\boldsymbol{\xi}_i^T$$
$$\boldsymbol{\alpha}_i^T(\mathbf{x}_i, t) \triangleq \frac{\partial V_i^T}{\partial \boldsymbol{\sigma}_i} B_i \tag{9}$$

where β is the learning gain. Given any matrix $A \in R^{n\times m}$, the operator $proj(\cdot)$ is defined as

$$proj(A) = \{proj(a_{ij})\}_{n\times m}$$
$$proj(a_{ij}) = \left\{ \begin{array}{ll} a_{ij} & |a_{ij}| \le G \\ G \cdot sign(a_{ij}) & |a_{ij}| > G \end{array} \right. .$$

Remark 3. The projection bound should be sufficiently large to ensure the learnability. The bound can be either determined from the physical process limitation or simply using a virtual bound which can be arbitrarily large but finite.

We consider tracking performance from the first learning iteration $i = 1$ and $\hat{\Theta}_0(t) = 0\ \forall t \in [0, T]$. The convergence of the proposed control scheme is given by the following theorem.

Theorem 1. For system (1), under the Assumptions 1-3, the learning control law (8) and the updating law (9), guarantee the uniform convergence of tracking error over $[0, T]$, when the learning repetition approaches to infinity.

Proof:

The proof consists of two parts. Part A derives the difference of the composite energy function; Part B proves the uniform convergence of the tracking error.

Part A: Difference of Composite Energy Function

Define a non-negative energy function at the ith learning cycle as:

$$E_i(t) = \frac{1}{2\beta}\int_0^t trace[(\hat{\Theta}_i - \Theta)^T(\hat{\Theta}_i - \Theta)]d\tau$$
$$+ V_i(\boldsymbol{\sigma}_i, t) \tag{10}$$

where $V_i(\boldsymbol{\sigma}_i, t)$ is a Lyapunov function which satisfies Assumption 1.

Consider the difference of the energy function E_i at the ith learning iteration.

$$\Delta E_i(t) = E_i(t) - E_{i-1}(t)$$
$$= V_i(\boldsymbol{\sigma}_i, t) + \frac{1}{2\beta}\int_0^t \{trace[(\hat{\Theta}_i - \Theta)^T(\hat{\Theta}_i - \Theta)]$$
$$-trace[(\hat{\Theta}_{i-1} - \Theta)^T(\hat{\Theta}_{i-1} - \Theta)]\}d\tau - V_{i-1}(\boldsymbol{\sigma}_{i-1}, t).$$

According to Assumption 1, the initial resetting condition, control law (8) and updating law (9), the following can be derived.

$$V_i(\boldsymbol{\sigma}_i, t) = \int_0^t \left(\frac{\partial V_i^T}{\partial t} + \frac{\partial V_i^T}{\partial \boldsymbol{\sigma}_i}\dot{\boldsymbol{\sigma}}_i\right) d\tau + V_i(\boldsymbol{\sigma}_i(0), 0)$$
$$= \int_0^t \left[\frac{\partial V_i^T}{\partial t} + \frac{\partial V_i^T}{\partial \boldsymbol{\sigma}_i}\mathbf{g}_i(\boldsymbol{\sigma}_i, \tau)\right] d\tau + \int_0^t \frac{\partial V_i^T}{\partial \boldsymbol{\sigma}_i} \cdot$$
$$B(\mathbf{x}_i, \tau)[-\hat{\Theta}_i(\tau)\boldsymbol{\xi}_i + \Theta(\tau)\boldsymbol{\xi}_i]d\tau$$
$$\le -\int_0^t \gamma_{3,i}(\|\boldsymbol{\sigma}_i\|) + \int_0^t \boldsymbol{\alpha}_i^T(\mathbf{x}_i, \tau)[\Theta(\tau) - \hat{\Theta}_i(\tau)]\boldsymbol{\xi}_i d\tau$$
$$\triangleq -\int_0^t \gamma_{3,i}(\|\boldsymbol{\sigma}_i\|) + \int_0^t \varsigma(\tau)d\tau. \tag{11}$$

Similarly, using initial resetting condition, we can obtain

$$\dot{V}_i(\boldsymbol{\sigma}_i, t) \le -\gamma_{3,i}(\|\boldsymbol{\sigma}_i\|) + \varsigma(t) \tag{12}$$

According to updating law (9)

$$
\begin{aligned}
&\frac{1}{2\beta}\{trace[(\hat{\Theta}_i-\Theta)^T(\hat{\Theta}_i-\Theta)]\\
&-trace[(\hat{\Theta}_{i-1}-\Theta)^T(\hat{\Theta}_{i-1}-\Theta)]\}\\
\le&\frac{1}{2\beta}\{trace[(\hat{\Theta}_i-\Theta)^T(\hat{\Theta}_i-\Theta)]-\\
&trace[(proj(\hat{\Theta}_{i-1})-\Theta)^T(proj(\hat{\Theta}_{i-1})-\Theta)]\}\\
=&\frac{1}{2\beta}trace[(\hat{\Theta}_i-proj(\hat{\Theta}_{i-1}))^T\\
&\cdot(\hat{\Theta}_i+proj(\hat{\Theta}_{i-1})-2\Theta)]\\
=&\frac{1}{2}\boldsymbol{\alpha}_i^T[\hat{\Theta}_i+proj(\hat{\Theta}_{i-1})-2\Theta]\boldsymbol{\xi}_i\\
=&\frac{1}{2}\boldsymbol{\alpha}_i^T[2\hat{\Theta}_i-\beta\boldsymbol{\alpha}_i\boldsymbol{\xi}_i^T-2\Theta]\boldsymbol{\xi}_i\\
=&-\varsigma(t)-\frac{\beta}{2}\|\boldsymbol{\alpha}_i(\mathbf{x}_i,t)\|^2\|\boldsymbol{\xi}_i\|^2 \qquad (13)
\end{aligned}
$$

According to (11), (13) and the positiveness of $\frac{\beta}{2}\|\boldsymbol{\alpha}_i(\mathbf{x}_i,t)\|^2\|\boldsymbol{\xi}_i\|^2$, we can obtain

$$
\begin{aligned}
&\Delta E_i(t)\\
=&V_i(\boldsymbol{\sigma}_i,t)+\frac{1}{2\beta}\int_0^t\{trace[(\hat{\Theta}_i-\Theta)^T(\hat{\Theta}_i-\Theta)]\\
&-trace[(\hat{\Theta}_{i-1}-\Theta)^T(\hat{\Theta}_{i-1}-\Theta)]\}d\tau-V_{i-1}(\boldsymbol{\sigma}_i,t)\\
=&-\int_0^t\gamma_{3,i}(\|\boldsymbol{\sigma}_i\|)d\tau-\int_0^t\frac{\beta}{2}\|\boldsymbol{\alpha}_i(\mathbf{x}_i,\tau)\|^2\|\boldsymbol{\xi}_i\|^2d\tau\\
&-V_{i-1}(\boldsymbol{\sigma}_{i-1},t)\\
\le&\,0. \qquad (14)
\end{aligned}
$$

From (14), it can be derived that the finiteness of $E_i(t)$ is ensured for any learning iteration provided that $E_1(t)$ is finite. In the following, we will show both $E_1(t)$ and $\mathbf{x}_1(t)$ are bounded.

$$
\begin{aligned}
&E_1(t)=\\
&V_1(\boldsymbol{\sigma}_1,t)+\frac{1}{2\beta}\int_0^t trace[(\hat{\Theta}_1-\Theta)^T(\hat{\Theta}_1-\Theta)]d\tau.
\end{aligned}
$$

Take the derivative of the above energy function.

$$
\begin{aligned}
&\dot{E}_1(t)\\
=&\frac{\partial V_1^T}{\partial t}+\frac{\partial V_1^T}{\partial\boldsymbol{\sigma}_1}\dot{\boldsymbol{\sigma}}_1+\frac{1}{2\beta}trace[(\hat{\Theta}_1-\Theta)^T(\hat{\Theta}_1-\Theta)]\\
=&\frac{\partial V_1^T}{\partial t}+\frac{\partial V_1^T}{\partial\boldsymbol{\sigma}_1}\dot{\boldsymbol{\sigma}}_1+\frac{1}{2\beta}\{trace[(\hat{\Theta}_1-\Theta)^T(\hat{\Theta}_1-\Theta)]\\
&-trace[(\hat{\Theta}_0-\Theta)^T(\hat{\Theta}_0-\Theta)]\}\\
&+\frac{1}{2\beta}trace[(\hat{\Theta}_0-\Theta)^T(\hat{\Theta}_0-\Theta)]. \qquad (15)
\end{aligned}
$$

When $i=1$, according to (12) and (13) , it can be derived that

$$
\begin{aligned}
\dot{E}_1(t)\le&-\gamma_{3,1}(\|\boldsymbol{\sigma}_1\|)-\frac{\beta}{2}\|\boldsymbol{\alpha}_1(\mathbf{x}_1,t)\|^2\|\boldsymbol{\xi}_1\|^2\\
&+\frac{1}{2\beta}trace(\Theta^T\Theta)\\
\le&\frac{1}{2\beta}trace(\Theta^T\Theta). \qquad (16)
\end{aligned}
$$

Because $\Theta(t)$ is continuous over $[0,T]$, it is bounded in the time interval $[0,T]$. Therefore, we can define

$$
L=\max_{t\in[0,T]}[\frac{1}{2\beta}trace(\Theta^T\Theta)]. \qquad (17)
$$

Then

$$
\begin{aligned}
|E_1(t)|\le&|E_1(0)|+|\int_0^t\dot{E}_1(\tau)d\tau|\\
\le&\int_0^t|\dot{E}_1(\tau)|d\tau\le\int_0^t Ld\tau\le LT.
\end{aligned}
$$

Therefore, $E_1(t)$ is finite, which implies $\mathbf{x}_{j,1}$ $(j=1,\cdots,m)$ is bounded. Moreover, $\mathbf{x}_{j,1}$ belongs to a compact set K.

Part B: Uniform Convergence of Tracking Error Using (14) repeatedly we have

$$
\begin{aligned}
E_i(t)=&E_1(t)+\sum_{k=2}^{i}\Delta E_k(t)\\
\le&E_1(t)-\sum_{k=2}^{i}V_{i-1}(\boldsymbol{\sigma}_{i-1},t).
\end{aligned}
$$

and

$$
\begin{aligned}
&0\le\gamma_{1,i}(\|\boldsymbol{\sigma}_i\|)\le V_i(\boldsymbol{\sigma}_i,t)\le E_i(t)\le\\
&E_1(t)\le|E_1(t)|\le LT.
\end{aligned}
$$

Obviously, $\|\mathbf{x}_{j,i}\|$ $(j=1,\cdots,m)$ is bounded and $\forall i\in N_+$, $\mathbf{x}_{j,i}$ belongs to the compact set $K=\{\mathbf{x}_{j,i}|\gamma_{1,i}(\|\boldsymbol{\sigma}_i\|)\le LT\}$. According to control laws (8)-(9) and system dynamics (1), the boundedness of $\mathbf{x}_i$ ensures the finiteness of $\hat{\Theta}(t)$, $\mathbf{u}_i(t)$ and $\dot{\mathbf{x}}_i(t)$. Consequently, the boundedness of $\dot{\mathbf{x}}_i(t)$ implies the uniform continuity of $\mathbf{x}_i(t)$.

$E_i(t)$ is a non-increasing series with a finite upper bound,

$$
\lim_{i\to\infty}E_i(t)\le E_1(t)-\lim_{i\to\infty}\sum_{k=2}^{i}V_{k-1}(\boldsymbol{\sigma}_{k-1},t).
$$

Therefore $\lim_{i\to\infty}E_i(t)$ exists. Since $E_1(t)$ is finite and $E_i(t)$ is positive, $\sum_{k=2}^{\infty}V_{k-1}(\boldsymbol{\sigma}_{k-1},t)$ converges. From the convergence theorem of the sum of series, $\lim_{i\to\infty}V_i(\boldsymbol{\sigma}_i,t)=0$, $\forall t\in[0,T]$, is guaranteed. It can be seen from Assumption 1,

$$0 \le \gamma_{1,i}(\|\boldsymbol{\sigma}_i(t)\| \le V_i(\boldsymbol{\sigma}_i, t)$$
$$\Rightarrow 0 \le \lim_{i\to\infty} \gamma_{1,i}(\|\boldsymbol{\sigma}_i(t)\|) \le \lim_{i\to\infty} V_i(\boldsymbol{\sigma}_i, t) = 0$$
$$\Rightarrow \lim_{i\to\infty} \gamma_{1,i}(\|\boldsymbol{\sigma}_i(t)\|) = 0.$$

Since $\gamma_{1,i}$ is a class-**KR** function,

$$\lim_{i\to\infty} \gamma_{1,i}(\|\boldsymbol{\sigma}_i(t)\|) = 0$$

implies $\lim_{i\to\infty} \|\boldsymbol{\sigma}_i(t)\| = 0$, i.e., pointwise convergence.

As $\mathbf{x}_{j,i}(t)$ is uniformly continuous on $[0, T]$,

$$\lim_{i\to\infty} \|\mathbf{e}_{j,i}(t)\| = 0 \Rightarrow \lim_{i\to\infty} \|\mathbf{e}_{j,i}(t)\|_s = 0. \quad (18)$$

As i approaches infinity, $\mathbf{x}_{j,i}$ uniformly converges to $\mathbf{x}_{jd,i}$ and the tracking error $\mathbf{e}_{j,i}$ uniformly converges to 0.

Remark 4. From practical point of view, Assumption 2 is difficult to be guaranteed. Here let us consider the initial shift problem. Assume that $\mathbf{e}_i(0) \neq 0$, but can be measured accurately since system states $\mathbf{x}_i$ are measurable. Because it is impossible to change values of $\mathbf{x}_i(0)$, when $\mathbf{x}_{d,i}(0) \neq \mathbf{x}_i(0)$, the first point ($t = 0$) cannot be learned at all. The most effective and practical way is to adjust the desired trajectories as below

$$\mathbf{x}'_{d,i} = \begin{cases} \mathbf{h}_{d,i}(t) & 0 \le t \le t_s \\ \mathbf{x}_{d,i}(t) & t \ge t_s \end{cases}$$

where t_s is the time required for the modified trajectory to reach the original one. Here $\mathbf{h}_i(t)$ is an interpolation to ensure that $\mathbf{h}_i(0) = \mathbf{x}_{d,i}(0)$, $\mathbf{h}_i(t_s) = \mathbf{x}_{d,i}(t_s)$ and $\dot{\mathbf{h}}_i(t_s) = \dot{\mathbf{x}}_{d,i}(t_s)$. Consequently, the modified $\mathbf{x}'_{d,i}$ meets the Assumption 2. According to Theorem 1, the tracking error, $\mathbf{e}_i(t) \triangleq \mathbf{x}_i(t) - \mathbf{x}'_{d,i}(t)$, uniformly convergent to zero as iteration time i approach infinity. Moreover, the system states can follow the desired trajectories in the specified time t_s.

4. ILLUSTRATIVE EXAMPLE

In this section, the following nonlinear dynamic system is considered

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= f + b[u + d_1] \end{aligned} \quad (19)$$

where $b = e^{sinx_1}$ and $f = 9(1 - cost)$ are known functions. System uncertainties $d_1 = 4(1 - e^{sint})cosx_1$ is unknown. The extended tracking error is defined as

$$\sigma_i = x_{2,i} - x_{2d,i} + 5(x_{1,i} - x_{1d,i}).$$

Then

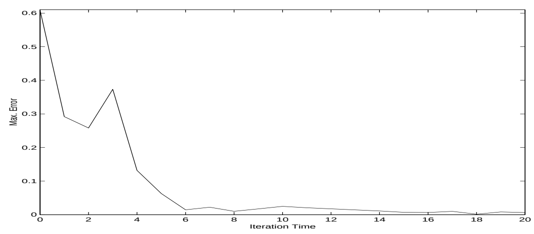

Fig. 1. Convergence of the extended tracking error.

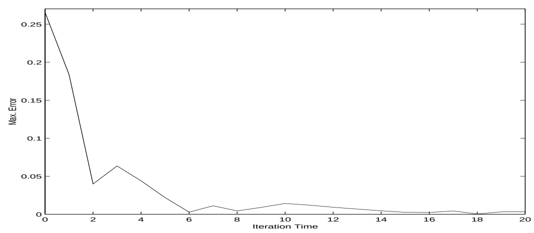

Fig. 2. Convergence of the extended tracking error.

$$\begin{aligned} \dot{\sigma}_i &= \dot{x}_{2,i} - \dot{x}_{2d,i} + 5(\dot{x}_{1,i} - \dot{x}_{1d,i}) \\ &= g_i + b_i[u_i + d_i - b^{-1}g_i] \end{aligned}$$

where the system unknown part $d_i = d_{1,i} + b_i^{-1}(f + 5x_2 - 5x_{2d} - \dot{x}_{2d})$ can be factored as $\Theta\boldsymbol{\xi}_i$. Here $\Theta = [4(1 - e^{sint}) \;\; 9(1 - cost) - 5x_{2d} - \dot{x}_{2d} \;\; 5]$ and $\boldsymbol{\xi}_i = [cosx_1 \;\; e^{-sinx_1} \;\; x_2 e^{-sinx_1}]^T$.

The simulations are performed for two classes of trajectories:

Class 1 $\;x_1 = \kappa_i sin^3(t)$
Class 2 $\;x_1 = \kappa_i e^{-3t}(\pi t^3 - t^4)$

In both classes, $t \in [0, \pi]$ and different values of κ_i will lead to different desired trajectories.

Case 1 Trajectory of Class 1 is used. κ_i is chosen randomly from the interval of $[-1, 0) \cup (0, 1]$ for each iteration. Here g is constructed as $g_i = -6\sigma_i$ and V_i is chosen as $V_i = 5\sigma_i^2$. $\beta = 2$. The maximum extended tracking error σ_i is recorded for each iteration and Fig. 1 shows the convergence property.

Case 2 Trajectory of Class 2 is used. κ_i is also chosen randomly from the interval of $[-1, 0) \cup (0, 1]$ for each iteration. $g_i = -5\sigma_i$, $V_i = 6\sigma_i^2$ and $\beta = 3$. Fig. (2) shows the convergence of the extended tracking error.

Case 3 Trajectories of Class 1 and Class 2 are used alternatively. $\mathbf{g}_i$, V_i and β are defined respectively as in *Case 1* and *Case 2*. κ_i is chosen randomly from $[-1, 0) \cup (0, 1]$ for each iteration. Fig. (3) shows the convergence of the extended tracking error.

The above simulations clearly demonstrate the ability of the proposed algorithm to learn from different motion patterns.

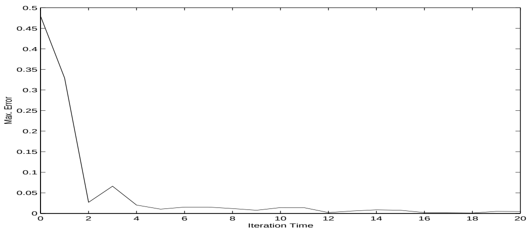

Fig. 3. Convergence of the extended tracking error.

5. CONCLUSION

A novel ILC scheme for nonlinear systems is developed in this paper to deal with non-uniform trajectory tracking problems. The parametric uncertainties, as the learnable part, are separated from the known system dynamics. Rigorous proof based on CEF shows the tracking error uniformly convergences to zero as iteration time approaches infinity.

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